Large-Scale Inhomogeneities May Improve the Cosmic Concordance of Supernovae

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We reanalyze the supernova data from the Union Compilation including the weak-lensing effects caused by inhomogeneities. We compute the lensing probability distribution function for each background solution described by the parameters Ω_M, Ω_Λ, and w in the presence of inhomogeneities, approximately modeled with a single-mass population of halos. We then perform a likelihood analysis in the parameter space of Friedmann-Lemaître-Robertson-Walker models and compare our results with the standard approach. We find that the inclusion of lensing can move the best-fit model significantly towards the cosmic concordance of the flat Lambda-Cold Dark Matter model, improving the agreement with the constraints coming from the cosmic microwave background and baryon acoustic oscillations.

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Introduction.—In the standard approach supernova (SNe) observations are analyzed in the framework of homogenous Friedmann-Lemaître-Robertson-Walker (FLRW) models. However, the Universe is known to be inhomogenous, showing a distribution of large galaxy clusters and filamentary structures surrounding much emptier voids of size $\approx$ 10–100 Mpc. A known effect of these structures on any set of standard candles is weak gravitational lensing [1]. Weak lensing can cause either brightening or dimming of the source depending on whether the matter column density along the line of sight is larger or smaller than the FLRW value.

The fundamental quantity describing this statistical magnification is the lensing probability distribution function (PDF). The lensing PDF is specific both to the given FLRW model, and to the particular spectrum of inhomogeneities introduced. It is not currently possible to extract the lensing PDF from the observational data and we have to resort to theoretical models. Two possible alternatives have been followed in the literature. A first approach (e.g., Ref. [2]) relates a "universal" form of the lensing PDF to the variance of the convergence, which in turn is fixed by the amplitude of the power spectrum, σ_8. Moreover, the coefficients of the proposed PDF are trained on some specific N-body simulations. A second approach (e.g., Ref. [3]) is to build a model for the inhomogeneous universe and directly compute the relative lensing PDF, usually through time-consuming ray-tracing techniques. The flexibility of this method is therefore penalized by the increased computational time.

In this work we use another approach, based on the stochastic modeling of the inhomogeneities introduced in Ref. [4]. This method combines the flexibility in modelling with a fast performance in obtaining the lensing PDF. To compute one lensing PDF, the numerical implementation TURBOGL 0.4 [4] takes, with an ordinary desktop computer, a time of an order of a second. This speed performance makes it feasible to do an *ab initio* likelihood analysis in the space of FLRW models endowed with inhomogeneities. In this Letter we will perform such an analysis for the Union SNe Compilation [5].

Setup.—We will treat inhomogeneities as perturbations over the FLRW model which is parametrized as usual by the present Hubble expansion rate $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, the present matter density parameter Ω_M and the present dark energy density parameter Ω_Λ and a constant equation of state w. We fix the radiation density to $\Omega_R = 4.2 \times 10^{-5} h^{-2}$. For inhomogeneities we use a "meatball" model [6] consisting of randomly placed spherical halos made of ordinary and dark matter. In principle these halos need not be virialized, and the spherical symmetry assumption is not very restrictive. As was explained in Ref. [4], the weak-lensing properties of a given universe model can be described by a set of matter distribution projections (z-dependent column densities) on a small number of independent redshift slices. For such projections any local density contrast, such as a long filament seen edge on, looks roughly like a spherical halo.

Here we use a simple single-mass halo model which is completely parametrized by the comoving distance between halos λ_c, the halo proper radius R_p and the density profile. We choose the latter to be the Navarro-Frenk-White profile [7] with a concentration parameter $c \simeq 6.7$ and we assume that the halos have virialized with a contrast of 200 at a redshift $z_{\rm vir}$, whereby (for a given $z_{\rm vir}$) the corresponding R_p can be taken constant. The halo mass is related to the comoving density $n_c \equiv \lambda_c^{-3}$ by $\rho_c \Omega_M = M n_c$. For numerical values we explored the range $\lambda_c = (5.4, 9.0, 12.6) h^{-1}$ Mpc and, correspondingly, $M = (0.44, 2.0, 5.6) 10^{14} h^{-1} \Omega_M M_\odot$ for $z_{\rm vir} = 0.8$, and

$z_{\rm vir} = (0, 0.8, 1.6)$ for $\lambda_c = 12.6h^{-1}$ Mpc. The numerical value of R_p depends on the background matter density at $z_{\rm vir}$. For the Λ CDM model the previous range of $z_{\rm vir}$ corresponds to $R_p \simeq (0.9, 0.7, 0.5)h^{-1}$ Mpc.

*Lensing.—*The meatball model incorporates quantitatively the crucial feature that photons can travel through voids and miss the localized overdensities. This feature is not present, for example, in "swiss-cheese" models where the bubble boundaries are designed to have compensating overdensities. Such models have indeed been shown to have on average little lensing effects [8,9]. The key quantity in all our analysis is the lens convergence κ, which in the weak-lensing approximation is given by

$$\kappa(z_s) = \int_0^{r_s} drG(r, r_s)\delta_M(r, t(r)). \qquad (1)$$

Here, $\delta_M(r, t)$ is the matter density contrast and $G(r, r_s) = \frac{3H_0^2\Omega_M}{2c^2}\frac{f_k(r)f_k(r_s-r)}{f_k(r_s)}\frac{1}{a(t(r))}$, where the functions $a(t)$ and $t(r)$ correspond to the FLRW model, $r_s = r(z_s)$ is the comoving position of the source at redshift z_s and the integral is evaluated along the unperturbed light path. Also, $f_k(r) = \sin(r\sqrt{k})/\sqrt{k}$, r, $\sinh(r\sqrt{-k})/\sqrt{-k}$ depending on the curvature $k>, =, <0$, respectively.

Neglecting the second-order contribution of the shear, the shift in the distance modulus caused by lensing is expressed solely in terms of κ:

$$\Delta m(z) = 5\log_{10}(1 - \kappa(z)). \qquad (2)$$

Eqs. (1) and (2) show that for a lower-than-FLRW column density the light is demagnified (e.g., empty beam $\delta = -1$), while in the opposite case it is magnified.

In Ref. [4] a fast and easy way to obtain the convergence PDF for these meatball models was derived. In short, the formula for the convergence Eq. (1) is replaced by a discretized probabilistic expression:

$$\kappa(\{k_{im}\}) = \sum_{i=1}^{N_S}\sum_{m=1}^{N_R} \kappa_{1im}(k_{im} - \Delta N_{im}). \qquad (3)$$

Here, κ_{1im} is the convergence due to one halo, at a comoving distance r_i, which the photon path intercepts with an impact parameter b_m, $\kappa_{1im} = G(r_i, r_s)\int_{b_m}^{R(t_i)}\frac{2xdx}{(x^2-b_m^2)^{1/2}}\frac{\rho_i(x)}{\bar\rho_M}$, where $\rho_i(x)$ is the local halo density and $\bar\rho_M$ is the FLRW matter density. In practice one divides the comoving distance r_s to the source and the radius R of the halo into bins of widths $R \ll \Delta r_i \ll r_s$ and $\Delta b_m \ll R$ and lets the centers of these bins define the allowed values for r and b. The quantity k_{im} in Eq. (3) is a Poisson random variable of parameter $\Delta N_{im} = n_c \Delta V_{im}$, which gives the expected number of halos within the phase space volume $\Delta V_{im} = 2\pi b_m \Delta b_m \Delta r_i$. That is, Eq. (3) defines a convergence as a function of a *configuration* $\{k_{im}\}$ of halos along an arbitrary line of sight from the observer to the source. The lensing PDF in the distance modulus $P_{\rm wl}(\Delta m, z_s)$ is then constructed from a large sample of random configurations $\{k_{im}\}$ using Eqs. (2) and (3). Note that the expected convergence computed from Eq. (3) is zero, consistent with photon conservation in weak lensing, because for a Poisson distributed variable the expected value coincides with its parameter.

*Likelihood function.—*After the raw lensing PDF $P_{\rm wl}(\Delta m)$ has been computed for a given set of FLRW-parameters and redshifts, it still has to be convolved with the intrinsic source brightness distribution $P_{\rm in}$: $P(\Delta m, z_s) = \int dy P_{\rm wl}(y, z_s)P_{\rm in}(\Delta m - y)$. We take $P_{\rm in}$ to be a gaussian in the distance moduli. The actual intrinsic distribution should be a universal function if the SN are similar at all distances. However, following Ref. [5], we will combine all observational (gaussian by assumption) uncertainties in quadrature with the intrinsic distribution, whereby $P_{\rm in}$ becomes an effective distribution specific for each SN event $P_{\rm in}(x) \rightarrow P_{\rm SN}(x, \sigma_i)$. The likelihood function for a single SN observation is then

$$L_i(\mu) = \int dy P_{\rm wl}(y, z_i)P_{\rm SN}(\Delta m_i - \mu - y, \sigma_i), \qquad (4)$$

where $\Delta m_i = m_{o,i} - m_{t,i}$, $m_{o,i}$ is the observed magnitude and the corresponding FLRW prediction is related to the luminosity distance d_L by $m_{t,i} = 5\log_{10} d_L(z_i)/10$ pc. The parameter μ is an unknown offset sum of the SNe absolute magnitudes, of k corrections and other possible systematics. Note also that L_i inherits the vanishing mean of $P_{\rm wl}$ and that its variance is simply given by the sum of the variances of the convolving PDFs.

We define the total likelihood function as the product of all independent likelihood functions in the data sample, further marginalized over μ:

$$L(\Omega_M, \Omega_\Lambda, w) = \int d\mu \Pi_i L_i(\mu). \qquad (5)$$

Since μ is degenerate with $\log_{10} H_0$ we are effectively marginalizing also over the expansion rate of the universe. A replacement of $P_{\rm wl}(y, z)$ by a *cosmology-independent* Gaussian with a variance [3] $\sigma \equiv 0.093z$, would reduce Eq. (5) to the form used in the analysis of Ref. [5]. Typical forms of $P_{\rm wl}$, $P_{\rm SN}$ and $L_i(\mu = 0)$ have been illustrated in Fig. 1. Also shown for later use is G_i, which is a Gaussian with the same variance of $L_i(0)$.

*Results.—*We run a global likelihood analysis using the formula (5) for two different setups: first in the (Ω_M, w) space for flat $(\Omega_k = 0)$ wCDM models and second in the $(\Omega_M, \Omega_\Lambda)$ space for a nonflat ΛCDM model $(w = -1)$ using the Union SNe Compilation of Ref. [5]. We show our results in Figs. 2 and 3 as confidence level contours for $\chi^2 = -2\log L$. For comparison we have performed the analysis also using the standard $P_{\rm SN}$ distribution (as done in Ref. [5]) and the distribution G_i. The idea for using G_i is that it takes into account the *cosmology-dependent* extra dispersion coming from lensing, but neglects the skewness of the true distribution. So, the contours relative to G_i give an idea of how much of the difference from the standard

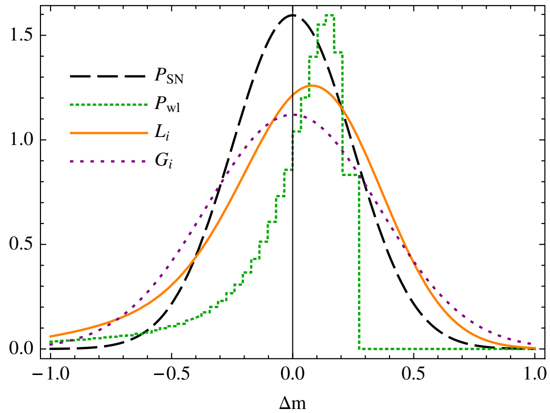

FIG. 1 (color online). PDFs for a SN with $\sigma = 0.25$ mag at $z_s = 1.5$ in the ΛCDM model endowed with halos specified by $z_{\mathrm{vir}} = 0.8$ and $\lambda_c = 12.6h^{-1}$ Mpc. The dotted histogram represents the lensing PDF, the dashed line the SN PDF and the solid line the full likelihood. The dotted curve is described in the text.

analysis comes from the widening of the intrinsic distribution, and how much from the skewness of the actual PDF. As it is evident from Figs. 2 and 3, the 1σ contours are basically determined by the cosmology-dependent widening, whereas the skewness starts to be relevant between the 2 and 3σ levels. We point out that our results are essentially unaffected if we add a constant σ_{sys} to the σ_i of Eq. (4) in order to have the same reduced χ^2 using G_i and P_{SN}.

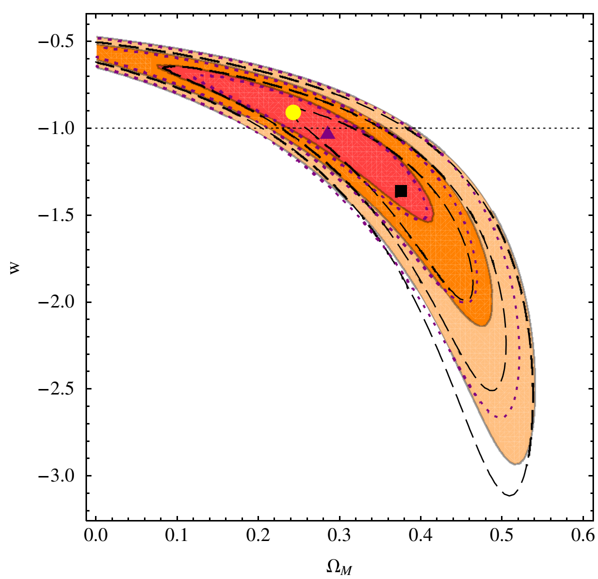

FIG. 2 (color online). 1, 2, and 3σ confidence level contours on w and Ω_M, for a flat wCDM universe with halos specified by $z_{\mathrm{vir}} = 0.8$ and $\lambda_c = 12.6h^{-1}$ Mpc. The results using the full likelihood of Eq. (5) are shown as filled contours and the best-fit model with a circle. The results using the gaussian G_i are shown as dotted lines with a triangle, the ones using the unlensed P_{SN} are shown as dashed lines with a square and correspond to the ones of Ref. [5] (without systematics).

Our most important result, clearly evident from Figs. 2 and 3, is that the inclusion of lensing effects in the likelihood analysis significantly moves the best-fit model, from $(\Omega_M^V, w^V) = (0.38, -1.4)$ and $(\Omega_M^V, \Omega_\Lambda^V) = (0.41, 0.94)$, towards the cosmic concordance of the flat ΛCDM model, therefore improving the agreement with the constraints coming from cosmic microwave background (CMB) and baryon acoustic oscillations (BAO).

To further explore this behavior we studied how the new best-fit model positions (Ω_M^*, w^*) and $(\Omega_M^*, \Omega_\Lambda^*)$ depend on the halos mass M and the virialization redshift z_{vir}. For a fixed halo mass, higher values of z_{vir} give denser halos with smaller radius R_p and higher lensing corrections to the likelihood. As explained before, the numerical value of R_p depends on the background model and we use the ΛCDM as a reference model to convert z_{vir} into R_p. A fit for $\lambda_c = 12.6h^{-1}$ Mpc then gives

$$\begin{aligned}(\Omega_M^*, w^*) &= (\Omega_M^V[1 - 1.5e^{-2.3R_p}], w^V[1 - 0.94e^{-1.7R_p}])\\ (\Omega_M^*, \Omega_\Lambda^*) &= (\Omega_M^V[1 - 0.8e^{-1.2R_p}], \Omega_\Lambda^V[1 - 0.54e^{-1.1R_p}])\end{aligned} \tag{6}$$

where $R_p \geq 0.5$ is in units of h^{-1} Mpc. If we fix z_{vir}, higher values of M (or equivalently λ_c) give a universe made of larger clumps with larger voids giving therefore stronger lensing corrections. A fit for $z_{\mathrm{vir}} = 0.8$ gives

$$\begin{aligned}(\Omega_M^*, w^*) &= (\Omega_M^V - 0.13M^{0.47}, w^V + 0.45M^{0.33})\\ (\Omega_M^*, \Omega_\Lambda^*) &= (\Omega_M^V - 0.15M^{0.29}, \Omega_\Lambda^V - 0.25M^{0.26})\end{aligned} \tag{7}$$

where $M \leq 1$ is in units of $5.6 \times 10^{14}h^{-1}\ \Omega_M M_\odot$.

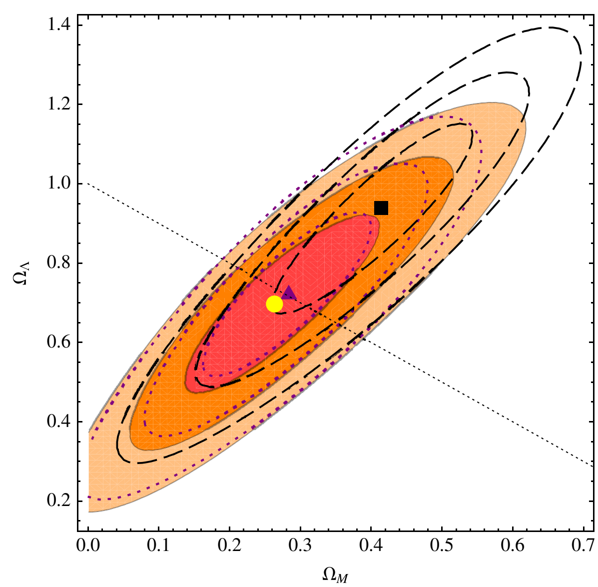

FIG. 3 (color online). 1, 2 and 3σ confidence level contours on Ω_M and Ω_Λ (i.e., allowing for nonzero curvature) for ΛCDM ($w = -1$) with halos as in Fig. 2. Note that, as also in the previous plot, the new best-fit points lie on the 1σ confidence level contour relative to P_{SN}. Labeling as in Fig. 2.

The general trend favoring models with smaller Ω_M follows from the fact that lensing effects in general make the fit slightly worse with than without lensing [10]. The effect comes both from the cosmology-dependent widening and from the skewness of the distributions, and it is obviously more pronounced for larger matter densities. This can be seen directly from Eq. (1), where the magnitude of the lensing effects is explicitly seen to be proportional to Ω_M. The overall movement of the best-fit model then follows the degeneracy of the FLRW models.

Discussion.—Our halo model was designed to capture the most important effects of the weak gravitational lensing by the nonlinear large-scale structures. In particular the large voids that dominate the late-time universe were imposed in the model by concentrating all matter into halos. Accordingly, we chose the halos to have the mass of a very large cluster, i.e., of order $10^{14}h^{-1}M_\odot$, which then corresponds to an interhalo distance of order $10h^{-1}$ Mpc.

Given this result, it is natural to ask if our toy model could also give a reasonable approximation to the observed power spectrum. This is not entirely obvious, because weak lensing and power spectrum probe somewhat different aspects of the inhomogeneities. We found that our single-mass halo model tends to concentrate too much power onto the interhalo distance scale, when compared to the nonlinear correction to the ΛCDM spectrum provided, for example, by the halo model of Ref. [11].

It will clearly be interesting to improve the modeling of the power spectrum by adopting a more realistic halo distribution function $f(M, z)$, and we plan to pursue this in future work. However, this Letter was devoted to explore the extent to which lensing can change the supernovae results and this is best done by adopting the simplest single-halo model with its few parameters. In any case, the choice of the mass function $f(M, z)$ is not that simple; even after fitting the power spectrum well, an efficient lensing requires modelling the voids and filaments that are described by higher order correlation terms. This can in principle be done in the current approach by introducing additional large-scale variations to the background density from which standard halo functions are drawn.

Finally, given a large enough SNe data set one could in principle measure the lensing PDF. However, to do this properly one would have to understand the selection effects that could, for example, cut the high magnification tail of the PDF, sizably biasing the average convergence, variance, and skewness.

Conclusions.—We have presented a reanalysis of the supernova data from the Union Compilation including the lensing effects caused by inhomogeneities. Unlike in the analysis of Refs. [5,12], where the lensing effects are accounted for by adding in quadrature a small z-dependent variance to the other statistical and systematic errors, we compute the actual probability distribution functions for each different FLRW model with a spectrum of inhomogeneities designed to mimic the observed large-scale structures. In particular, large voids that dominate the late-time universe are imposed on the model by concentrating all matter into halos of mass of order $10^{14}h^{-1}M_\odot$. We found that including inhomogeneities significantly changes the likelihood contours (the likelihood peaks, for instance, move of around 1σ) and clearly improves the concordance of the supernova data with the CMB and the BAO, which may be used to strengthen the case for the standard ΛCDM model.

One should be reminded that our findings could change if other effects caused by large-scale inhomogeneities are introduced, e.g., selection or redshift effects. It also remains to be seen how a more realistic inhomogeneous distribution, providing a better fit to the matter power spectrum, would affect these weak-lensing corrections to the SNe contours.

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