

## Dark Energy from Primordial Inflationary Quantum Fluctuations

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We show that current cosmic acceleration can be explained by an almost massless scalar field experiencing quantum fluctuations during primordial inflation. Provided its mass does not exceed the Hubble parameter today, this field has been frozen during the cosmological ages to start dominating the Universe only recently. By using supernovae data, completed with baryonic acoustic oscillations from galaxy surveys and cosmic microwave background anisotropies, we infer the energy scale of primordial inflation to be around a few TeV, which implies a negligible tensor-to-scalar ratio of the primordial fluctuations. Moreover, our model suggests that inflation lasted for an extremely long period. Dark energy could therefore be a natural consequence of cosmic inflation close to the electroweak energy scale.

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*Introduction.*—In the past decade, various cosmological observations have accumulated evidence that the Universe is currently undergoing accelerated expansion [1,2]. Although cosmic acceleration can be triggered by a non-vanishing cosmological constant, one still has to explain why it is so small and why it has started to dominate the energy content of the Universe only recently [3]. These questions have motivated the exploration of alternative explanations all referred to as dark energy. Among them, the quintessence models consider a scalar field rolling down a potential in a way similar to the mechanism at work during primordial inflation [4]. A quintessence field yields a time-dependent equation of state  $w(t) = P/\rho > -1$ ,  $P$  and  $\rho$  being its pressure and energy density, respectively. A cosmological constant being equivalent to  $w = -1$ , quintessence models generically predict a different expansion history of the Universe, and this can be tested by observations. According to Ref. [5], the quintessence models can be divided into “freezing” and “thawing” types. The former yields a decreasing  $w(t)$  which approaches  $-1$  at low redshifts. This type of model has been intensively used in the literature to address the coincidence problem. Indeed, assuming the potential to be of the Ratra-Peebles kind, the quintessence field tracks a

cosmological attractor which erases memory of the initial conditions [6]. Once the field energy scale is adjusted to the current dark energy value, its recent domination is automatic. On the other hand, the thawing type gives  $w \simeq -1$  at high redshifts but can evolve and start deviating from this value at low redshifts, exactly as during the inflationary graceful exit. These models have been less explored than the freezing type due to their dependence on the initial field values. Initial conditions are not washed out by an attractor mechanism and therefore constitute additional and unwanted model parameters. In this Letter, we show that inflationary cosmology solves this problem by giving natural initial values for the quintessence field which can explain the current acceleration. They are set by the quantum generation of field fluctuations during inflation. Assuming the accelerated expansion today to be sourced by the quintessence field, we use the supernovae data to infer the energy scale of inflation which ends up being at a few TeV, i.e., close to the electroweak symmetry breaking energy scale. Such a scenario implies a negligible tensor-to-scalar ratio of the primordial cosmological perturbations and a reheating temperature also around a few TeV. Combined with cosmic microwave background (CMB) bounds, the class of

allowed inflationary models is thus severely constrained [7]. Moreover, the total number of  $e$ -folds of inflation has to be extremely large, as it could be in a self-reproducing inflationary scenario.

Linking primordial inflation and dark energy has been originally discussed in the context of anthropic selection effects [8–11]. In these approaches, inflationary quantum fluctuations are used to randomize the possible dark energy density values while anthropic selection effects favor the ones compatible with our own existence. Inflationary stochastic effects in freezing-type quintessence models have been explored in Ref. [12] to determine how inflation influences the likely initial conditions of the field. As shown in Ref. [13], they indeed have a tendency to push the freezing-type quintessence field away from the region where the tracker behavior can efficiently wash out the initial conditions. As a result, freezing quintessence on the tracker today suggests that inflation lasted a low number of  $e$ -folds.

In our model, the initial value of the thawing quintessence field is directly determined by the energy scale of inflation and keeps its initial value until low redshifts. Therefore, today's energy density of the quintessence field is almost equal to its initial energy density and, hence, is directly related to the inflationary energy scale. Checking the consistency of the model provides us a way to probe primordial inflation from dark energy observations. This is our main point and an essential difference from the freezing-type quintessence scenario.

*Initial field values from inflation.*—Let us consider a (real) free massive scalar field  $\varphi$  of mass  $m$  such that  $m$  is less than the present Hubble scale  $H_0$ . Clearly, since the inflationary Hubble parameter  $H_{\text{inf}} \gg H_0$ , we are in the presence of an almost massless scalar field which will acquire quantum fluctuations during inflation. By decomposing the scalar field as a homogeneous mode plus linear perturbations (at the onset of inflation)  $\varphi(\mathbf{x}, t) = \phi(t) + \delta\phi(\mathbf{x}, t)$  and assuming an almost constant value for  $H_{\text{inf}}$ , its Fourier modes after Hubble exit read [14]

$$|\delta\phi_{\mathbf{k}}|^2 \simeq \frac{H_{\text{inf}}^2}{2k^3} \left( \frac{k}{aH_{\text{inf}}} \right)^{2m^2/(3H_{\text{inf}}^2)}. \quad (1)$$

The comoving wave vector is  $\mathbf{k}$ , and  $a$  stands for the scale factor. If inflation starts at  $a = a_s$ , after  $N = \ln(a/a_s)$   $e$ -folds, super-Hubble fluctuations induce a real space field variance given by

$$\begin{aligned} \langle \delta\phi^2 \rangle &\simeq \int_{a_s H_{\text{inf}}}^{a H_{\text{inf}}} \frac{d^3\mathbf{k}}{(2\pi)^3} |\delta\phi_{\mathbf{k}}|^2 \\ &= \frac{3H_{\text{inf}}^4}{8\pi^2 m^2} \left[ 1 - \exp\left(-\frac{2m^2}{3H_{\text{inf}}^2} N\right) \right] \rightarrow \frac{3H_{\text{inf}}^4}{8\pi^2 m^2}, \end{aligned} \quad (2)$$

where the last limit is reached if inflation lasts long enough for the exponential term to cancel. This result can be reproduced by using the stochastic inflation formalism [15]. The homogeneous mode evolves as

$\phi = \phi_s \exp[-Nm^2/(3H_{\text{inf}}^2)]$  and becomes completely suppressed compared to the field fluctuations. The same holds for the mean squared field derivative (in  $e$ -folds) which is suppressed by a factor  $m^4/H_{\text{inf}}^4$  compared to Eq. (2). For  $m \ll H_{\text{inf}}$  the required number of  $e$ -folds could actually be extremely large. Here, one should notice that the long-wave fluctuation  $\delta\phi$  is almost homogeneous and determines the typical value of the classical field  $\varphi$ . Notice that backreaction effects induced by the field fluctuations over the expansion rate can produce  $\mathcal{O}(1)$  variations in the total number of  $e$ -folds. However, one can show that they induce a correction factor to Eq. (2) given by  $1 + \mathcal{O}(H_{\text{inf}}^2/M_p^2)$  which ends up being completely negligible as soon as  $H_{\text{inf}} \ll M_p$ . Since after inflation  $H \gg m$ , Hubble damping prevents the field from rolling down the potential until the time at which  $m \simeq H \lesssim H_0$ . Before going into a detailed comparison with observations, let us derive some order of magnitude results. The present energy density of this quintessence field is roughly

$$V(\phi) \simeq \frac{1}{2} m^2 \langle \delta\phi^2 \rangle \simeq \frac{3H_{\text{inf}}^4}{16\pi^2} \quad (3)$$

and does not depend on  $m$ . In order to explain dark energy, one needs  $H_{\text{inf}} \simeq (\Omega_\Lambda)^{1/4} \sqrt{4\pi H_0 M_p}$ , where  $M_p$  stands for the reduced Planck mass and  $\Omega_\Lambda$  is the current density parameter associated with a cosmological constant. Using fiducial values for  $H_0$  and  $\Omega_\Lambda$ , one gets  $H_{\text{inf}} \simeq 6 \times 10^{-3}$  eV [16]. The energy scale of inflation is thus [17]

$$E_{\text{inf}} \equiv \rho_{\text{inf}}^{1/4} = (3M_p^2 H_{\text{inf}}^2)^{1/4} \simeq 5 \text{ TeV}. \quad (4)$$

As a result, in the context of our scenario, we rephrase the question on the smallness of the dark energy density into the relatively small energy scale (TeV) of inflation, which may be more tractable to address and surprisingly close to the electroweak energy scale. Notice that, although the mass  $m$  does not appear in Eqs. (3) and (4), it still determines when the quintessence field starts rolling down the potential and how the equation of state of dark energy will deviate from  $w = -1$ . For TeV scale inflation, our model predicts a negligible tensor-to-scalar ratio. By using the above estimates, with  $m \lesssim H_0$ , the number of  $e$ -folds required to reach the Bunch-Davies fluctuations is  $N \simeq H_{\text{inf}}^2/m^2 \gtrsim 10^{60}$ , which may therefore suggest a self-reproducing inflationary model [18,19]. In the following, we assume that  $N > H_{\text{inf}}^2/m^2 > 10^{60}$ . For such a large  $N$ , the homogeneous mode of the scalar field is also suppressed significantly so that the typical value of the classical field  $\varphi$  is completely determined by the long-wave fluctuation  $\delta\phi$ .

*Observational constraints.*—In order to constrain our model from current observations, we have numerically solved the Einstein and Klein-Gordon equations

$$H^2 = \frac{1}{3M_p^2} (\rho_r + \rho_m + \rho_\phi), \quad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \quad (5)$$

for various values of the parameters ( $m$ ,  $\phi_{\text{ini}}$ ,  $H_0$ ). The value  $\phi_{\text{ini}}$  denotes one realization of the quintessence field fluctuations in our Hubble patch, and  $\rho_r$  and  $\rho_m$  are the energy density associated with radiation and matter, respectively. The integration is started at high redshift, when the dark energy is subdominant, to the time at which  $H = H_0$ . We have fixed the current radiation energy density to  $\Omega_r h^2 = 4.15 \times 10^{-5}$  and dropped the decaying mode for the field. By solving the evolution equations, we obtain  $H$  as a function of the redshift  $z$  which can be used to compare the model with observational data. We have used type Ia supernovae (SN) redshift-distance modulus relations given in Ref. [2] complemented with the redshift-distance relations at  $z = 0.2$  and  $z = 0.35$  coming from the baryon acoustic oscillations (BAO) data [20]. Finally, we have used the angular scale and height of the first peak of the CMB power spectrum given in Ref. [16]. Our likelihood has therefore been defined as the product of those three, namely, by summing the chi-squared of SN, BAO, and CMB data.

In order to test how the model can fit cosmic acceleration, let us first consider a Jeffreys' prior for the initial field values  $\phi_{\text{ini}} \geq M_P$ . Here we are not yet assuming that the initial conditions are set during inflation but simply requiring super-Planckian field values (necessary for triggering acceleration). Together with flat priors on  $m \leq 100$  km/s/Mpc and  $H_0$  around the current measured value, we have explored the three-dimensional parameter space with gridding methods. In Fig. 1, we have represented the one- and two-sigma contours of the two-dimensional marginalized probability distribution (over  $H_0$ ) in the plane ( $\phi_{\text{ini}}$ ,  $m$ ). By marginalizing over  $\phi_{\text{ini}}$ , we find the mass of the quintessence field to be constrained by  $m < 75$  km/s/Mpc (at 95% of confidence), which is consistent with our earlier estimation that it cannot exceed  $H_0$  too much. We also find, as expected, that the data favor arbitrarily high super-Planckian initial field values. In order to check our results, we have also derived the field energy

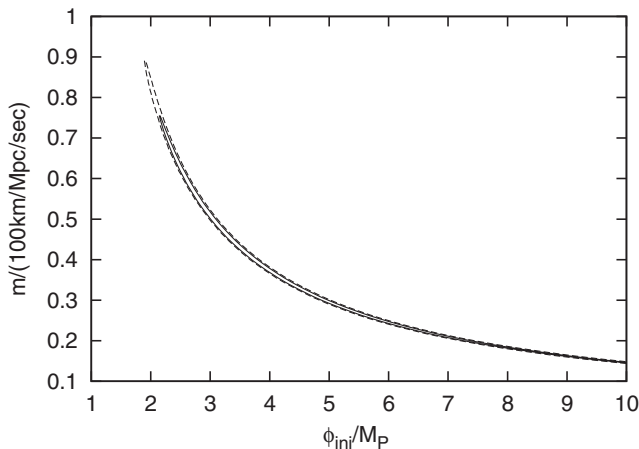


FIG. 1. One- and two-sigma contours of the two-dimensional marginalized probability distribution in the plane ( $\phi_{\text{ini}}$ ,  $m$ ) from SN + BAO + CMB data.

density parameter  $\Omega_\phi$  and its equation of state  $w_\phi$ , both evaluated at the present time. The contours plotted in Fig. 1 end up being centered around the cosmological constant case  $\Omega_\phi = 0.73$ ,  $w_\phi = -1$  and are compatible with the results of Ref. [2].

We are now in a position to infer the energy scale of inflation when the initial field values are set by inflationary quantum fluctuations. Compared to the above data analysis, our mechanism gives a prior probability distribution of  $\phi_{\text{ini}}$  which is Gaussian [21] with a standard deviation given by Eq. (2):  $\sigma(H_{\text{inf}}, m) = \sqrt{3/8} \frac{H_{\text{inf}}^2}{\pi m}$ . Denoting by  $D$  our data sets and  $I$  the prior space, using Bayes' theorem, and marginalizing over  $\phi_{\text{ini}}$  and  $H_0$  yields

$$P(H_{\text{inf}}, m|D, I) \propto P(H_{\text{inf}}, m|I) \iint dH_0 \frac{d\phi_{\text{ini}}}{\sqrt{2\pi}\sigma} \times \exp\left(-\frac{\phi_{\text{ini}}^2}{2\sigma^2}\right) P(H_0|I) \mathcal{L}(D|m, \phi_{\text{ini}}, H_0, I), \quad (6)$$

where  $\mathcal{L}$  is the overall SN + BAO + CMB likelihood and  $P(H_{\text{inf}}, m|I)$  the prior probability distribution on  $H_{\text{inf}}$  and  $m$ . The scale of inflation being *a priori* unknown, we have chosen a flat prior on the logarithm of  $H_{\text{inf}}$  while the other parameters assume the same prior as before. In Fig. 2, we have represented the one- and two-sigma confidence intervals of this two-dimensional posterior, up to the change of variable  $H_{\text{inf}} \rightarrow E_{\text{inf}}$ . The lower panel is the fully marginalized probability distribution for  $E_{\text{inf}}$ , and the energy scale of inflation verifies

$$3.8 \text{ TeV} < E_{\text{inf}} < 12.1 \text{ TeV}, \quad (7)$$

at 95% of confidence. Though this constraint depends on the priors on  $H_{\text{inf}}$  and  $m$ , the dependence is small. As discussed below, our model works for a more general potential, which can slightly change the constraint. Thus, it is safe to say that our model suggests the energy scale of inflation  $E_{\text{inf}}$  to be around a few TeV for a wide class of thawing quintessential models.

*Discussion.*—Although we have focused on a free scalar field so far, the mechanism discussed here could also be applied to any potential having an absolute minimum by using the inflationary stochastic formalism [15]. However, the prior probability distribution for the field  $\phi_{\text{ini}}$  will no longer be Gaussian, and this could therefore shift the favored values of  $E_{\text{inf}}$ . Since our scenario needs an extremely large  $e$ -folding number  $N \simeq H_{\text{inf}}^2/m^2 \geq 10^{60}$ , the running of  $H_{\text{inf}}$  along the inflaton potential must be extremely small. Thus, as a candidate of such a TeV scale inflation, new inflation might be preferred, which can indeed realize a self-reproducing era. We also would like to mention the case that the total number of  $e$ -folds is much smaller than  $H_{\text{inf}}^2/m^2$ , in which case Eq. (2) yields  $H_{\text{inf}}/M_P \propto \sqrt{\Omega_\Lambda/N}$ . In this limit, we have only a lower bound  $E_{\text{inf}} > \text{TeV}$  and would have to explain why the

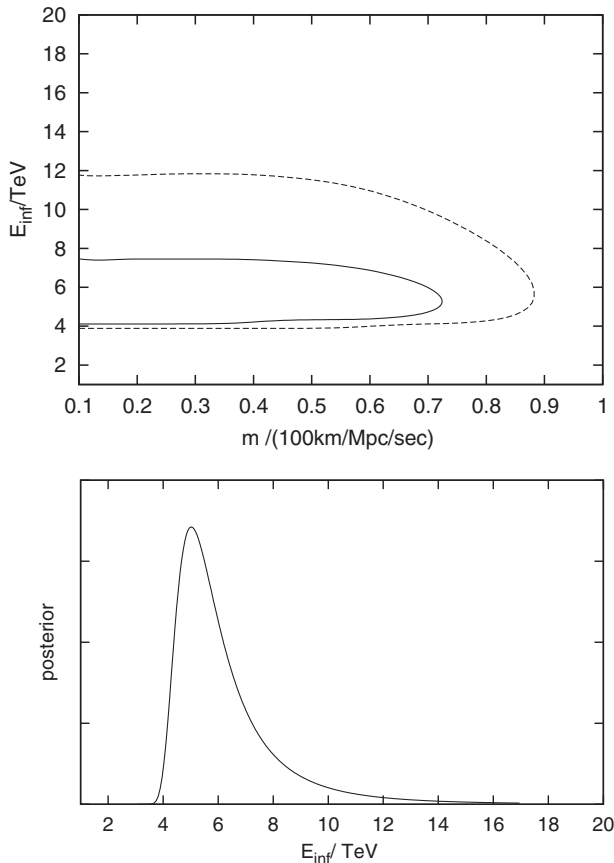


FIG. 2. One- and two-sigma contours in the plane  $(m, E_{\text{inf}})$  from SN + BAO + CMB data. The lower panel shows the marginalized posterior for  $E_{\text{inf}}$ . At 95% of confidence, we find  $3.8 \text{ TeV} < E_{\text{inf}} < 12.1 \text{ TeV}$ .

homogeneous value  $\phi_s \approx 0$  after inflation. Notice that with CMB anisotropies imposing  $H_{\text{inf}}/M_P < 10^{-5}$ , we get an absolute lower limit for the total number of  $e$ -folds  $N > 10^9$ , under which the scale of inflation would actually be too high. Finally, it is worth stressing that we are in the presence of a transient dark energy model. By rolling down its potential, the thawing quintessence field will acquire kinetic energy and the current cosmic acceleration will come to an end.

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