Cosmology of a Covariant Galileon Field

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We study the cosmology of a covariant scalar field respecting a Galilean symmetry in flat space-time. We show the existence of a tracker solution that finally approaches a de Sitter fixed point responsible for cosmic acceleration today. The viable region of model parameters is clarified by deriving conditions under which ghosts and Laplacian instabilities of scalar and tensor perturbations are absent. The field equation of state exhibits a peculiar phantomlike behavior along the tracker, which allows a possibility to observationally distinguish the Galileon gravity from the cold dark matter model with a cosmological constant.

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The problem of dark energy being responsible for cosmic acceleration today has motivated the idea that the gravitational law may be modified from general relativity (GR) on large scales (see Ref. [1] for reviews). On the other hand, one needs to recover Newton gravity at short distances for compatibility with solar-system experiments. Besides the chameleon mechanism [2] based on the density-dependent matter coupling with a scalar field [used also in $f(R)$ theories [3]], there is another way to recover GR at short distances: the Vainshtein mechanism [4] based on nonlinear field self-interactions such as $\Box\phi(\nabla\phi)^2$, where $(\nabla\phi)^2 \equiv \partial^\mu\phi\partial_\mu\phi$. This nonlinear effect has been employed for the brane-bending mode of the self-accelerating branch in the Dvali-Gabadadze-Porrati braneworld [5], but the Dvali-Gabadadze-Porrati model is unfortunately plagued by a ghost problem [6].

In order to avoid the appearance of ghosts, it is important to keep the field equations up to second order in time derivatives. A scalar field ϕ called "Galileon" [7], whose action is invariant under the Galilean symmetry $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ in flat space-time, allows five field Lagrangians that give rise to derivatives up to second order (see Refs. [8–10] for related works). If the analysis in [7] is extended to the curved space-time, one needs to introduce couplings between the field ϕ and the curvature tensors for constructing the Lagrangians free from higher-order derivatives in the equations of motion [8].

The five covariant Lagrangians that respect the Galilean symmetry in flat space-time are given by

$$\begin{aligned}&\mathcal{L}_1 = M^3\phi, \qquad \mathcal{L}_2 = (\nabla\phi)^2, \qquad \mathcal{L}_3 = (\Box\phi)(\nabla\phi)^2/M^3,\\ &\mathcal{L}_4 = (\nabla\phi)^2[2(\Box\phi)^2 - 2\phi_{;\mu\nu}\phi^{;\mu\nu} - R(\nabla\phi)^2/2]/M^6,\\ &\mathcal{L}_5 = (\nabla\phi)^2[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu}\\ &\qquad + 2\phi_{;\mu}^{\ \nu}\phi_{;\nu}^{\ \rho}\phi_{;\rho}^{\ \mu} - 6\phi_{;\mu}\phi^{;\mu\nu}\phi^{;\rho}G_{\nu\rho}]/M^9, \end{aligned} \tag{1}$$

where a semicolon represents a covariant derivative, M is a constant having a dimension of mass, and $G_{\nu\rho}$ is the Einstein tensor. In this Letter we study the cosmology based on the action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \frac{1}{2}\sum_{i=1}^{5} c_i\mathcal{L}_i\right] + \int d^4x\,\mathcal{L}_M, \tag{2}$$

where $M_{\rm pl}$ is the reduced Planck mass and c_i are constants. For the matter Lagrangian $\mathcal{L}_M$ we take into account perfect fluids of radiation (density ρ_r) and nonrelativistic matter (density ρ_m). Although the cosmological dynamics up to $\mathcal{L}_4$ were discussed in Ref. [10], we will show that inclusion of $\mathcal{L}_5$ is crucially important to determine the full Galileon dynamics. Moreover the viable parameter space will be clarified for such a full theory.

In the flat Friedmann-Lemaître-Robertson-Walker universe with a scale factor $a(t)$, the variation of the action (2) leads to

$$3M_{\rm pl}^2H^2 = \rho_{\rm DE} + \rho_m + \rho_r, \tag{3}$$

$$3M_{\rm pl}^2H^2 + 2M_{\rm pl}^2\dot{H} = -P_{\rm DE} - \rho_r/3, \tag{4}$$

where $H = \dot{a}/a$ is the Hubble parameter (a dot represents a derivative with respect to cosmic time t), and

$$\begin{aligned}\rho_{\rm DE} \equiv& -c_1M^3\phi/2 - c_2\dot{\phi}^2/2 + 3c_3H\dot{\phi}^3/M^3\\ &- 45c_4H^2\dot{\phi}^4/(2M^6) + 21c_5H^3\dot{\phi}^5/M^9,\end{aligned} \tag{5}$$

$$\begin{aligned}P_{\rm DE} \equiv& \, c_1M^3\phi/2 - c_2\dot{\phi}^2/2 - c_3\dot{\phi}^2\ddot{\phi}/M^3\\ &+ 3c_4\dot{\phi}^3[8H\ddot{\phi} + (3H^2 + 2\dot{H})\dot{\phi}]/(2M^6)\\ &- 3c_5H\dot{\phi}^4[5H\ddot{\phi} + 2(H^2 + \dot{H})\dot{\phi}]/M^9.\end{aligned} \tag{6}$$

The matter fluids obey the continuity equations $\dot{\rho}_m + 3H\rho_m = 0$ and $\dot{\rho}_r + 4H\rho_r = 0$. From Eqs. (3) and (4) the dark component also satisfies $\dot{\rho}_{\rm DE} + 3H(\rho_{\rm DE} + P_{\rm DE}) = 0$. We define the dark energy equation of state $w_{\rm DE}$ and the effective equation of state $w_{\rm eff}$, as $w_{\rm DE} \equiv P_{\rm DE}/\rho_{\rm DE}$ and $w_{\rm eff} \equiv -1 - 2\dot{H}/(3H^2)$. The latter is known by the background expansion history of the Universe.

Since we are interested in the case where the late-time cosmic acceleration is driven by field kinetic terms without a potential, we set $c_1 = 0$ in the following discussion.

We introduce the following quantities useful to describe the cosmological dynamics:

$$r_1 \equiv \dot{\phi}_{\rm dS} H_{\rm dS}/(\dot{\phi} H), \qquad r_2 \equiv (\dot{\phi}/\dot{\phi}_{\rm dS})^4/r_1, \tag{7}$$

where $\dot{\phi}_{\rm dS}$ and $H_{\rm dS} \approx 10^{-60} M_{\rm pl}$ are the field velocity and the Hubble parameter at the de Sitter (dS) solution, respectively. The mass M is related to $H_{\rm dS}$ via $M^3 = M_{\rm pl} H_{\rm dS}^2$. At the dS point one has $r_1 = 1$ and $r_2 = 1$. Equation (3) can be written as $\Omega_m + \Omega_r + \Omega_{\rm DE} = 1$, where $\Omega_m = \rho_m/(3M_{\rm pl}^2 H^2)$, $\Omega_r = \rho_r/(3M_{\rm pl}^2 H^2)$, and

$$\Omega_{\rm DE} = -\frac{1}{6} c_2 x_{\rm dS}^2 r_1^3 r_2 + c_3 x_{\rm dS}^3 r_1^2 r_2 - \frac{15}{2} c_4 x_{\rm dS}^4 r_1 r_2 + 7 c_5 x_{\rm dS}^5 r_2, \tag{8}$$

where $x_{\rm dS} \equiv \dot{\phi}_{\rm dS}/(H_{\rm dS} M_{\rm pl})$. Since $\Omega_{\rm DE} = 1$ at the dS point, Eq. (8) gives a relation between the terms $c_2 x_{\rm dS}^2$, $c_3 x_{\rm dS}^3$, $\alpha \equiv c_4 x_{\rm dS}^4$, $\beta \equiv c_5 x_{\rm dS}^5$. Combining this with another relation coming from Eq. (4), we obtain

$$c_2 x_{\rm dS}^2 = 6 + 9\alpha - 12\beta, \qquad c_3 x_{\rm dS}^3 = 2 + 9\alpha - 9\beta. \tag{9}$$

It is useful to use α and β because the coefficients of physical quantities (such as $\Omega_{\rm DE}$) can be expressed in terms of those quantities thanks to Eq. (9). The relations (9) are not subject to change under a rescaling $c_i \to c_i/\gamma^i$ and $x_{\rm dS} \to \gamma x_{\rm dS}$, where γ is a real number. Therefore rescaled choices of c_i will lead to the same dynamics (as they have the same α and β) for both the background and the linear perturbation, which implies that redefining the coefficients c_i in terms of α and β is convenient.

The autonomous equations for the variables r_1, r_2, Ω_r follow from Eqs. (3) and (4), and fluid equations. One can show that there is an equilibrium point characterized by

$$r_1 = 1, \qquad \text{i.e.} \quad \dot{\phi} H = \text{const}, \tag{10}$$

at which the variables r_2 and Ω_r satisfy

$$r_2' = \frac{2r_2(3 - 3r_2 + \Omega_r)}{1 + r_2}, \quad \Omega_r' = \frac{\Omega_r(\Omega_r - 1 - 7r_2)}{1 + r_2}, \tag{11}$$

where a prime represents a derivative with respect to $N = \ln a$. This result is interesting because it shows the universality of the equations of motion without any dependence on α and β. Along the solution (10), the field velocity evolves as $\dot{\phi} \propto t$ during radiation and matter eras ($H \propto 1/t$). There is also a simple relation $\Omega_{\rm DE} = r_2$ along the solution $r_1 = 1$.

We have three fixed points: (a) $(r_1, r_2, \Omega_r) = (1, 0, 1)$ (radiation), (b) $(r_1, r_2, \Omega_r) = (1, 0, 0)$ (matter), (c) $(r_1, r_2, \Omega_r) = (1, 1, 0)$ (dS). The stability of these points can be analyzed by considering linear perturbations δr_1, δr_2, $\delta\Omega_r$ about them. The perturbation δr_1 satisfies

$$\delta r_1' = -\frac{9 + \Omega_r + 3r_2}{2(1 + r_2)} \delta r_1, \tag{12}$$

which shows that, in the regime $0 \le r_2 \le 1$ and $\Omega_r \ge 0$, the solution is stable in the direction of r_1. Since the dS point is stable in the other two directions, the solutions finally approach it. The points (a) and (b) are saddle because they are unstable in the direction of r_2.

Along the solution (10) we have $\rho_{\rm DE} = 3M^6/H^2$, $P_{\rm DE} = -3M^6(2 + w_{\rm eff})/H^2$, and

$$w_{\rm DE} = -2 - w_{\rm eff} = -\frac{\Omega_r + 6}{3(r_2 + 1)}, \quad w_{\rm eff} = \frac{\Omega_r - 6r_2}{3(r_2 + 1)}. \tag{13}$$

From the radiation era to the dS epoch the effective equation of state evolves as $w_{\rm eff} = 1/3 \to 0 \to -1$, whereas the dark energy equation of state exhibits a peculiar evolution: $w_{\rm DE} = -7/3 \to -2 \to -1$.

The evolution of dark energy is different depending on the initial conditions of (r_1, r_2, Ω_r). If they are chosen to be close to the fixed point (a) at the onset of the radiation era, then the solutions follow the sequence (a) $\to$ (b) $\to$ (c). If $r_1 \ll 1$ initially, the dominant contribution to $\Omega_{\rm DE}$ comes from the term $\mathcal{L}_5$, i.e., $\Omega_{\rm DE} \simeq 7\beta r_2$. In this case the solutions approach $r_1 = 1$ at late times with the increase of r_1. For the initial conditions with $r_1 \gg 1$ the term $\mathcal{L}_2$ gives the dominant contribution to $\Omega_{\rm DE}$, but this case is not viable because the field kinetic energy decreases rapidly as in quintessence without a potential. Numerical simulations show that if $r_1 \lesssim 2$ initially the solutions approach $r_1 = 1$, but in the opposite case the universe finally reaches the matter-dominated epoch.

Let us find the allowed parameter space in terms of (α, β) by deriving the conditions for the avoidance of ghosts and instabilities of scalar and tensor perturbations. Using the Faddeev-Jackiw method [11], the action (2) can be expanded at second order in perturbations. Following the similar procedure as in Ref. [12], the no-ghost condition for the scalar sector of the action (2) is given by

$$Q_S \equiv -s/(1 + \mu_3)^2 > 0, \tag{14}$$

where $s \equiv 6(1 + \mu_1)(\mu_1 + \mu_2 + \mu_1\mu_2 - 2\mu_3 - \mu_3^2)$, and

$$\mu_1 \equiv 3\alpha r_1 r_2/2 - 3\beta r_2, \tag{15}$$

$$\mu_2 \equiv (3\alpha - 4\beta + 2) r_1^3 r_2/2 - 2(9\alpha - 9\beta + 2) r_1^2 r_2 + 45\alpha r_1 r_2/2 - 28\beta r_2, \tag{16}$$

$$\mu_3 \equiv -(9\alpha - 9\beta + 2) r_1^2 r_2/2 + 15\alpha r_1 r_2/2 - 21\beta r_2/2. \tag{17}$$

The condition for the avoidance of Laplacian instabilities associated with the scalar field propagation speed is

$$c_S^2 = \{(1 + \mu_1)^2[2\mu_3' - (1 + \mu_3)(5 + 3w_{\rm eff}) + 3\Omega_m + 4\Omega_r] - 4\mu_1'(1 + \mu_1)(1 + \mu_2) + 2(1 + \mu_3)^2(1 + \mu_4)\}/s > 0, \tag{18}$$

where

$$\mu_4 \equiv -\alpha r_1 r_2/2 + 3\beta r_2(3 + 3w_{\rm eff} - 3r_1'/r_1 - r_2'/r_2)/2. \tag{19}$$

Similar calculations for the tensor perturbation lead to

$$Q_T \equiv 3\alpha r_1 r_2/4 - 3\beta r_2/2 + 1/2 > 0, \tag{20}$$

$$c_T^2 = \frac{2r_1(2 - \alpha r_1 r_2) - 3\beta(r_2 r_1' + r_1 r_2')}{2r_1(2 + 3\alpha r_1 r_2 - 6\beta r_2)} > 0. \quad (21)$$

We consider the following three different regimes.

(i) $r_1 \ll 1$, $r_2 \ll 1$—This characterizes the early cosmological epoch in which the term $\mathcal{L}_5$ dominates the dynamics of the field. For the scalar modes we have $Q_S \simeq 60\beta r_2$ and $c_S^2 \simeq (1 + \Omega_r)/40$. The sign change of r_2 implies the appearance of ghosts. For the initial conditions with $r_2 > 0$, it is required that $\beta > 0$. The Laplacian instabilities of the scalar modes can be avoided because $c_S^2 \simeq 1/20$ and $c_S^2 \simeq 1/40$ during radiation and matter eras, respectively. Since $Q_T \simeq 1/2$ and $c_T^2 \simeq 1 + 3\beta r_2(5 - 3\Omega_r)/8 \simeq 1$, the tensor modes do not provide additional constraints. We also have

$$w_{\rm DE} \simeq -(1 + \Omega_r)/8, \qquad w_{\rm eff} \simeq \Omega_r/3, \quad (22)$$

which is valid for $\Omega_r \gg \{r_1, r_2\}$.

(ii) $r_1 = 1$, $r_2 \ll 1$—This corresponds to the equilibrium point (10) during radiation or matter domination. The conditions (14) and (18) reduce to

$$Q_S \simeq 3(2 - 3\alpha + 6\beta)r_2 > 0, \quad (23)$$

$$c_S^2 \simeq \frac{8 + 10\alpha - 9\beta + \Omega_r(2 + 3\alpha - 3\beta)}{3(2 - 3\alpha + 6\beta)} > 0. \quad (24)$$

For the branch $r_2 > 0$ the first condition reduces to $2 - 3\alpha + 6\beta > 0$. For the tensor modes, we have $c_T^2 \simeq 1 - r_2(4\alpha + 3\beta + 3\beta\Omega_r)/2 \simeq 1$ and $Q_T > 0$.

(iii) $r_1 = 1$, $r_2 = 1$—This corresponds to the dS point, at which the conditions (14), (20), (18), and (21) are given by

$$Q_S = \frac{4 - 9(\alpha - 2\beta)^2}{3(\alpha - 2\beta)^2} > 0, \quad (25)$$

$$Q_T = (2 + 3\alpha - 6\beta)/4 > 0, \quad (26)$$

$$c_S^2 = \frac{(\alpha - 2\beta)(4 + 15\alpha^2 - 48\alpha\beta + 36\beta^2)}{2[4 - 9(\alpha - 2\beta)^2]} > 0, \quad (27)$$

$$c_T^2 = \frac{2 - \alpha}{2 + 3\alpha - 6\beta} > 0. \quad (28)$$

If $\beta > 0$, c_T^2 can have a minimum during the transition from the regime $r_2 \ll 1$ to $r_2 \simeq 1$. This value tends to decrease as β approaches 1. Imposing that $c_T^2 > 0$ at the minimum, we obtain $\alpha < 12\sqrt{\beta} - 9\beta - 2$. In Fig. 1 we illustrate the region in which this condition as well as the conditions (23)–(28) are satisfied for $r_2 > 0$. Numerical simulations confirm that for the parameters inside the shaded region in Fig. 1 the no-ghost and stability conditions are not violated even in the intermediate cosmological epoch.

In Fig. 2 we plot the variation of $w_{\rm DE}$ and $w_{\rm eff}$ versus the redshift z for several different model parameters and initial conditions. In the case (A) the initial condition is chosen to be $r_1 = 1$, so that $w_{\rm DE}$ and $w_{\rm eff}$ evolve according to

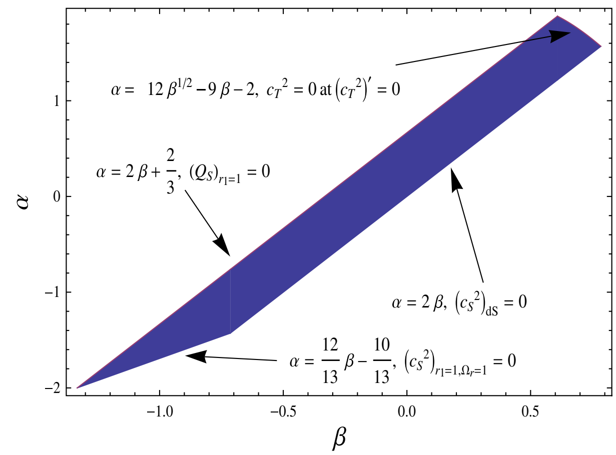

FIG. 1 (color online). The viable parameter space in the (α, β) plane for the branch $r_2 > 0$. We also show several conditions that determine the border between the allowed and excluded regions.

Eq. (13) with the variation of r_2 and Ω_r. While the evolution of $w_{\rm eff}$ is similar to that in the ΛCDM model, the dark energy equation of state evolves from the regime $w_{\rm DE} < -1$ to the dS attractor with $w_{\rm DE} = -1$. The cases (B) and (B') in Fig. 2 correspond to the initial conditions in regime (i). As estimated by Eq. (22), $w_{\rm DE}$ starts to evolve from $-1/4$ and reaches the value $-1/8$ during the matter era. The evolution of $w_{\rm DE}$ is different depending on the epoch at which r_1 grows to the order of 1. In case (B) the solutions reach the regime $r_1 \sim 1$ only recently, whereas in case (B') the approach of this regime occurs much earlier. The equilibrium point (10) can be regarded as a tracker that attracts solutions with different initial conditions to a common trajectory. Before approaching the tracker, the solutions cross the boundary $w_{\rm DE} = -1$ without any unstable behavior of perturbations. Note that there are no significant

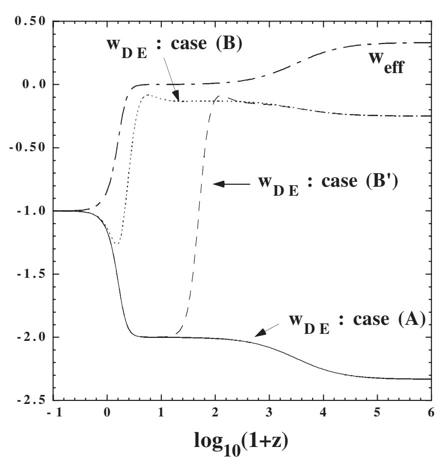

FIG. 2. Evolution of $w_{\rm eff}$ and $w_{\rm DE}$ for cases: (A) $\alpha = -1.4$, $\beta = -0.8$, $x_{\rm dS} = 1$ with initial conditions $r_1 = 1$, $r_2 = 10^{-60}$, $\Omega_r = 0.999\,99$ at the redshift $z = 3.11 \times 10^8$; (B) $\alpha = 0.1$, $\beta = 0.049$, $x_{\rm dS} = 1$ with initial conditions $r_1 = 5 \times 10^{-11}$, $r_2 = 8 \times 10^{-12}$, $\Omega_r = 0.999\,995$ at $z = 6.44 \times 10^8$; and (B') the same α, β, $x_{\rm dS}$ as in case (B) but with different initial conditions $r_1 = 5 \times 10^{-7}$, $r_2 = 8 \times 10^{-16}$, $\Omega_r = 0.9995$ at $z = 6.72 \times 10^6$.

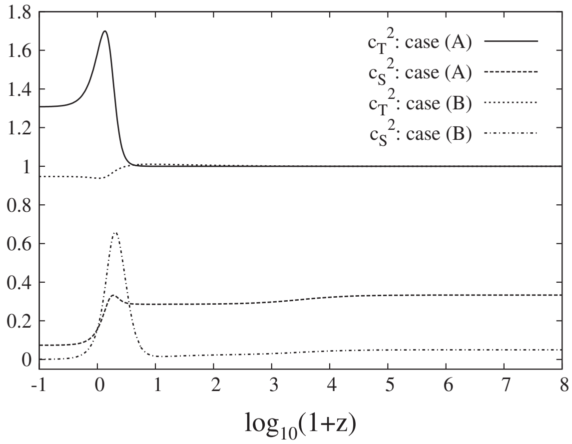

FIG. 3. Evolution of c_S^2 and c_T^2 for cases (A) and (B) as in Fig. 2.

differences for the variation of $w_{\rm eff}$ between cases (A) and (B) [also (B')].

In Fig. 3 we show the evolution of c_S^2 and c_T^2 for the same model parameters and initial conditions as in Fig. 2. In case (A) the scalar propagation speed remains subluminal, as estimated by Eqs. (24) and (27). For $\alpha = -1.4$ and $\beta = -0.8$, Eq. (28) shows that at the dS point the tensor mode becomes superluminal. However, both the scalar and tensor modes can be subluminal at the dS point, as in case (B) of Fig. 3 ($\alpha = 0.1$, $\beta = 0.049$).

For the initial conditions starting from regime (i) we require $\beta > 0$ to avoid ghosts. Under the conditions (23), (25), and (27) with $\beta > 0$, one can show that c_S^2 in Eq. (24) gets larger than 1. If the solutions approach the tracker in regime (ii) long before the dS epoch, there is a period in which c_S^2 exceeds 1. This superluminal propagation can be avoided if r_1 grows to the order of unity only recently. Case (B) in Fig. 3 corresponds to such an example for which c_S^2 has a peak smaller than 1 after the matter era. In this case, the tensor mode is slightly superluminal in regime (i). In general, there is a period in which the propagation speed of either scalar or tensor modes exceeds 1. However, this does not necessarily imply the inconsistency of theory because of the possibility for the absence of closed causal curves [10].

In summary, we have studied the cosmology for the full Galileon action (2) and derived all conditions for the consistency of such theory. We have shown that, under the conditions (9), there exist stable dS solutions responsible for dark energy. In spite of the complexities of Galileon Lagrangians, the conditions for the avoidance of ghosts and Laplacian instabilities constrain the allowed parameter space in terms of the variables α and β in a simple way. While the evolution of $w_{\rm DE}$, c_S^2, and c_T^2 is different depending on the model parameters and the initial conditions of r_1, we have derived convenient analytic formulas to evaluate those quantities in three distinct regimes.

There are several interesting applications of Galileon gravity. First, the study of cosmological perturbations may provide some signatures for the modification of gravity from GR. The last term of $\mathcal{L}_4$ in Eq. (1), for example, gives rise to a correction of the order $\alpha r_1 r_2$ to the bare gravitational constant. This affects the effective gravitational coupling $G_{\rm eff}$ that appears in the equation of matter perturbations. Also it will be possible to constrain the Galileon models from the time variation of $G_{\rm eff}$. Second, the study of spherically symmetric solutions in both weak and strong gravitational backgrounds can allow us to understand how the Vainshtein mechanism works in general. We expect that such analyses will provide us deep insight on the possible modification of gravity and that it will shed new light on the nature of dark energy.

[1] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006); R. Durrer and R. Maartens, Gen. Relativ. Gravit. **40**, 301 (2008); T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010); A. De Felice and S. Tsujikawa, Living Rev. Relativity **13**, 3 (2010).

[2] J. Khoury and A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004); Phys. Rev. D **69**, 044026 (2004).

[3] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007); A. A. Starobinsky, JETP Lett. **86**, 157 (2007); S. Tsujikawa, Phys. Rev. D **77**, 023507 (2008).

[4] A. I. Vainshtein, Phys. Lett. B **39**, 393 (1972).

[5] G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).

[6] A. Nicolis and R. Rattazzi, J. High Energy Phys. 06 (2004) 059; K. Koyama and R. Maartens, J. Cosmol. Astropart. Phys. 01 (2006) 016.

[7] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D **79**, 064036 (2009).

[8] C. Deffayet, G. Esposito-Farese, and A. Vikman, Phys. Rev. D **79**, 084003 (2009); C. Deffayet, S. Deser, and G. Esposito-Farese, Phys. Rev. D **80**, 064015 (2009).

[9] F. P. Silva and K. Koyama, Phys. Rev. D **80**, 121301 (2009); N. Chow and J. Khoury, Phys. Rev. D **80**, 024037 (2009); T. Kobayashi, H. Tashiro, and D. Suzuki, Phys. Rev. D **81**, 063513 (2010); C. de Rham and A. J. Tolley, J. Cosmol. Astropart. Phys. 05 (2010) 015; A. De Felice and S. Tsujikawa, J. Cosmol. Astropart. Phys. 07 (2010) 024.

[10] R. Gannouji and M. Sami, Phys. Rev. D **82**, 024011 (2010).

[11] L. D. Faddeev and R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988).

[12] A. De Felice and T. Suyama, J. Cosmol. Astropart. Phys. 06 (2009) 034; A. De Felice, S. Mukohyama and S. Tsujikawa, Phys. Rev. D **82**, 023524 (2010).