Quasiprobabilistic Interpretation of Weak Measurements in Mesoscopic Junctions

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The impossibility of measuring noncommuting quantum mechanical observables is one of the most fascinating consequences of the quantum mechanical postulates. Hence, to date the investigation of quantum measurement and projection is a fundamentally interesting topic. We propose to test the concept of weak measurement of noncommuting observables in mesoscopic transport experiments, using a quasiprobabilistic description. We derive an inequality for current correlators, which is satisfied by every classical probability but violated by high-frequency fourth-order cumulants in the quantum regime for experimentally feasible parameters.

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Every measurement in quantum mechanics is in principle described by the projection postulate [1]. However, in practice perfect projective detectors often do not exist and the measurement encounters a finite error. This can be resolved by replacing the projection by Kraus operators defining a positive operator-valued measure (POVM) [2,3]. The Kraus operator can be continuously changed from projection—strong measurement (exact)—to almost identity operator—weak measurement (with huge random error). Effectively, a POVM means that we take detector's degrees of freedom as part of the considered Hilbert space and make a projective measurement on the detector. Obviously, in that case the detector-system coupling defines the strength of the measurement of the system. The equivalence of a POVM and the projective measurement follows from Naimark theorem [4]. The actual modeling of a detection scheme by POVM is a long-standing problem [5–7].

The interpretation of the results of a weak measurement can lead to paradoxes. For instance, if a weak measurement of $\hat{A}$ performed on the state $\hat{\rho}$ is followed by a projection $\hat{B}$ then the *weak value* can be defined $B\langle A\rangle_\rho = \mathrm{Tr}(\hat{B}\hat{A}\hat{\rho})/\mathrm{Tr}(\hat{B}\hat{\rho})$ [8]. The unusual feature of the weak value is that it can exceed the spectrum of $\hat{A}$, which obviously contradicts our classical intuition. The strange properties of weak measurements have been confirmed experimentally in quantum optics [9], while experiments in solid state physics are proposed [10]. The interpretation of recent experiments on current fluctuations in mesoscopic junctions in the quantum regime [11] is impossible in terms of a usual probability [12]. Instead, we proposed to consider a weak current measurement, which implies a large background noise, but avoids the paradox of a certain average square to become negative. The necessity of a weak measurement lead to corrections in the observed finite-frequency noise; however, a direct experimental proof of this scheme was not feasible.

In this Letter, we first construct a general formula for a quantum quasiprobability, which does not depend on the details of the measurement apparatus and confirms the previously used formulas [12]. Second, we propose a scheme to test experimentally its negativity in frequency domain. To this end, we will derive a classical inequality for high-frequency current correlators of the form

$$C^2_{\omega\omega'} \le (C_{\omega\omega} + 2\pi S^2_\omega/\delta\omega)(C_{\omega'\omega'} + 2\pi S^2_{\omega'}/\delta\omega'). \quad (1)$$

Here $C_{\omega\omega'} = \langle\langle I(\omega)I(-\omega)I(\omega')I(-\omega')\rangle\rangle/t_0$ is a fourth-order correlator and $S_\omega = \langle\langle I(\omega)I(-\omega)\rangle\rangle/t_0$ the frequency-dependent current noise, where t_0 is the total (long) measurement time and $\delta\omega$ is the bandwidth of the detector (S and C are independent of t_0). Inequality (1) is satisfied by every classical stochastic process, but can be violated by high-frequency correlators in the quantum regime of a mesoscopic junction for experimentally accessible parameters. We believe our proposed violation of the classical inequality (1) can be realized with the existing techniques [11]. This violation will be a proof of negative values of the quasiprobability. Although it is not necessary to explain the strange features of weak values [8], it offers an alternative test of nonclassicality similar to the Wigner function [13]. Moreover, the quasiprobabilistic interpretation can be easily generalized to an arbitrary sequence of measurements. This interpretation facilitates the transfer to mesoscopic junctions and we present an example, how the negativity of the quasiprobability can be proven in a tunnel contact and discuss the experimental feasibility.

We will construct the quasiprobability by a deconvolution from a suitable POVM. The real parts of weak values can then be expressed as averages with respect to the quasiprobability. Let us begin with the basic properties of a POVM. The Kraus operators $\hat{K}(A)$ for an observable described by $\hat{A}$ with continuous outcome A need only to satisfy $\int dA \hat{K}^\dagger(A)\hat{K}(A) = \hat{1}$. The act of measurement on the state defined by the density matrix $\hat{\rho}$ results in the new state $\hat{\rho}(A) = \hat{K}(A)\hat{\rho}\hat{K}^\dagger(A)$. The new state yields a normalized and positive definite probability density $\rho(A) = \mathrm{Tr}\hat{\rho}(A)$. The procedure can be repeated recursively for an arbitrary sequence of (not necessarily commuting) operators $\hat{A}_1, \ldots, \hat{A}_n$ [14],

$$\hat{\rho}(A_1, \ldots, A_n) = \hat{K}(A_n)\hat{\rho}(A_1, \ldots, A_{n-1})\hat{K}^\dagger(A_n). \quad (2)$$

The corresponding probability density is given by $\rho(A_1, \ldots, A_n) = \mathrm{Tr}\hat{\rho}(A_1, \ldots, A_n)$. We now define a family of Kraus operators, namely $\hat{K}_\lambda(A) = (2\lambda/\pi)^{1/4} \times \exp(-\lambda(\hat{A} - A)^2)$. It is clear that $\lambda \to \infty$ should correspond to exact, strong, projective measurement, while $\lambda \to 0$ is a weak measurement and gives a large error. We also see that strong projection changes the state (by collapse), while $\lambda \to 0$ gives $\hat{\rho}(A) \sim \hat{\rho}$, and hence this case corresponds to the weak measurement. However, the repetition of the same measurement k times effectively means one measurement with $\lambda \to k\lambda$ so, with $k \to \infty$, even a weak coupling $\lambda \ll 1$ results in a strong measurement. For an arbitrary sequence of measurements, we can write the final density matrix as the convolution

$$\hat{\rho}_{\boldsymbol{\lambda}}(\boldsymbol{A}) = \int DA' \hat{\varrho}_{\boldsymbol{\lambda}}(\boldsymbol{A}') \prod_k g_k(A_k - A'_k) \quad (3)$$

with $g_k(A) = e^{-2\lambda_k A^2}\sqrt{2\lambda_k/\pi}$. Here, $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ $\boldsymbol{A} = (A_1, \ldots, A_n)$, and $DA = dA_1 \ldots dA_n$. The quasidensity matrix $\hat{\varrho}$ is given recursively by

$$\hat{\varrho}_{\boldsymbol{\lambda}}(\boldsymbol{A}) = \int \frac{d\chi}{2\pi} e^{-i\chi A_n} \int \frac{d\phi}{\sqrt{2\pi\lambda_n}} e^{-\phi^2/2\lambda_n} \times e^{i(\chi/2+\phi)\hat{A}_n} \hat{\varrho}_{\boldsymbol{\lambda}}(A_1, \ldots, A_{n-1}) e^{i(\chi/2-\phi)\hat{A}_n} \quad (4)$$

with the initial density matrix $\hat{\varrho} = \hat{\rho}$ for $n = 0$. We can interpret g in (3) as some internal noise of the detectors which, in the limit $\lambda \to 0$, should not influence the system. We *define* the quasiprobability $\varrho_{\boldsymbol{\lambda}} = \mathrm{Tr}\hat{\varrho}_{\boldsymbol{\lambda}}$ and abbreviate $\varrho \equiv \varrho_{\boldsymbol{0}}$. In this limit (4) reduces to

$$\hat{\varrho}(\boldsymbol{A}) = \int \frac{d\chi}{2\pi} e^{-i\chi A_n} e^{i\chi\hat{A}_n/2} \hat{\varrho}(A_1, \ldots, A_{n-1}) e^{i\chi\hat{A}_n/2}. \quad (5)$$

Note that $\varrho_{0\ldots0,\lambda} = \varrho$, so the last measurement does not need to be weak (it can be even a projection), and marginal distributions are consistent with absence of a measurement, $\int dA_k \varrho(\boldsymbol{A}) = \varrho(\ldots, A_{k-1}, A_{k+1}, \ldots)$. In the case of commuting operators the quasiprobability reduces to the usual probability $\varrho = \rho_\infty$. For $\hat{A}_1 = \hat{x}$ and $\hat{A}_2 = \hat{p}$ with $[\hat{x}, \hat{p}] = i\hbar$ we obtain the Wigner function $\varrho(x, p) = \varrho(p, x) = W(x, p)$ [13]. The definition preserves locality—for $(\hat{\boldsymbol{A}}, \hat{\rho}_a)$ and $(\hat{\boldsymbol{B}}, \hat{\rho}_b)$ acting in two separate Hilbert spaces we have $\hat{\rho} = \hat{\rho}_a\hat{\rho}_b \to \varrho(\boldsymbol{A}, \boldsymbol{B}) = \varrho_a(\boldsymbol{A})\varrho_b(\boldsymbol{B})$. The averages with respect to ϱ are easily calculated by means of the generating function (5), e.g., $\langle A\rangle_\varrho = \mathrm{Tr}\hat{A}\hat{\rho}$, $\langle AB\rangle_\varrho = \mathrm{Tr}\{\hat{A}, \hat{B}\}\hat{\rho}/2$, $\langle ABC\rangle_\varrho = \mathrm{Tr}\hat{C}\{\hat{B}, \{\hat{A}, \hat{\rho}\}\}/4$ for $\boldsymbol{A} = (A, B, C)$. This ordering of operators is called time symmetric [15,16]. To relate the quasiprobability to weak values, we have to consider two measurements: $\hat{A}$ and $\hat{B}$. The real part of the weak value is just the average $\mathrm{Re}_B\langle A\rangle_\rho = \langle A\rangle_{\varrho|B}$ with respect to the conditional quasiprobability $\varrho(A|B) = \varrho(A, B)/\varrho(B)$. The complex weak values require a different interpretation [8], which can also be generalized to sequential measurement [17].

We shall apply the above scheme to the measurement of current $I(t)$ through a mesoscopic junction in a stationary state. For a moment, we forget about quantum mechanics and recall basic properties of stochastic processes [18], applying them to $\delta I = I - \langle I\rangle$. It is convenient to define the noise (second cumulant), $\tilde{S}(\alpha, \beta) = 2\pi\delta(\alpha + \beta)S_\alpha = \langle\delta I(\alpha)\delta I(\beta)\rangle$, and the fourth cumulant $\tilde{C}(\alpha, \beta, \gamma, \eta) = 2\pi\delta(\alpha + \beta + \gamma + \eta)C(\alpha, \beta, \gamma, \eta)$ with

$$\tilde{C}(\alpha, \beta, \gamma, \eta) = \langle\delta I(\alpha)\delta I(\beta)I(\gamma)\delta I(\eta)\rangle - \tilde{S}(\alpha, \beta)\tilde{S}(\gamma, \eta) - \tilde{S}(\alpha, \gamma)\tilde{S}(\beta, \eta) - \tilde{S}(\alpha, \eta)\tilde{S}(\gamma, \beta). \quad (6)$$

Here and throughout the text we use Latin arguments in time domain and Greek ones in frequency domain, related by $a(\omega) = \int dt e^{i\omega t}a(t)$. Note, that the delta function of the frequency sum has a cutoff of the order of the measuring time t_0 (larger than all relevant time scales of the system), which in some following expressions is a simple prefactor and does not enter final conclusions.

Let us define the fluctuating noise spectral density $X_\omega = \int_{\omega_-}^{\omega_+} \delta I(\alpha)\delta I(-\alpha)d\alpha$ with $\omega_\pm = \omega \pm \delta\omega/2$, for which we obtain the average fluctuations

$$\begin{aligned} \langle(\delta X_\omega)^2\rangle/t_0 &= \int_{\omega_-}^{\omega_+} C_{\alpha\beta}d\alpha d\beta + 2\pi\int_{\omega_-}^{\omega_+} S_\alpha^2 d\alpha, \\ \langle\delta X_\omega \delta X_{\omega'}\rangle/t_0 &= \int_{\omega_-}^{\omega_+} d\alpha \int_{\omega'_-}^{\omega'_+} C_{\alpha\beta}d\beta, \end{aligned} \quad (7)$$

where $\delta X = X - \langle X\rangle$ and $C_{\alpha\beta} = C(\alpha, -\alpha, \beta, -\beta)$. The intervals $[\omega_-, \omega_+]$ and $[\omega'_-, \omega'_+]$ are nonoverlapping. Considering classical correlators of δX at different frequencies we obtain the Cauchy-Bunyakovsky-Schwarz inequality

$$B = \frac{\langle\delta X_\omega \delta X_{\omega'}\rangle^2}{\langle(\delta X_\omega)^2\rangle\langle(\delta X_{\omega'})^2\rangle} \le 1. \quad (8)$$

If we choose, e.g., $0 \le \omega'_- < \omega'_+ < \omega_-$, the correlators correspond to a low- and high-frequency measurement. Furthermore, assuming that S and C are constant within the bandwidth $\delta\omega$, $\delta\omega'$, the inequality (8) takes the form (1) mentioned in the introduction. It is interesting to note, that for frequency-independent (classical) noise the inequality is always satisfied.

Turning to the quantum case we stress that continuous measurement cannot be strong as we would end up with the quantum Zeno effect and suppress the dynamics of the system completely [19]. This follows from Eq. (4) for Heisenberg operators $\hat{A}_k = \hat{A}(t_k = k\Delta)$ and a finite $t_n = n\Delta$. In the limit $\sum \lambda_k \to \infty$ and $n \to \infty$, $\hat{\varrho}$ becomes diagonal in the eigenbasis of $\hat{A}$ and freezes. Nonclassical behavior of quantum correlations in the limit of weak measurement can also be shown using the Leggett-Garg inequality [20], which involves time-resolved second order correlations assuming that the observables are bounded. Our inequality is more general as we do not require bounded observables but we need fourth-order correlations in the frequency domain instead, which is more suitable for electric current measurements.

We denote the measured current operator in the Heisenberg picture by $\hat{I}(t)$. Strong, projective measurement can be performed only if we are interested in the long-time limit. The finer the time resolution we want the weaker the measurement must be as $[\hat{I}(t), \hat{I}(t')] \neq 0$. We define a generating functional of the quasiprobability in the weak measurement limit by

$$\varrho[I] = \int D\chi e^{-i\int \chi(t)I(t)dt} \times \mathrm{Tr}[\mathcal{T} e^{\int i\chi(t)\hat{I}(t)dt/2} \hat{\rho}\, \tilde{\mathcal{T}}\, e^{\int i\chi(t)\hat{I}(t)dt/2}] \quad (9)$$

with $\mathcal{T}$ ($\tilde{\mathcal{T}}$) denoting (anti)time ordering. This represents a straightforward generalization of the generating function obtained from the trace of (5), in which the time-variable labels the subsequent measurements. The averages of current powers (noise and third cumulant) have been already calculated [7,21–25] with respect to the quasiprobability (without introducing this notion) and measured [11]. In experiments, the large measured offset noise plays the role of g in (3) preventing paradoxical results [12]. We emphasize that in long-time averages the quasiprobability becomes a conventional probability and reproduces the formula for the usual full counting statistics [26], also confirmed experimentally [27,28].

We consider a quantum point contact described by fermionic operators around the Fermi level [21]. Each operator $\hat{\psi}_{A\bar{n}}$ with $\bar{n} = (n, \sigma)$ is denoted by mode number $n \in \{1..N\}$ and spin orientation σ and $A = L, R$ for left and right going electrons, respectively. Each mode can have its own Fermi velocity v_n and transmission coefficient T_n (reflection $R_n = 1 - T_n$). We will assume noninteracting electrons and energy- and spin-independent transmission through the junction. The Hamiltonian including a voltage bias V reads

$$\hat{H} = \sum_{\bar{n}} \int dx\{i\hbar v_n[\hat{\psi}^\dagger_{L\bar{n}}(x)\partial_x \hat{\psi}_{L\bar{n}}(x) - L \leftrightarrow R] + q_n\delta(x)[\hat{\psi}^\dagger_{L\bar{n}}(x)\hat{\psi}_{R\bar{n}}(-x) + \hat{\psi}^\dagger_{R\bar{n}}(x)\hat{\psi}_{L\bar{n}}(-x)] - eV\theta(x)[\hat{\psi}^\dagger_{L\bar{n}}(x)\hat{\psi}_{L\bar{n}}(x) + \hat{\psi}^\dagger_{R\bar{n}}(x)\hat{\psi}_{R\bar{n}}(x)]\}. \quad (10)$$

The fermionic operators satisfy anticommutation relations $\{\hat{\psi}_a(x), \hat{\psi}_b(x')\} = 0$ and $\{\hat{\psi}_a(x), \hat{\psi}^\dagger_b(x')\} = \delta_{ab}\delta(x - x')$ for $a, b = L\bar{n}, R\bar{m}$. The transmission coefficients are $T_n = \cos^2(q_n/\hbar v_n)$. We apply (9) to the current operator $\hat{I} = \sum_{\bar{n}} ev_n \hat{\psi}^\dagger_{L\bar{n}}(0_+)\hat{\psi}_{L\bar{n}}(0_+) - L \leftrightarrow R$ and the density matrix $\hat{\rho} \propto \exp(-\hat{H}/k_BT)$.

To find conditions in which the inequality (1) is violated, it is enough to consider the case $V = 0$. Using Eq. (9) we obtain $S_\alpha = \hbar G w(\alpha)$ and $C_{\alpha\beta} = \hbar F G e^2(w(\alpha) + w(\beta))/2$ [21–23]. Here we denote $w(\omega) = \omega \coth(\hbar\omega/2k_BT)$, conductance $G = \sum_n e^2T_n/\pi\hbar$ and Fano factor $F = \sum_n R_nT_n/\sum_n T_n$. In the tunneling limit, $T_n \ll 1$, for a finite bias voltage V we only need to replace $w(\omega)$ with $(w(\omega + eV/\hbar) + w(\omega - eV/\hbar))/2$ and $F = 1$. For $T = 35$ mK, $\delta\omega = \delta\omega' = 2\omega' = 2\pi \times 200$ MHz, $\omega = 2\pi \times 6$ GHz, $G^{-1} = 500$ kΩ and $V = 0$, we get $B = 1.4$, which contradicts our classical expectation (8) and clearly shows that the quasiprobability ϱ must take negative values. Generally, the violation occurs for sufficiently small $G < G_{\min}$, as shown in Fig. 1(a). At $eV = 0$ and $\delta\omega \simeq 2\omega \gg \delta\omega' \simeq 2\omega' \gtrsim k_BT/\hbar$, we have $G_{\min} = 3\sqrt{\omega/\omega'}Fe^2/2h$. For larger conductance, one can still find a reasonable range of parameters for the violation at $G^{-1} = 5$ kΩ, as shown in Fig. 1(b). The strongest violation occurs at low temperature and voltage but at large bandwidth. Unfortunately, the typical experimental conductance is with $G^{-1} \approx 50\ \Omega$ [29] even larger and would require either $\omega \sim 2\pi \times 1$ THz or a temperature $\sim$1 mK. However, we can make the reasonable assumption that all modes of the junction are independent and replace the inequality (1)—valid for the whole junction—by the same one for a single mode. If we can assume that the modes are independent and have similar transmission coefficient ($T_n \ll 1$) we can simply divide C and S by the number of modes in (1). Note that C enters there linearly while S enters quadratically, so effectively we weaken the contribution from the second cumulant. For a tunnel junction we can thus replace G by G/N_0 where N_0 is a lower bound of

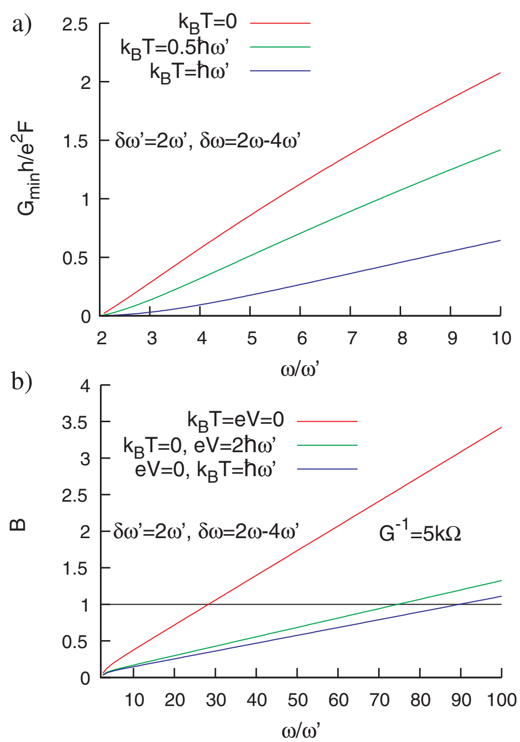

FIG. 1 (color online). (a) The minimal value of the conductance G which satisfies the inequality (8) for the mesoscopic junction at zero voltage. The nonclassicality occurs for $G < G_{\min}$. (b) The dependence of inequality parameter B [given by (8)] for a the tunnel junction as function of the high frequency. Below the line at 1 is the classical regime. Either finite voltage and/or temperature lead to suppression of B and result in the necessity to measure at higher frequencies.

the contributing modes ($T_n > 0$), which must be larger than $G\pi\hbar/e^2$. The results in Fig. 1(b) are valid, for example, for $N_0 = 100 < (h/2e^2)/50\ \Omega \approx 258$. In this case, it is necessary to ensure that most of the modes contributing to the transport have small transmission eigenvalues.

The cumulants are never measured directly. The second cumulant always contains a large offset noise generated by detector and amplifier and effectively described by λ in g. The offset noise can presumably be subtracted as it is due to detector's amplifier. The offset noise may be smaller in the case of a many-mode tunnel junction and one way around is to measure cross correlations by different detectors and amplifiers [12]. The fourth cumulant should also contribute to photon counting statistics [30,31], but in the limit $\delta\omega \to 0$ the photon statistics is dominated by the second cumulants in (1).

We have shown that the unusual properties of weak measurements can be interpreted in terms of a real quasiprobability, which can take negative values. Our interpretation agrees well with predictions and measurements of the current fluctuations in mesoscopic junctions. Its direct confirmation would be the measurement of high-frequency fourth-order averages of the current though the junction. By a violation of the inequality (1), the negativity of the quasiprobability could be directly demonstrated. Finally, the separation between detector and the system is somewhat arbitrary. One could argue that simply adding some noise can restore the positive probability. This is why an experimental estimate of the detector noise g or the strength of the measurement λ is also desirable.

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[1] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton U.P., Princeton, 1932).
[2] K. Kraus, *States, Effects and Operations* (Springer, Berlin, 1983).
[3] See, e.g., J. Preskill, Quantum Information and Computation (www.theory.caltech.edu/people/preskill/ph229)
[4] M. A. Naimark, Izvestiia Akademii nauk SSSR Ser. Mat. **4**, 277 (1940).
[5] V. B. Braginsky and F. Ya. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, England, 1992).
[6] A. N. Jordan and M. Büttiker, Phys. Rev. B **71**, 125333 (2005); A. N. Jordan, A. N. Korotkov, and M. Büttiker, Phys. Rev. Lett. **97**, 026805 (2006); A. N. Jordan and A. N. Korotkov, Phys. Rev. B **74**, 085307 (2006); A. N. Korotkov in Quantum Noise in Mesoscopic Physics, edited by Y. V. Nazarov (Kluwer, Dordrecht, 2003)..
[7] K. V. Bayandin, A. V. Lebedev, and G. B. Lesovik, JETP **106**, 117 (2008).
[8] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988); Y. Aharonov and L. Vaidman, Phys. Rev. A **41**, 11 (1990); Y. Aharonov and D. Rohrlich, *Quantum Paradoxes* (Wiley-VCH, Weinheim, Germany, 2005).
[9] G. J. Pryde *et al.*, Phys. Rev. Lett. **94**, 220405 (2005); A. M. Steinberg, Phys. Rev. Lett. **74**, 2405 (1995); H. M. Wiseman, Phys. Rev. A **65**, 032111 (2002).
[10] N. S. Williams and A. N. Jordan, Phys. Rev. Lett. **100**, 026804 (2008); A. Romito, Y. Gefen, and Y. M. Blanter, Phys. Rev. Lett. **100**, 056801 (2008); V. Shpitalnik, Y. Gefen, and A. Romito, *ibid.* **101**, 226802 (2008).
[11] E. Zakka-Bajjani *et al.*, Phys. Rev. Lett. **99**, 236803 (2007); **104**, 206802 (2010); J. Gabelli and B. Reulet, Phys. Rev. Lett. **100**, 026601 (2008); J. Stat. Mech. (2009) P01049.
[12] A. Bednorz and W. Belzig, Phys. Rev. Lett. **101**, 206803 (2008).
[13] E. P. Wigner, Phys. Rev. **40**, 749 (1932); M. Hillery *et al.*, Phys. Rep. **106**, 121 (1984).
[14] A. Schmid, Ann. Phys. (N.Y.) **173**, 103 (1987); C. Anastopoulos and N. Savvidou, J. Math. Phys. (N.Y.) **47**, 122106 (2006); **48**, 032106 (2007).
[15] B. Berg *et al.*, Phys. Rev. A **80**, 033624 (2009).
[16] H. F. Hofmann, Phys. Rev. A **81**, 012103 (2010).
[17] G. Mitchison, R. Jozsa, and S. Popescu, Phys. Rev. A **76**, 062105 (2007).
[18] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 2007).
[19] B. Misra and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) **18**, 756 (1977).
[20] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985); A. J. Leggett, J. Phys. Condens. Matter **14**, R415 (2002).
[21] Y. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[22] A. V. Galaktionov, D. S. Golubev, and A. D. Zaikin, Phys. Rev. B **68**, 235333 (2003); D. S. Golubev, A. V. Galaktionov, and A. D. Zaikin, Phys. Rev. B **72**, 205417 (2005).
[23] A. Bednorz and W. Belzig Phys. Rev. B **81**, 125112 (2010).
[24] J. Salo, F. W. J. Hekking, and J. P. Pekola, Phys. Rev. B **74**, 125427 (2006); T. T. Heikkilä and T. Ojanen, Phys. Rev. B **75**, 035335 (2007).
[25] S. Bachmann, G. M. Graf, and G. B. Lesovik, J. Stat. Phys. **138**, 333 (2010).
[26] L. S. Levitov and G. B. Lesovik, JETP Lett. **58**, 230 (1993); L. S. Levitov, H. W. Lee, and G. B. Lesovik, J. Math. Phys. (N.Y.) **37**, 4845 (1996); W. Belzig and Y. V. Nazarov, Phys. Rev. Lett. **87**, 197006 (2001); Y. V. Nazarov and M. Kindermann, Eur. J. Phys. B **35**, 413 (2003); M. Kindermann and Y. V. Nazarov in Quantum Noise in Mesoscopic Physics, edited by Y. V. Nazarov (Kluwer, Dordrecht, 2003).
[27] M. I. Reznikov *et al.*, Phys. Rev. Lett. **75**, 3340 (1995); A. Kumar *et al.*, Phys. Rev. Lett. **76**, 2778 (1996); R. J. Schoelkopf *et al.*, Phys. Rev. Lett. **78**, 3370 (1997).
[28] B. Reulet, J. Senzier, and D. E. Prober, Phys. Rev. Lett. **91**, 196601 (2003); Y. Bomze *et al.*, Phys. Rev. Lett. **95**, 176601 (2005); **101**, 016803 (2008).
[29] B. Reulet *et al.*, in *Perspectives of Mesoscopic Physics*, edited by A. Aharony and O. Entin-Wohlman (World Scientific, Singapore, 2010).
[30] C. W. J. Beenakker and H. Schomerus, Phys. Rev. Lett. **86**, 700 (2001); **93**, 096801 (2004); A. V. Lebedev, G. B. Lesovik, and G. Blatter, Phys. Rev. B **81**, 155421 (2010).
[31] J. Gabelli *et al.*, Phys. Rev. Lett. **93**, 056801 (2004).