Short-Range Force Detection Using Optically Cooled Levitated Microspheres

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We propose an experiment using optically trapped and cooled dielectric micro-spheres for the detection of short-range forces. The center-of-mass motion of a microsphere trapped in vacuum can experience extremely low dissipation and quality factors of 10^{12}, leading to yoctonewton force sensitivity. Trapping the sphere in an optical field enables positioning at less than 1 μm from a surface, a regime where exotic new forces may exist. We expect that the proposed system could advance the search for non-Newtonian gravity forces via an enhanced sensitivity of 10^5–10^7 over current experiments at the 1 μm length scale. Moreover, our system may be useful for characterizing other short-range physics such as Casimir forces.

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Since the pioneering work of Ashkin and co-workers in the 1970s [1], optical trapping of dielectric objects has become an extraordinarily rich area of research. Optical tweezers are used extensively in biophysics to study and manipulate the dynamics of single molecules, and in soft condensed-matter physics to study macromolecular interactions [2,3]. Much recent work has focused on trapping in solution where strong viscous damping dominates particle motion. There has also been interest in extending the regime that Ashkin and co-workers opened, namely, trapping subwavelength particles in vacuum where particle motion is strongly decoupled from a room-temperature environment [1,4].

Recent theoretical studies have suggested that nanoscale dielectric objects trapped in ultrahigh vacuum might be cooled to their ground state of (center-of-mass) motion via radiation pressure forces of an optical cavity [5,6]. This remarkable result is made possible by isolation from the thermal bath, robust decoupling from internal vibrations, and lack of a clamping mechanism. In fact, a trapped dielectric nanosphere has been predicted to attain an ultrahigh mechanical quality factor Q exceeding 10^{12} for the center-of-mass mode, limited by background gas collisions. Such large Q factors enable cavity cooling, for which the lowest possible phonon occupation of the mechanical oscillator is n_T/Q, where n_T is the number of room-temperature thermal phonons. Although such Q factors have yet to be observed in experiment, optically levitated microspheres have been trapped in vacuum for lifetimes exceeding 1000 s [1] and electrically levitated microspheres have exhibited pressure-limited damping that is consistent with theoretical predictions down to $\sim 10^{-6}$ Torr [7].

In addition to being beneficial for ground-state cooling and studies of quantum coherence in mesoscopic systems, mechanical oscillators with high quality factors also enable sensitive resonant force detection [8,9]. Optically levitated microspheres in vacuum have been studied theoretically in the context of reaching and exceeding the standard quantum limit of position measurement [10]. In this Letter, we discuss the force sensing capability of a microsphere trapped inside a medium-finesse optical cavity at ultrahigh vacuum and propose an experiment that could extend the search for non-Newtonian gravitylike forces that may occur at micron scale distances. Such forces could be mediated by particles residing in submillimeter scale extra spatial dimensions [11] or by moduli in the case of gauge-mediated supersymmetry breaking [12]. The apparatus we propose is also well suited to studying Casimir forces [13] and may be useful for studying radiative heat transfer at the nanoscale [14].

Corrections to Newtonian gravity at short range are generally parametrized according to a Yukawa-type potential

$$V = -\frac{G_N m_1 m_2}{r}[1 + \alpha e^{-r/\lambda}], \tag{1}$$

where m_1 and m_2 are two masses interacting at distance r, α is the strength of the potential relative to gravity, and λ is the range of the interaction. For two objects of mass density ρ and linear dimension λ with separation $r \approx \lambda$, a Yukawa force scales roughly as $F_Y \sim G_N \rho^2 \alpha \lambda^4$, decreasing rapidly with smaller λ. For example, taking gold masses, for an interaction potential with $\alpha = 10^5$ and $\lambda = 1\ \mu$m, $F_Y \sim 10^{-21}$ N. As we will discuss, the thermal-noise-limited force sensitivity of micron scale, optically levitated silica microspheres at 300 K with $Q = 10^{12}$ can be of order $\sim 10^{-21}$ N$/\sqrt{\text{Hz}}$, and therefore allows probing deep into unexplored regimes. For instance, current experimental limits at $\lambda = 1\ \mu$m have ruled out interactions with $|\alpha|$ exceeding 10^{10}.

The proposed experimental setup is shown schematically in Fig. 1. A dielectric microsphere of radius $a = 150$ nm is optically levitated and cooled in an optical cavity of length L by use of two light fields of wave numbers $k_t = 2\pi/\lambda_{\text{trap}}$ and $k_c = 2\pi/\lambda_{\text{cool}}$, respectively. The silica microsphere has density $\rho = 2300$ kg/m^3, dielectric constant $\epsilon = 2$, and is trapped near the position of the closest antinode of the cavity trapping field to a gold mirror surface. The mirror is a 200 nm thick SiN mem-

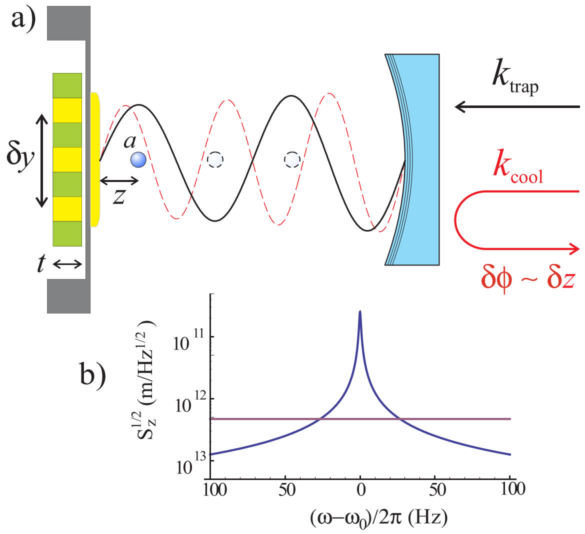

FIG. 1 (color online). (a) Proposed experimental geometry. A subwavelength dielectric microsphere of radius a is trapped with light in an optical cavity. The sphere is positioned at an antinode occurring at distance z from a gold-coated SiN membrane. Light of a second wavelength $\lambda_{\rm cool} = 2\lambda_{\rm trap}/3$ is used to simultaneously cool and measure the center-of-mass motion of the sphere. The sphere displacement δz results in a phase shift $\delta\phi$ in the cooling light reflected from the cavity. For the short-range gravity measurement, a source mass of thickness t with varying density sections is positioned on a movable element behind the mirror surface that oscillates harmonically with an amplitude δy. The source mass is coated with a thin layer of gold to provide an equipotential. (b) Displacement spectral density (blue curve) due to thermal noise and shot-noise limited displacement sensitivity (flat line, red) for parameters discussed in the text.

brane coated with 200 nm of gold. A source mass of thickness $t = 5\ \mu$m and length 20 μm with varying density sections of width 2 μm (e.g., Au and Si) is positioned at edge-to-edge separation $d = 1\ \mu$m from the sphere. Below we describe trapping and cooling of the microsphere's center-of-mass motion, detection of Casimir forces between the microsphere and gold mirror, and the search for gravitylike forces on the microsphere due to the source mass.

Following Ref. [5], the subwavelength dielectric particle has a center-of-mass resonance frequency $\omega_0 = [\frac{6k_t^2 I_t}{\rho c}\,\mathrm{Re}\frac{\epsilon-1}{\epsilon+2}]^{1/2}$, where I_t is the intracavity intensity of the trapping light. The trap depth is $U = \frac{3I_t V}{c}\frac{\epsilon-1}{\epsilon+2}$, where V is the volume of the microsphere. For concreteness, we consider a cavity of length $L = 0.15$ m, finesse $\mathcal{F} = 200$, driven with a trapping laser of power $P_t = 2$ mW and wavelength $\lambda_{\rm trap} = 1.5\ \mu$m. We choose a cavity mode waist $w = 15\ \mu$m. The Gaussian profile of the trapping beam near the mode waist provides transverse confinement, with an oscillation frequency of $\sim$1 kHz. Tighter transverse confinement could be established by use of a transverse standing wave potential. The cooling light has input power $P_c = 48\ \mu$W, and an optimized red detuning of $\delta = -0.23\kappa$, where the cavity decay rate is $\kappa = \pi c/L\mathcal{F}$. The cooling light causes a slight shift z_0 in the axial equilibrium position of the microsphere, given by $z_0 = \frac{1}{2}\frac{k_c}{k_t^2}\frac{I_c}{I_t} \approx 2$ nm, where I_c is the intracavity intensity of the cooling mode. The optomechanical coupling of the cooling mode is $g = \frac{3V}{4V_c}\frac{\epsilon-1}{\epsilon+2}\omega_c$, where $V_c = \pi w^2 L/4$ is the cavity mode volume [5], and $\omega_c = k_c c$. The optimum detuning is determined by minimizing the final phonon occupancy n_f, which depends on the laser-cooling rate and heating due to photon recoil from light scattered by the sphere. Additional cavity loss due to photon scattering is negligible: $\sim 10^{-3}\kappa$ for our parameters. Values of the trapping and cooling parameters appear in Table I.

A mechanical oscillator with frequency $\sim$37 kHz and $Q \sim 10^{12}$ will respond to perturbations with a characteristic time scale of $2Q/\omega_0 \sim 10^7$ s. The cooling serves both to damp the Q factor so that perturbations to the system ring down within reasonably short periods of time and to localize the sphere by reducing the amplitude of the thermal motion. Because of the low cavity finesse, the cooling is not in the resolved sideband regime. Still, for the parameters discussed above, the phonon occupation of the microsphere oscillation is reduced by a factor of over 10^5. This corresponds to operating with an effective $Q_{\rm eff} \approx 10^5$ and ring down time of $\approx$1 s. Cooling of the transverse motion is also required, as the rms position spread must be maintained to be less than $\sim 0.1\ \mu$m. We imagine this can be done with active feedback to modulate the power of a transverse trapping laser using the signal from a transverse position measurement, for example, generated by measuring scattered light incident on a quadrant photodiode. A modest cooling factor of $\approx$1000 in the transverse directions is sufficient to yield the required localization.

The cooling light is also used to detect the position of the sphere. The phase of the cooling light reflected from the cavity is modulated by microsphere motion through the optomechanical coupling $\partial\omega_c/\partial z = 2k_c g$. Photon shot noise limits the minimum detectable phase shift to $\delta\phi \approx 1/(2\sqrt{I})$, where $I \equiv P_c/(\hbar\omega_c)$ [15]. The corresponding photon shot-noise limited displacement sensitivity is

TABLE I. Parameters for trapping and cooling a silica sphere with radius $a = 150$ nm.

Parameter	Units	Value
$\lambda_{\rm trap}$	μm	1.5
U/k_B	K	3.7×10^3
$\omega_0/2\pi$	Hz	3.7×10^4
$T_{\rm int}$	K	900
Q, ($Q_{\rm eff}$)	...	6.1×10^{11}, (1.0×10^5)
δ/κ	...	-0.23
n_T, (n_f)	...	1.7×10^8, (510)
$\sqrt{S_z}$	m/$\sqrt{\rm Hz}$	4.7×10^{-13}
$F_{\rm min}$	N/$\sqrt{\rm Hz}$	1.9×10^{-21}
$z_{\rm th}$	m/$\sqrt{\rm Hz}$	2.6×10^{-11}

$\sqrt{S_z(\omega)} = \frac{\kappa}{4k_c g}\frac{1}{\sqrt{I}}\sqrt{1+4\omega^2/\kappa^2}$ [15], for an impedance matched cavity. This displacement sensitivity is generally well below the thermal-noise-limited sensitivity, as shown in Fig. 1(b). We assume that substrate vibrational noise, electronics noise, and laser noise can be controlled at a level comparable to the photon shot noise. The minimum detectable force due to thermal noise at temperature T_{eff} is $F_{\min} = \sqrt{4kk_B T_{\mathrm{eff}} b/\omega_0 Q_{\mathrm{eff}}}$, where k is the center-of-mass mode spring constant and b is the bandwidth of the measurement. We assume an initial center-of-mass temperature $T_{\mathrm{c.m.}} = 300$ K, and that $Q \approx \omega_0/\gamma_g$ is limited by background gas collisions, with loss rate $\gamma_g = 16P_{\mathrm{gas}}/(\pi\bar{v}\rho a)$ [16], for a background air pressure of $P_{\mathrm{gas}} = 10^{-10}$ Torr and rms gas velocity $\bar{v}$. Cooling the center-of-mass mode to $T_{\mathrm{eff}} = 0.9$ mK results in $F_{\min} \sim 10^{-21}\ \mathrm{N}/\sqrt{\mathrm{Hz}}$ as shown in Table I. In this regime $F_{\min}$ scales linearly with the sphere radius a.

The microsphere absorbs optical power from both the trapping and cooling light in the cavity, which results in an increased internal temperature T_{int}. Assuming negligible cooling due to gas collisions, the absorbed power is reradiated as blackbody radiation. T_{int} is listed in Table I for fused silica with dielectric response $\epsilon = \epsilon_1 + i\epsilon_2$, with $\epsilon_1 = 2$ and $\epsilon_2 = 1.0 \times 10^{-5}$ as in Ref. [17], and $\epsilon_{\mathrm{bb}} = 0.1$ as in Ref. [5], for an environmental temperature $T_{\mathrm{ext}} = 300$ K. We assume T_{int} and $T_{\mathrm{c.m.}}$ are not significantly coupled over the time scale of the experimental measurements at $P_{\mathrm{gas}} \sim 10^{-10}$ Torr.

Casimir force.—The Casimir force between a dielectric sphere and metal plane can be written using the proximity force approximation (PFA) as [13] $F_c = -\eta\frac{\pi^3 a\hbar c}{360(z-a)^3}$ in the limit that $(z-a) \ll a$. The prefactor η characterizes the reduction in the force compared with that between two perfect conductors [18]. For $z \gg a$, the force takes the Casimir-Polder [19] form $F_{\mathrm{cp}} = -\frac{3\hbar c\alpha_V}{8\pi^2\epsilon_0}\frac{1}{z^5}$, where $\alpha_V = 3\epsilon_0 V\frac{\epsilon-1}{\epsilon+2}$ is the electric polarizability. Our setup is capable of probing the unexplored transition between these two regimes, and of testing the PFA, which is expected to be valid for $z - a \lesssim a$ [20]. To estimate η, we adopt a similar approach to that taken in Ref. [18] to determine the force between a metal and dielectric plate. We assume dielectric spheres separated from a metal mirror will have a similar prefactor. Taking an infinite plate with $\epsilon = 2$ and thickness $2a$, and another with gold of thickness 200 nm, we find $\eta \approx 0.13$ at $(z-a) = 225$ nm.

For a sphere located at the position of the first antinode of the trapping field, the Casimir force displaces the equilibrium position by approximately -3 nm. The gradient of the Casimir force near the static mirror surface produces a fractional shift in the resonance frequency of the sphere given by $|\delta\omega_0/\omega_0| = \frac{|\partial F_c/\partial z|}{2k}$. A similar frequency shift has been measured for an atomic sample [21]. The shift is shown in Fig. 2 as a function of mirror separation $(z-a)$ for $\eta = 0.13$. The minimum detectable frequency shift due to thermal noise is given by $|\delta\omega_0/\omega_0|_{\min} = \sqrt{k_B T_{\mathrm{eff}} b/k\omega_0 Q_{\mathrm{eff}} z_{\mathrm{rms}}^2}$. For $z_{\mathrm{rms}} = 5$ nm, $|\delta\omega_0/\omega_0| \approx 10^{-7}$ can be detected in $\sim$1 s. Other sources of systematic frequency shifts near the surface, for example, from variation of the cavity finesse with bead position or from diffuse scattered light on the gold surface, would need to be experimentally characterized. Also, surface roughness of the microsphere can modify the Casimir force [22]. Rotation of the microsphere may lead to an effective averaging over surface inhomogeneities.

Search for non-Newtonian gravity.—To generate a modulation of any Yukawa-type force at the resonance frequency of the center-of-mass mode along z, the source mass is mounted on a cantilever beam that undergoes a lateral tip displacement of 2.6 μm at a frequency of $\omega_0/3$. The mechanical motion occurs at a subharmonic of the microsphere resonance to avoid direct vibrational coupling. To estimate the force on the sphere a numerical integration over the geometry of the masses is performed. For $b = 10^{-5}$ Hz, the estimated search reach is shown in Fig. 3, curve (a). Several orders of magnitude of improvement are possible between 0.1 nm and a few microns, due to the proximity of the masses and high force sensitivity.

The source mass surface is coated with 200 nm of gold in order to screen the differential Casimir force, depending on which material is directly beneath the microsphere. Following the method of Ref. [18], the differential Casimir force is 9×10^{-24} N, which is comparable to the sensitivity of the experiment at 10^{-5} Hz bandwidth. The gold coating on the cavity mirror membrane attenuates this Casimir interaction even further. Patch potentials on the mirror surface and any electric charge on the sphere can

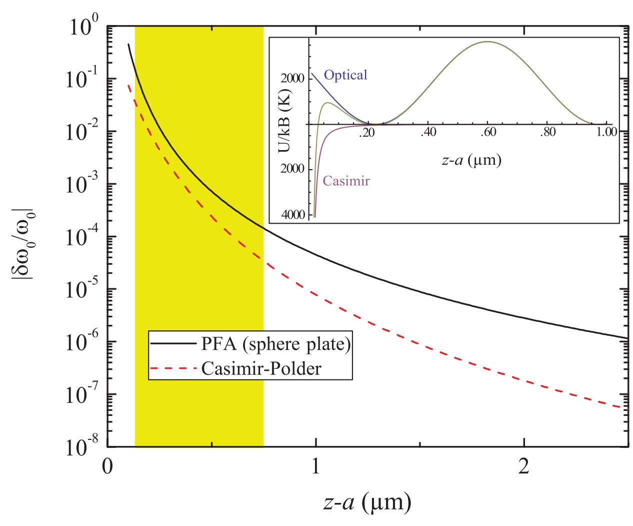

FIG. 2 (color online). Fractional frequency shift due to Casimir interaction of microspheres of radius $a = 150$ nm at distance z from the gold surface. The PFA is expected to be valid at short distances, while the Casimir-Polder form is expected at large distances, and the transition region is shown as a shaded area. Inset: Optical and Casimir contributions to the cavity trapping potential.

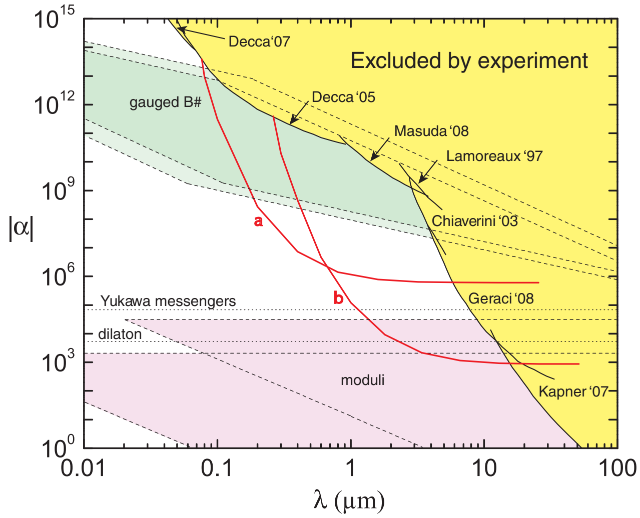

FIG. 3 (color online). Experimental constraints [23–29] and theoretical predictions [30] for short-range forces due to an interaction potential of Yukawa form $V = -\frac{G_N m_1 m_2}{r} \times [1 + \alpha e^{-r/\lambda}]$. Lines ($a$) and ($b$) denote the projected improved search reach for microspheres of radius $a = 150$ nm and $a = 1500$ nm, respectively.

produce spurious forces on the sphere. By translating the position of the optical trap along the surface, these and other backgrounds, e.g., vibration, can be distinguished from a Yukawa-type signal, as any Yukawa-type signal should exhibit a spatial periodicity associated with the alternating density pattern, similar to the system discussed in Ref. [23].

Increasing the radius of the sphere can significantly enhance the search for non-Newtonian effects at longer range. Curve (b) in Fig. 3 shows the estimated search reach that would be obtained by scaling the sphere size by a factor of 10 and positioning it at edge-to-edge separation of 3.8 μm from a source mass with thickness $t = 10$ μm consisting of sections of width 10 μm driven at an amplitude of 13 μm. Such a larger sphere could be trapped in an optical lattice potential with the incident beams at a shallow angle, instead of in an optical cavity, to enable subwavelength confinement. In this case cooling could be performed by use of active feedback. Alternatively it may be possible to trap the larger 1.5 μm sphere in a cavity by use of longer wavelength light (e.g., $\lambda_{\mathrm{trap}} = 10.6$ μm) by choosing a sphere material such as ZnSe with lower optical loss at this wavelength.

The experiment we have proposed may allow improvement by several orders of magnitude in the search for non-Newtonian gravity below the 10 μm length scale. An experimental challenge will be to capture and cool individual microspheres and precisely control their position near a surface. Previous experimental work has been successful at optically trapping 1.5 μm radius spheres [4], and similar techniques may work for the setup proposed here. Extrapolating the results of Ref. [7] at 10^{-6} Torr for the system we consider would yield a pressure-limited $Q \sim 10^9$. In the absence of additional damping mechanisms, we expect that $Q \approx 10^{12}$ could be achieved at lower pressure. Further improvements in force sensitivity may be possible in a cryogenic environment.

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