Collective Motion of Vibrated Polar Disks

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We experimentally study a monolayer of vibrated *disks* with a built-in polar asymmetry which enables them to move quasiballistically on a large persistence length. Alignment occurs during collisions as a result of self-propulsion and hard core repulsion. Varying the amplitude of the vibration, we observe the onset of large-scale collective motion and the existence of giant number fluctuations with a scaling exponent in agreement with the predicted theoretical value.

The recent surge of theoretical and numerical activity about the collective properties of interacting self-propelled particles has produced some striking results, even in the simplest situations where local alignment, the only interaction, is competing with some noise: for instance, true long-range order may arise in two dimensions, yielding collectively-moving ordered phases endowed with generic long-range correlations and anomalous "giant" number fluctuations [1–4]. Despite the ubiquity of the collective motion observed at all scales in more or less complex situations ranging from the cooperative action of molecular motors [5], the collective displacement of cells [6], to the behavior of large animal, human, or robot groups [7], there is, as of now, a lack of well-controlled experiments to which this theoretical progress can be seriously confronted. Indeed, working with large animal groups usually implies that experiments are just observations, with the unavoidable difficulties to track trajectories and without much of a control parameter to vary [8]. Similarly, experiments on living cells (during development or wound healing or within bacteria and amoeba colonies) often involve the presence of external (chemical) gradients, genetic factors, etc., which are hard to evaluate and known to have a possibly strong influence. There is, hence, a crucial need for *model experiments* using man-made objects with rather well understood interactions. Swimmers [9] (from chemically-powered nanorods to microscopic and macroscopic size mechanical devices) offer an interesting direction but the intrinsically long-range nature of hydrodynamic interactions may appear as a unnecessary complication. In this context, vibrated, dry, inert, "granular" particles appear as an attractive case where much control can be exerted on the system, in the absence of long-range interactions or unwanted additional features, so that the onset of collective motion would then be a bona fide spontaneous symmetry breaking phenomenon.

Various objects can be set in fairly regular motion on a flat surface when vibrated properly: Yamada, Hondou, and Sano were pioneers in demonstrating that an axisymmetric polar object vibrated between two plates can move quasiballistically [10]. At the collective level, Kudrolli's group studied the behavior of polar rods [11] and, more recently, of short snakelike chains [12], but was unable to observe genuine long-range orientational order, i.e., collective motion. A few other works have dealt with the collective properties of shaken elongated apolar particles (a realization of so-called "active nematics liquid crystals") [13], but there no net collective motion is expected anyway.

Thus, to our knowledge, no well-controlled experiment has produced a fluctuating, collectively moving ordered phase of the type frequently observed in simple numerical models. This may be just due to the scarcity of attempts, but recent results might provide a deeper reason: it was found that self-propelled particles with apolar (nematic) alignment interactions cannot give rise to polar order, i.e., to collective motion [14]. (They may give rise, however, to nematic order.) The few experiments mentioned above all dealt with elongated objects ("self-propelled rods"), and fall into this class because of their shape.

In this Letter, we report on experiments conducted on vibrated *disks* with a built-in polar asymmetry which enables them to move coherently [Fig. 1]. The isotropic shape of the particles prevents strong nematic alignment. On the contrary, we shall see that polar alignment is favored as an effective result of hard core repulsion and polar motion. Varying the amplitude of the vibration, we observe the onset of large-scale collective motion and the existence of giant number fluctuations with a scaling exponent in agreement with the predicted theoretical value. We discuss the difficulties in characterizing collective motion in a finite domain and the possible key differences with the simple models usually considered at the theoretical level.

Experiments with shaken granular particles are notoriously susceptible to systematic deviations from pure vertical vibration [15]. We use a 110 mm thick truncated cone of expanded polystyren sandwiched between two nylon disks. The top disk (diameter 425 mm) is covered by a glass plate on which lay the particles. The bottom one (diameter 100 mm) is mounted on the slider of a stiff square air-bearing (C40-03100-100254,IBSPE), which provides virtually friction-free vertical motion and submicron amplitude residual horizontal motion. The vertical
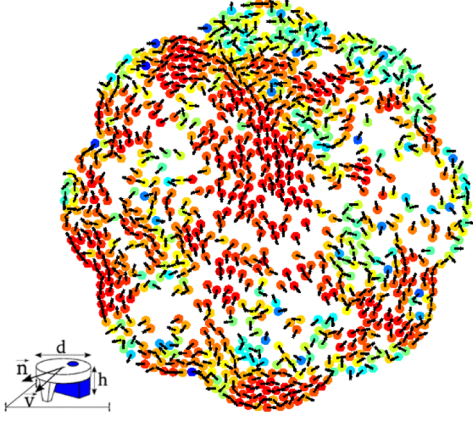
FIG. 1 (color online). Collective motion of self-propelled disks. Bottom left panel: a sketch of our polar particles. Main panel: a snapshot of an ordered regime observed in our flower-shape domain. The dark gray reveals the local alignment between particles {both perfect alignment [light grey (red)] and pergect antialignment [dark grey (blue)]}. The intrinsic polarity of the particles is indicated by the black arrows.

alignment is controlled by set screws. The vibration is produced with an electromagnetic servo-controlled shaker (V455/6-PA1000L,LDS), the accelerometer for the control being fixed at the bottom of the top vibrating disk, embedded in the expanded polystyren. A 400 mm long brass rod couples the air-bearing slider and the shaker. It is flexible enough to compensate for the alignment mismatch, but stiff enough to ensure mechanical coupling. The shaker rests on a thick wooden plate ballasted with 460 kg of lead bricks and isolated from the ground by rubber mats (MUSTshock $100 \times 100 \times EP5$, Musthane). We have measured the mechanical response of the whole setup and found no resonances in the window 70–130 Hz. Here, we use a sinusoidal vibration of frequency $f = 115$ Hz and vary the relative acceleration to gravity $\Gamma = 2\pi a f^2 / g$. The vibration amplitude a at a peak acceleration of 1 g at this frequency is 25 μm. Using a triaxial accelerometer (356B18,PCB Electronics), we checked that the horizontal to vertical ratio is lower than 10^{-2} and that the spatial homogeneity of the vibration is better than 1%.

Our polar particles are micro-machined copper-beryllium disks (diameter $d = 4$ mm) with an off-center tip and a glued rubber skate located at diametrically opposite positions [Fig. 1]. These two "legs," which have different mechanical response under vibration, endow the particles with a polar axis which can be determined from above thanks to a black spot located on their top. Under proper vibration, they can be set in directed motion (see below). Of total height $h = 2.0$ mm, they are sandwiched between two thick glass plates separated by a gap of $H = 2.4$ mm. We also used, to perform "null case experiments," plain rotationally invariant disks (same metal, diameter, and height), hereafter called the "symmetric" particles. We laterally confined the particles in a flower-shaped arena of internal diameter $D = 160$ mm [Fig. 1].

The petals avoid the stagnation and accumulation of particles along the boundaries as reported, for instance, in [11] by "reinjecting" them into the bulk. A CCD camera with a spatial resolution of 1728×1728 pixels and standard tracking software is used to capture the motion of the particles at a frame rate of 20 Hz. In the following, the unit of time is set to be the period of vibration and the unit length is the particle diameter. Within these units, the resolution on the position $\vec{r}$ of the particles is better than 0.1, that on the orientation $\vec{n}$ is of the order of 0.05 rad and the lag separating two images is $\tau_0 = 5.75$. Measuring the long-time averaged spatial density map (for various numbers of particles), we find that this density field slightly increases near the boundaries, but is constant to a few percent in a region of interest (ROI) of diameter $20d$. This provides an additional check of the spatial homogeneity of our setup.

We first performed experiments with 50 particles, i.e., at a surface fraction small enough so that collisions are rare and the individual dynamics can be investigated. For large acceleration, the polar particles describe random-walk-like trajectories with short persistence length. Decreasing Γ, they show more and more directed motion, and the persistence length quickly exceeds the system size. This is in contrast with the symmetric particles which retain the same shortly correlated individual walk dynamics for all Γ values [Figs. 2(a) and 2(b)].

More precisely, individual velocities $\vec{v}_i(t) \equiv [\vec{r}_i(t + \tau_0) - \vec{r}_i(t)]/\tau_0$ measured within the ROI have a well-defined most probable or mean value $v_{\text{typ}} \simeq 0.025$
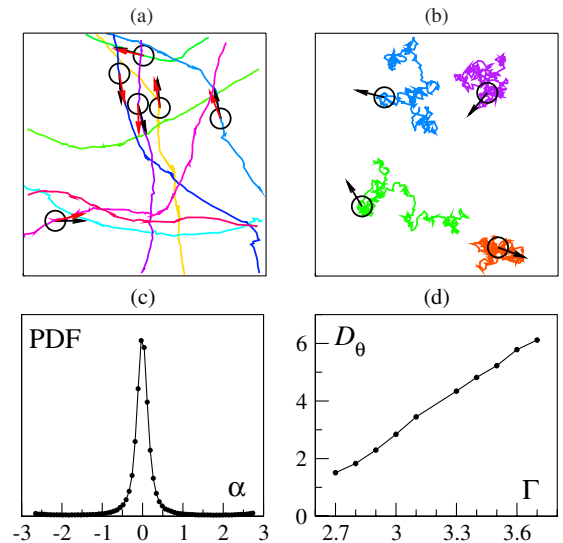

FIG. 2 (color online). Individual dynamics for $\Gamma = 2.7$. (a) Typical portions of polar particles trajectories inside the ROI. Black and grey (red) arrows indicate $\vec{v}_i^t$ and $\vec{n}_i^t$ at selected times. The domain area is about $15 \times 15d$. (b) The same for symmetric particles. (c) Probability distribution function (PDF) with counterpropagating waves with a common linear polarization (lin-lin) of α, the angle between $\vec{v}_i^t$ and $\vec{n}_i^t$. (d) Variation of angular diffusion coefficient D_θ with Γ.

which changes by only 6% over the interval $\Gamma \in [2.7, 3.7]$ (not shown). For smaller values of Γ the velocity decreases suddenly and the particles come to an almost complete stop around $\Gamma = 2.4$. The local displacements of our polar particles are overwhelmingly taking place along $\vec{n}_i(t)$, their instantaneous polarity [Fig. 2(c)]. The distribution of the angle $\theta_i(t, t + \tau_0)$ by which they turn during an interval τ_0 [defined using the polarity $\vec{n}_i(t)$] is an exponential distribution of zero-mean and variance $2D_\theta/\tau_0$. The angular diffusion constant D_θ decreases fast and linearly for $\Gamma \in [2.7, 3.7]$ [Fig. 2(d)]. In contrast again, D_θ is about 1 order of magnitude larger for our isotropic particles, and varies little with Γ (not shown). A persistence length can then be defined as $\xi = \frac{1}{2}\pi^2 v_{\mathrm{typ}}/D_\theta$ (i.e., the length traveled over the time needed to turn by π, assuming a constant speed v_{typ}). Its typical value decreases from above 100 for $\Gamma = 2.7$ to around 20 for $\Gamma = 3.7$ whereas it stays around 1 for the symmetric particles.

We now turn to the collective dynamics of our polar particles. As seen above, the relative acceleration Γ has a strong influence on their individual dynamics, controlling the persistence length of their trajectories via the angular diffusion constant D_θ. During collisions, they typically bounce against each other several times, yielding, on average, some degree of polar alignment [Fig. 3]. All this is reminiscent of Vicsek-like models, for which one of the main control parameters is the strength of the angular noise competing with the alignment interaction [3,4]. Thus Γ is not only an easy control parameter, but also a natural one, which we use in the following. The surface fraction ϕ of particles is another natural control parameter in collective motion and granular media studies, but it is somewhat more tedious to vary, and, more importantly, one should avoid to deal with too few, respectively, too many, particles in order to prevent loss of statistical quality, respectively, jamming effects. Below, we present results obtained with $N = 890$ particles, which gives a surface fraction $\phi \simeq 0.38$ in the ROI where an average of 160 particles (slightly dependent on Γ) is found. Similar results were obtained at nearby densities. To characterize orientational order, we use the modulus of the average velocity-defined polarity $\Psi(t) = |\langle \vec{u}_i(t) \rangle|$ where $\vec{u}_i(t)$ is the unit vector along $\vec{v}_i(t)$ and the average is overall particles inside the ROI at time t [16].

At low Γ values, for which the directed motion of our polar particles is most persistent, we observe spectacular large-scale collective motion, with jets and swirls as large as the system size [Fig. 1 and [17]]. Of course, because our boundary conditions are not periodic, the collective motion observed is not sustained at all times. Large moving clusters form, then breakdown, etc., As a result, the times series of the order parameter Ψ presents strong variations, but can take a rather well-defined order one value for long periods of time [Fig. 4(a)]. At high Γ values (large noise) no large-scale ordering is found. Decreasing Γ, the PDF of Ψ becomes wider and wider, with a mean and a most prob-
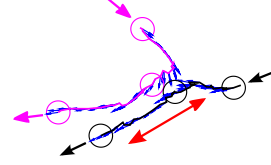

FIG. 3 (color online).   Trajectories of two particles "during" a collision: they first collide almost head on, but repeated contacts [all along the grey (red) double-headed arrow] finally leave them almost aligned, despite their isotropic shape.

able value increasing sharply [Figs. 4(b) and 4(c)]. Note that the most probable value corresponds, at small Γ, to the plateau value found in time series of Ψ. Thus, we observe the clear emergence of long-range orientational order over the range of usable Γ values. In contrast, the same experiments realized with our symmetric particles do not give rise to any collective motion [Fig. 4(c)], which ultimately indicates that our observations with polar particles are not due to some residual large-scale component of our shaking apparatus [15].

Unfortunately, we could not observe the saturation of the order parameter expected deep in the ordered phase, because the "self-propulsion" of our polar particles deteriorates for $\Gamma \lesssim 2.7$. Nevertheless, large Ψ values were observed, signalling that our lowest usable Γ values are already in the ordered phase, albeit not quite surely out of the critical-transitional region. We thus investigate the emergence of the so-called "giant number fluctuations" (GNF) which have been shown theoretically and numerically to be a landmark of orientationally ordered phases for active particles [1,2]. To this aim, we recorded, along time, the number $n(t)$ of particles present in boxes of various sizes located within the ROI. GNF are characterized by the
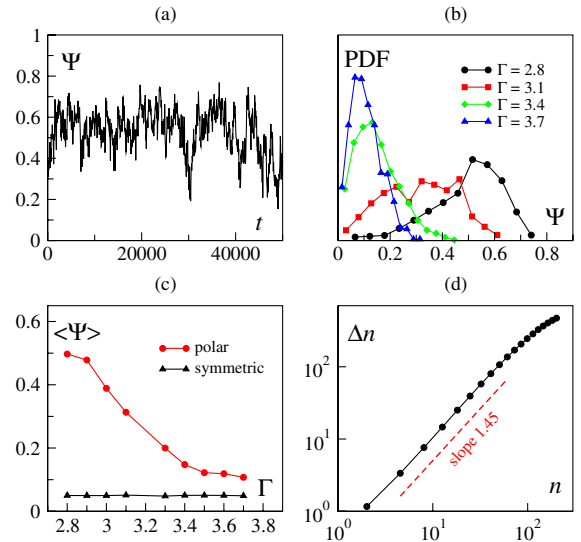

FIG. 4 (color online).   Collective dynamics. (a) Time series of order parameter Ψ at $\Gamma = 2.8$. (b) PDF (lin-lin) of $\Psi(t)$ at various Γ values. (c) $\langle \Psi \rangle$ vs Γ for polar and symmetric particles. (d) Δn vs n at $\Gamma = 2.8$.

fact that the variance Δn of this number scales faster than the mean n. This is indeed what we find: over a range of scales, Δn grows like n^α with $\alpha \sim 1.45 \pm 0.05$. For larger scales, one feels the finite system size and Δn levels off. In fact, according to the prediction derived from the work of Tu and Toner [18] and confirmed in simulations [4], this number should be 1.6. Thus our finding is quite consistent with the predicted value, all the more so since α is expected to converge from below as the system size increases [2]. Although this will require confirmation by experiments performed in larger dishes, this result constitutes the first experimental evidence for GNF in collections of polar active particles [19].

To summarize, we have shown that shaken particles with a polarity not related to their shape can exhibit collective motion on scales or the order of the domain in which they evolve. In the most ordered regimes reachable, we recorded giant number fluctuations with a scaling exponent consistent with that of polar active phases.

That we observe dominant *polar* order is worth discussing. On one hand it was recently shown that if their alignment interaction is *nematic*, polar particles cannot order polarly and only nematic order arises [14]. However self-propulsion enhances polar correlations and this nematic order is made of polar packets [20], which could dominate the global order in a small domain such as our ROI. On the other hand, preliminary studies of the somewhat unusual collisions which often consist of series of repeated shocks [Fig. 3] reveal a systematic bias in favor of polar alignment in line with [21]. Whether such a bias could drive the transition towards polar order is still theoretically an open question. It is not clear at this point in what class our system falls.

In Vicsek-style models (and their continuous descriptions) no GNF proper exist near the transition, where high-order, high-density bands emerge [4,22]. Here, we found GNF in our most-ordered regimes, with approximately the expected exponent. But it is impossible, at this stage, to disentangle fluctuations due to the proximity of the transition, those due to the frustration induced by our boundaries (which would break bands), and "genuine" GNF. Also, further complications could arise from the influence of the near-jammed clusters forming when order sets in. From that perspective, our results pave the way for the study of jamming in active matter.

Thus performing experiments in larger domains and investigating deeper into the ordered phase is of utmost importance. Very recent results obtained in a twice larger system indicate an anomalous density fluctuation exponent even closer to the theoretical value. More work is under progress.

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[17] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.105.098001 for a movie.

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