Measurement of D^0-$\bar{D}^0$ Mixing Parameters Using $D^0 \to K^0_S \pi^+ \pi^-$ and $D^0 \to K^0_S K^+ K^-$ Decays

P. del Amo Sanchez,[1] J. P. Lees,[1] V. Poireau,[1] E. Prencipe,[1] V. Tisserand,[1] J. Garra Tico,[2] E. Grauges,[2] M. Martinelli,[3a,3b]
A. Palano,[3a,3b] M. Pappagallo,[3a,3b] G. Eigen,[4] B. Stugu,[4] L. Sun,[4] M. Battaglia,[5] D. N. Brown,[5] B. Hooberman,[5]
L. T. Kerth,[5] Yu. G. Kolomensky,[5] G. Lynch,[5] I. L. Osipenkov,[5] T. Tanabe,[5] C. M. Hawkes,[6] A. T. Watson,[6] H. Koch,[7]
T. Schroeder,[7] D. J. Asgeirsson,[8] C. Hearty,[8] T. S. Mattison,[8] J. A. McKenna,[8] A. Khan,[9] A. Randle-Conde,[9] V. E. Blinov,[10]
A. R. Buzykaev,[10] V. P. Druzhinin,[10] V. B. Golubev,[10] A. P. Onuchin,[10] S. I. Serednyakov,[10] Yu. I. Skovpen,[10]
E. P. Solodov,[10] K. Yu. Todyshev,[10] A. N. Yushkov,[10] M. Bondioli,[11] S. Curry,[11] D. Kirkby,[11] A. J. Lankford,[11]
M. Mandelkern,[11] E. C. Martin,[11] D. P. Stoker,[11] H. Atmacan,[12] J. W. Gary,[12] F. Liu,[12] O. Long,[12] G. M. Vitug,[12]
C. Campagnari,[13] T. M. Hong,[13] D. Kovalskyi,[13] J. D. Richman,[13] A. M. Eisner,[14] C. A. Heusch,[14] J. Kroseberg,[14]
W. S. Lockman,[14] A. J. Martinez,[14] T. Schalk,[14] B. A. Schumm,[14] A. Seiden,[14] L. O. Winstrom,[14] C. H. Cheng,[15]
D. A. Doll,[15] B. Echenard,[15] D. G. Hitlin,[15] P. Ongmongkolkul,[15] F. C. Porter,[15] A. Y. Rakitin,[15] R. Andreassen,[16]
M. S. Dubrovin,[16] G. Mancinelli,[16] B. T. Meadows,[16] M. D. Sokoloff,[16] P. C. Bloom,[17] W. T. Ford,[17] A. Gaz,[17]
J. F. Hirschauer,[17] M. Nagel,[17] U. Nauenberg,[17] J. G. Smith,[17] S. R. Wagner,[17] R. Ayad,[18,*] W. H. Toki,[18] T. M. Karbach,[19]
J. Merkel,[19] A. Petzold,[19] B. Spaan,[19] K. Wacker,[19] M. J. Kobel,[20] K. R. Schubert,[20] R. Schwierz,[20] D. Bernard,[21]
M. Verderi,[21] P. J. Clark,[22] S. Playfer,[22] J. E. Watson,[22] M. Andreotti,[23a,23b] D. Bettoni,[23a] C. Bozzi,[23a] R. Calabrese,[23a,23b]
A. Cecchi,[23a,23b] G. Cibinetto,[23a,23b] E. Fioravanti,[23a,23b] P. Franchini,[23a,23b] E. Luppi,[23a,23b] M. Munerato,[23a,23b]
M. Negrini,[23a,23b] A. Petrella,[23a,23b] L. Piemontese,[23a] R. Baldini-Ferroli,[24] A. Calcaterra,[24] R. de Sangro,[24]
G. Finocchiaro,[24] M. Nicolaci,[24] S. Pacetti,[24] P. Patteri,[24] I. M. Peruzzi,[24,†] M. Piccolo,[24] M. Rama,[24] A. Zallo,[24]
R. Contri,[25a,25b] E. Guido,[25a,25b] M. Lo Vetere,[25a,25b] M. R. Monge,[25a,25b] S. Passaggio,[25a] C. Patrignani,[25a,25b]
E. Robutti,[25a] S. Tosi,[25a,25b] B. Bhuyan,[26] C. L. Lee,[27] M. Morii,[27] A. Adametz,[28] J. Marks,[28] S. Schenk,[28] U. Uwer,[28]
F. U. Bernlochner,[29] H. M. Lacker,[29] T. Lueck,[29] A. Volk,[29] P. D. Dauncey,[29] M. Tibbetts,[30] P. K. Behera,[31] U. Mallik,[31]
C. Chen,[32] J. Cochran,[32] H. B. Crawley,[32] L. Dong,[32] W. T. Meyer,[32] S. Prell,[32] E. I. Rosenberg,[32] A. E. Rubin,[32]
Y. Y. Gao,[33] A. V. Gritsan,[33] Z. J. Guo,[33] N. Arnaud,[34] M. Davier,[34] D. Derkach,[34] J. Firmino da Costa,[34] G. Grosdidier,[34]
F. Le Diberder,[34] A. M. Lutz,[34] B. Malaescu,[34] A. Perez,[34] P. Roudeau,[34] M. H. Schune,[34] J. Serrano,[34] V. Sordini,[34,‡]
A. Stocchi,[34] L. Wang,[34] G. Wormser,[34] D. J. Lange,[35] D. M. Wright,[35] I. Bingham,[36] J. P. Burke,[36] C. A. Chavez,[36]
J. P. Coleman,[36] J. R. Fry,[36] E. Gabathuler,[36] R. Gamet,[36] D. E. Hutchcroft,[36] D. J. Payne,[36] C. Touramanis,[36] A. J. Bevan,[37]
F. Di Lodovico,[37] R. Sacco,[37] M. Sigamani,[37] G. Cowan,[38] S. Paramesvaran,[38] A. C. Wren,[38] D. N. Brown,[39] C. L. Davis,[39]
A. G. Denig,[40] M. Fritsch,[40] W. Gradl,[40] A. Hafner,[40] K. E. Alwyn,[41] D. Bailey,[41] R. J. Barlow,[38] G. Jackson,[41]
G. D. Lafferty,[41] T. J. West,[41] J. Anderson,[42] R. Cenci,[42] A. Jawahery,[42] D. A. Roberts,[42] G. Simi,[42] J. M. Tuggle,[42]
C. Dallapiccola,[43] E. Salvati,[43] R. Cowan,[44] D. Dujmic,[44] P. H. Fisher,[44] G. Sciolla,[44] M. Zhao,[44] D. Lindemann,[45]
P. M. Patel,[45] S. H. Robertson,[45] M. Schram,[45] P. Biassoni,[46a,46b] A. Lazzaro,[46a,46b] V. Lombardo,[46a] F. Palombo,[46a,46b]
S. Stracka,[46a,46b] L. Cremaldi,[47] R. Godang,[47,§] R. Kroeger,[47] P. Sonnek,[47] D. J. Summers,[47] H. W. Zhao,[47] X. Nguyen,[48]
M. Simard,[48] P. Taras,[48] G. De Nardo,[49a,49b] D. Monorchio,[49a,49b] G. Onorato,[49a,49b] C. Sciacca,[49a,49b] G. Raven,[50]
H. L. Snoek,[50] C. P. Jessop,[51] K. J. Knoepfel,[51] J. M. LoSecco,[51] W. F. Wang,[51] L. A. Corwin,[52] K. Honscheid,[52] R. Kass,[52]
J. P. Morris,[52] A. M. Rahimi,[52] N. L. Blount,[53] J. Brau,[53] R. Frey,[53] O. Igonkina,[53] J. A. Kolb,[53] R. Rahmat,[53] N. B. Sinev,[53]
D. Strom,[53] J. Strube,[53] E. Torrence,[53] G. Castelli,[54a,54b] E. Feltresi,[54a,54b] N. Gagliardi,[54a,54b] M. Margoni,[54a,54b]
M. Morandin,[54a] M. Posocco,[54a] M. Rotondo,[54a] F. Simonetto,[54a,54b] R. Stroili,[54a,54b] E. Ben-Haim,[55] G. R. Bonneaud,[55]
H. Briand,[55] G. Calderini,[55] J. Chauveau,[55] O. Hamon,[55] Ph. Leruste,[55] G. Marchiori,[55] J. Ocariz,[55] J. Prendki,[55] S. Sitt,[55]
M. Biasini,[56a,56b] E. Manoni,[56a,56b] C. Angelini,[57a,57b] G. Batignani,[57a,57b] S. Bettarini,[57a,57b] M. Carpinelli,[57a,57b,||]
G. Casarosa,[57a,57b] A. Cervelli,[57a,57b] F. Forti,[57a,57b] M. A. Giorgi,[57a,57b] A. Lusiani,[57a,57c] N. Neri,[57a,57b] E. Paoloni,[57a,57b]
G. Rizzo,[57a,57b] J. J. Walsh,[57a] D. Lopes Pegna,[58] C. Lu,[58] J. Olsen,[58] A. J. S. Smith,[58] A. V. Telnov,[58] F. Anulli,[59a]
E. Baracchini,[59a,59b] G. Cavoto,[59a] R. Faccini,[59a,59b] F. Ferrarotto,[59a] F. Ferroni,[59a,59b] M. Gaspero,[59a,59b] L. Li Gioi,[59a]
M. A. Mazzoni,[59a] G. Piredda,[59a] F. Renga,[59a,59b] M. Ebert,[60] T. Hartmann,[60] T. Leddig,[60] H. Schröder,[60] R. Waldi,[60]
T. Adye,[61] B. Franek,[61] E. O. Olaiya,[61] F. F. Wilson,[61] S. Emery,[62] G. Hamel de Monchenault,[62] G. Vasseur,[62] Ch. Yèche,[62]
M. Zito,[62] I. J. R. Aitchison,[63,¶] M. T. Allen,[63] D. Aston,[63] D. J. Bard,[63] R. Bartoldus,[63] J. F. Benitez,[63] C. Cartaro,[63]
M. R. Convery,[63] J. Dorfan,[63] G. P. Dubois-Felsmann,[63] W. Dunwoodie,[63] R. C. Field,[63] M. Franco Sevilla,[63]
B. G. Fulsom,[63] A. M. Gabareen,[63] M. T. Graham,[63] P. Grenier,[63] C. Hast,[63] W. R. Innes,[63] M. H. Kelsey,[63] H. Kim,[63]
P. Kim,[63] M. L. Kocian,[63] D. W. G. S. Leith,[63] S. Li,[63] B. Lindquist,[63] S. Luitz,[63] V. Luth,[63] H. L. Lynch,[63]
D. B. MacFarlane,[63] H. Marsiske,[63] D. R. Muller,[63] H. Neal,[63] S. Nelson,[63] C. P. O'Grady,[63] I. Ofte,[63] M. Perl,[63]

T. Pulliam,[63] B. N. Ratcliff,[63] A. Roodman,[63] A. A. Salnikov,[63] V. Santoro,[63] R. H. Schindler,[63] J. Schwiening,[63] A. Snyder,[63] D. Su,[63] M. K. Sullivan,[63] S. Sun,[63] K. Suzuki,[63] J. M. Thompson,[63] J. Va'vra,[63] A. P. Wagner,[63] M. Weaver,[63] C. A. West,[63] W. J. Wisniewski,[63] M. Wittgen,[63] D. H. Wright,[63] H. W. Wulsin,[63] A. K. Yarritu,[63] C. C. Young,[63] V. Ziegler,[63] X. R. Chen,[64] W. Park,[64] M. V. Purohit,[64] R. M. White,[64] J. R. Wilson,[64] S. J. Sekula,[65] M. Bellis,[66] P. R. Burchat,[66] A. J. Edwards,[66] T. S. Miyashita,[66] S. Ahmed,[67] M. S. Alam,[67] J. A. Ernst,[67] B. Pan,[67] M. A. Saeed,[67] S. B. Zain,[67] N. Guttman,[68] A. Soffer,[68] P. Lund,[69] S. M. Spanier,[69] R. Eckmann,[70] J. L. Ritchie,[70] A. M. Ruland,[70] C. J. Schilling,[70] R. F. Schwitters,[70] B. C. Wray,[70] J. M. Izen,[71] X. C. Lou,[71] F. Bianchi,[72a,72b] D. Gamba,[72a,72b] M. Pelliccioni,[72a,72b] M. Bomben,[73a,73b] L. Lanceri,[73a,73b] L. Vitale,[73a,73b] N. Lopez-March,[74] F. Martinez-Vidal,[74] D. A. Milanes,[74] A. Oyanguren,[74] J. Albert,[75] Sw. Banerjee,[75] H. H. F. Choi,[75] K. Hamano,[75] G. J. King,[75] R. Kowalewski,[75] M. J. Lewczuk,[75] I. M. Nugent,[75] J. M. Roney,[75] R. J. Sobie,[75] T. J. Gershon,[76] P. F. Harrison,[76] J. Ilic,[76] T. E. Latham,[76] E. M. T. Puccio,[76] H. R. Band,[77] X. Chen,[77] S. Dasu,[77] K. T. Flood,[77] Y. Pan,[77] R. Prepost,[77] C. O. Vuosalo,[77] and S. L. Wu[77]

(*B*A*B*AR Collaboration)

[1]*Laboratoire d'Annecy-le-Vieux de Physique des Particules (LAPP), Université de Savoie, CNRS/IN2P3, F-74941 Annecy-Le-Vieux, France*
[2]*Universitat de Barcelona, Facultat de Fisica, Departament ECM, E-08028 Barcelona, Spain*
[3a]*INFN Sezione di Bari, I-70126 Bari, Italy*
[3b]*Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy*
[4]*University of Bergen, Institute of Physics, N-5007 Bergen, Norway*
[5]*Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA*
[6]*University of Birmingham, Birmingham, B15 2TT, United Kingdom*
[7]*Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany*
[8]*University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1*
[9]*Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom*
[10]*Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia*
[11]*University of California at Irvine, Irvine, California 92697, USA*
[12]*University of California at Riverside, Riverside, California 92521, USA*
[13]*University of California at Santa Barbara, Santa Barbara, California 93106, USA*
[14]*University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA*
[15]*California Institute of Technology, Pasadena, California 91125, USA*
[16]*University of Cincinnati, Cincinnati, Ohio 45221, USA*
[17]*University of Colorado, Boulder, Colorado 80309, USA*
[18]*Colorado State University, Fort Collins, Colorado 80523, USA*
[19]*Technische Universität Dortmund, Fakultät Physik, D-44221 Dortmund, Germany*
[20]*Technische Universität Dresden, Institut für Kern- und Teilchenphysik, D-01062 Dresden, Germany*
[21]*Laboratoire Leprince-Ringuet, CNRS/IN2P3, Ecole Polytechnique, F-91128 Palaiseau, France*
[22]*University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom*
[23a]*INFN Sezione di Ferrara, I-44100 Ferrara, Italy*
[23b]*Dipartimento di Fisica, Università di Ferrara, I-44100 Ferrara, Italy*
[24]*INFN Laboratori Nazionali di Frascati, I-00044 Frascati, Italy*
[25a]*INFN Sezione di Genova, I-16146 Genova, Italy*
[25b]*Dipartimento di Fisica, Università di Genova, I-16146 Genova, Italy*
[26]*Indian Institute of Technology Guwahati, Guwahati, Assam, 781 039, India*
[27]*Harvard University, Cambridge, Massachusetts 02138, USA*
[28]*Universität Heidelberg, Physikalisches Institut, Philosophenweg 12, D-69120 Heidelberg, Germany*
[29]*Humboldt-Universität zu Berlin, Institut für Physik, Newtonstrasse 15, D-12489 Berlin, Germany*
[30]*Imperial College London, London, SW7 2AZ, United Kingdom*
[31]*University of Iowa, Iowa City, Iowa 52242, USA*
[32]*Iowa State University, Ames, Iowa 50011-3160, USA*
[33]*Johns Hopkins University, Baltimore, Maryland 21218, USA*
[34]*Laboratoire de l'Accélérateur Linéaire, IN2P3/CNRS et Université Paris-Sud 11, Centre Scientifique d'Orsay, B. P. 34, F-91898 Orsay Cedex, France*
[35]*Lawrence Livermore National Laboratory, Livermore, California 94550, USA*
[36]*University of Liverpool, Liverpool L69 7ZE, United Kingdom*
[37]*Queen Mary, University of London, London, E1 4NS, United Kingdom*
[38]*University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom*

[39]University of Louisville, Louisville, Kentucky 40292, USA

[40]Johannes Gutenberg-Universität Mainz, Institut für Kernphysik, D-55099 Mainz, Germany

[41]University of Manchester, Manchester M13 9PL, United Kingdom

[42]University of Maryland, College Park, Maryland 20742, USA

[43]University of Massachusetts, Amherst, Massachusetts 01003, USA

[44]Massachusetts Institute of Technology, Laboratory for Nuclear Science, Cambridge, Massachusetts 02139, USA

[45]McGill University, Montréal, Québec, Canada H3A 2T8

[46a]INFN Sezione di Milano, I-20133 Milano, Italy

[46b]Dipartimento di Fisica, Università di Milano, I-20133 Milano, Italy

[47]University of Mississippi, University, Mississippi 38677, USA

[48]Université de Montréal, Physique des Particules, Montréal, Québec, Canada H3C 3J7

[49a]INFN Sezione di Napoli, I-80126 Napoli, Italy

[49b]Dipartimento di Scienze Fisiche, Università di Napoli Federico II, I-80126 Napoli, Italy

[50]NIKHEF, National Institute for Nuclear Physics and High Energy Physics, NL-1009 DB Amsterdam, The Netherlands

[51]University of Notre Dame, Notre Dame, Indiana 46556, USA

[52]The Ohio State University, Columbus, Ohio 43210, USA

[53]University of Oregon, Eugene, Oregon 97403, USA

[54a]INFN Sezione di Padova, I-35131 Padova, Italy

[54b]Dipartimento di Fisica, Università di Padova, I-35131 Padova, Italy

[55]Laboratoire de Physique Nucléaire et de Hautes Energies, IN2P3/CNRS, Université Pierre et Marie Curie-Paris6,
Université Denis Diderot-Paris7, F-75252 Paris, France

[56a]INFN Sezione di Perugia, I-06100 Perugia, Italy

[56b]Dipartimento di Fisica, Università di Perugia, I-06100 Perugia, Italy

[57a]INFN Sezione di Pisa, I-56127 Pisa, Italy

[57b]Dipartimento di Fisica, Università di Pisa, I-56127 Pisa, Italy

[57c]Scuola Normale Superiore di Pisa, I-56126 Pisa, Italy

[58]Princeton University, Princeton, New Jersey 08544, USA

[59a]INFN Sezione di Roma, I-00185 Roma, Italy

[59b]Dipartimento di Fisica, Università di Roma La Sapienza, I-00185 Roma, Italy

[60]Universität Rostock, D-18051 Rostock, Germany

[61]Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

[62]CEA, Irfu, SPP, Centre de Saclay, F-91191 Gif-sur-Yvette, France

[63]SLAC National Accelerator Laboratory, Stanford, California 94309 USA

[64]University of South Carolina, Columbia, South Carolina 29208, USA

[65]Southern Methodist University, Dallas, Texas 75275, USA

[66]Stanford University, Stanford, California 94305-4060, USA

[67]State University of New York, Albany, New York 12222, USA

[68]Tel Aviv University, School of Physics and Astronomy, Tel Aviv, 69978, Israel

[69]University of Tennessee, Knoxville, Tennessee 37996, USA

[70]University of Texas at Austin, Austin, Texas 78712, USA

[71]University of Texas at Dallas, Richardson, Texas 75083, USA

[72a]INFN Sezione di Torino, I-10125 Torino, Italy

[72b]Dipartimento di Fisica Sperimentale, Università di Torino, I-10125 Torino, Italy

[73a]INFN Sezione di Trieste, I-34127 Trieste, Italy

[73b]Dipartimento di Fisica, Università di Trieste, I-34127 Trieste, Italy

[74]IFIC, Universitat de Valencia-CSIC, E-46071 Valencia, Spain

[75]University of Victoria, Victoria, British Columbia, Canada V8W 3P6

[76]Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom

[77]University of Wisconsin, Madison, Wisconsin 53706, USA

We report a direct measurement of D^0-$\bar{D}^0$ mixing parameters through a time-dependent amplitude analysis of the Dalitz plots of $D^0 \to K_S^0 \pi^+ \pi^-$ and, for the first time, $D^0 \to K_S^0 K^+ K^-$ decays. The low-momentum pion π_s^+ in the decay $D^{*+} \to D^0 \pi_s^+$ identifies the flavor of the neutral D meson at its production. Using 468.5 fb^{-1} of $e^+ e^-$ colliding-beam data recorded near $\sqrt{s} = 10.6$ GeV by the *BABAR* detector at the PEP-II asymmetric-energy collider at SLAC, we measure the mixing parameters $x = [1.6 \pm 2.3(\text{stat}) \pm 1.2(\text{syst}) \pm 0.8(\text{model})] \times 10^{-3}$, and $y = [5.7 \pm 2.0(\text{stat}) \pm 1.3(\text{syst}) \pm 0.7(\text{model})] \times 10^{-3}$. These results provide the best measurement to date of x and y. The knowledge of the value of x, in particular, is crucial for understanding the origin of mixing.

Particle-antiparticle mixing and CP violation (CPV) in the charm sector are predicted to be very small in the standard model (SM) [1–5]. Evidence for D^0-$\bar{D}^0$ mixing has been found only recently [6–11] and CPV has not been observed. Although precise SM predictions for D^0-$\bar{D}^0$ mixing are difficult to quantify, recent calculations of the mixing parameters x and y allow for values as large as $\mathcal{O}(10^{-2})$ [1]. The analyses to date that have reported evidence for mixing have not been able to provide direct measurements of x and y. A time-dependent amplitude analysis of the Dalitz plot (DP) of D^0 mesons decaying into $K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$ self-conjugate final states offers a unique way to access the mixing parameters x and y directly. In this Letter we study the time evolution of these three-body decays as a function of the position in the DP of squared invariant masses $s_+ = m^2(K_S^0h^+)$, $s_- = m^2(K_S^0h^-)$, where h represents π or K, and report the most precise single measurements of x and y to date. The knowledge of the value of x, in particular, is crucial for understanding the origin of mixing and for determining whether contributions beyond the SM are present.

We use the complete data sample of 468.5 fb^{-1} recorded near $\sqrt{s} = 10.6$ GeV by the *BABAR* experiment [12] at the PEP-II asymmetric-energy e^+e^- collider. The flavor of the neutral D meson at production is identified through the charge of the π_s^+ ("slow pion") produced in the decay $D^{*+} \rightarrow D^0\pi_s^+$ [13]. The D^0 and $\bar{D}^0$ mesons evolve and decay as a mixture of the Hamiltonian eigenstates D_1 and D_2, with masses and widths m_1, Γ_1 and m_2, Γ_2, respectively. These mass eigenstates can then be written as linear combinations of flavor eigenstates, $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$, where $|p|^2 + |q|^2 = 1$. The mixing parameters are defined as $x = (m_1 - m_2)/\Gamma$ and $y = (\Gamma_1 - \Gamma_2)/2\Gamma$, where $\Gamma = (\Gamma_1 + \Gamma_2)/2$ is the average decay width.

Assuming no CPV in the decay, the relation $\overline{\mathcal{A}}(s_+, s_-) = \mathcal{A}(s_-, s_+)$ holds, where $\mathcal{A}$ and $\overline{\mathcal{A}}$ are the decay amplitudes for a D^0 or a $\bar{D}^0$ into the final state $K_S^0h^+h^-$ as a function of the position in the DP. The time-dependent decay amplitude for a charm meson tagged at $t = 0$ as D^0 or $\bar{D}^0$ can then be written as

$$\mathcal{M}(s_+, s_-, t) = \mathcal{A}(s_+, s_-)g_+(t) + \frac{q}{p}\mathcal{A}(s_-, s_+)g_-(t),$$

$$\overline{\mathcal{M}}(s_+, s_-, t) = \frac{q}{p}\overline{\mathcal{A}}(s_+, s_-)g_+(t) + \overline{\mathcal{A}}(s_-, s_+)g_-(t),$$

where $g_\pm(t) = 1/2[e^{-i(m_1-i\Gamma_1/2)t} \pm e^{-i(m_2-i\Gamma_2/2)t}]$ and $q/p = 1$ if CP is conserved in the mixing amplitude. The decay rates for D^0 and $\bar{D}^0$ are obtained by squaring $\mathcal{M}$ and $\overline{\mathcal{M}}$ respectively, and consist of a sum of terms depending on (s_+, s_-) and proportional to $\cosh(y\Gamma t)$, $\sinh(y\Gamma t)$, $\cos(x\Gamma t)$, and $\sin(x\Gamma t)$, all modulated by the exponential decay factor $e^{-\Gamma t}$. Assuming a model for $\mathcal{A}(s_+, s_-)$, it is possible to extract the mixing parameters x and y from the data, along with the amplitude model parameters and the proper-time resolution function. The

variation of the distribution of the events in the DP as a function of the proper D^0 decay time is the signature of D^0-$\bar{D}^0$ mixing. The sensitivity to x and y arises mostly from regions in the DP where Cabibbo-favored and doubly Cabibbo-suppressed amplitudes interfere and from regions populated by CP eigenstates [14]. This method was pioneered by CLEO [15] and extended to a significantly larger data sample by Belle [16].

The D^0 candidates are reconstructed in the $K_S^0\pi^+\pi^-$ ($K_S^0K^+K^-$) final state by combining K_S^0 candidates with two oppositely charged pions (kaons), with an invariant mass m_{D^0} between 1.824 and 1.904 GeV/c^2. In order to reduce combinatorial background and to remove D^0 candidates from B-meson decays, we require the momentum of the D^0 in the e^+e^- center-of-mass frame to be greater than 2.5 GeV/c. The difference Δm between the D^{*+} and D^0 reconstructed invariant masses is required to satisfy $0.143 < \Delta m < 0.149$ GeV/c^2. Each pion (kaon) track is identified using a likelihood particle identification algorithm based on dE/dx ionization energy loss and Cherenkov angle measurements. The K_S^0 candidates are selected by pairing two oppositely charged pions whose invariant mass is within 9 MeV/c^2 of the nominal K_S^0 mass [17]. We require the cosine of the angle between the K_S^0 flight direction (defined by the K_S^0 production and decay vertices) and the K_S^0 momentum to be greater than 0.99, and a decay length projected along the K_S^0 momentum to be greater than 10 times its error. These selection criteria suppress to a negligible level the background from $D^0 \rightarrow \pi^+\pi^-h^+h^-$ decays. For each charged track we require a transverse momentum with respect to the beam axis to be greater than 100 MeV/c, and for tracks from the D^0 decay we additionally require at least two hits in the two innermost layers of the silicon vertex tracker [12].

The D^0 proper time t, and its error σ_t, are obtained through a kinematic fit of the entire decay chain which constrains the K_S^0 and pion (kaon) tracks to originate from a common vertex and also requires the D^0 and the π_s^+ candidates to originate from a common vertex, constrained by the position and size of the e^+e^- interaction region. This reduces the contribution from $D^0 \rightarrow K_S^0K_S^0$ decays (affecting only $K_S^0\pi^+\pi^-$) to 3% of the total background. We retain candidates for which the χ^2 probability of the fit is greater than 0.01%, $|t| < 6$ ps, and $\sigma_t < 1$ ps. The most probable value for σ_t is about 0.2 (0.3) ps for $K_S^0\pi^+\pi^-$ ($K_S^0K^+K^-$) signal candidates. For events where multiple D^{*+} candidates share one or more tracks, we keep the D^{*+} candidate with the highest χ^2 probability. After applying all selection criteria, we find 744 000 (96 000) $K_S^0\pi^+\pi^-$ ($K_S^0K^+K^-$) candidates. Their m_{D^0} and Δm distributions are shown in Fig. 1 and in [18].

The mixing parameters x and y are determined from an unbinned, extended maximum-likelihood fit to the $K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$ samples over the observables m_{D^0}, Δm, s_+, s_-, t, and σ_t. First, the signal and back-
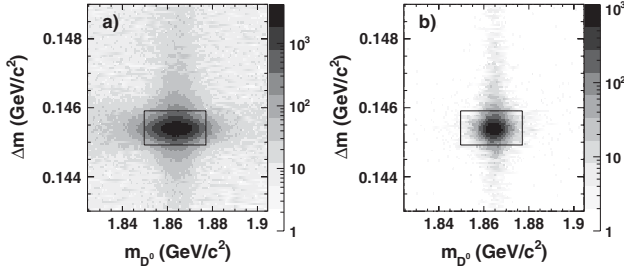
FIG. 1. Distributions of m_{D^0}-Δm for (a) $K_S^0 \pi^+ \pi^-$ and (b) $K_S^0 K^+ K^-$ data after all selection criteria. The gray scale indicates the number of events per bin. The rectangles enclose the signal region.

ground yields are determined from a fit to m_{D^0} and Δm distributions. For the subsequent fits, we restrict events to the signal region illustrated in Fig. 1, defined to lie within twice the measured resolution around the mean m_{D^0} and Δm values, and holding the signal and background yields fixed to their signal window rescaled values. In the mixing fit, the m_{D^0} and Δm shapes are excluded to minimize correlations with the rest of the observables. Our reference fit allows for mixing but assumes no CPV. We then allow for CPV as a cross-check of the mixing results. To avoid potential bias, the mixing results were examined only after the fitting and analysis procedures were finalized.

For each fit stage, different subsamples are characterized separately: $K_S^0 h^+ h^-$ signal, random π_s^+, misreconstructed D^0, $D^0 \to K_S^0 K_S^0$ events (for $K_S^0 \pi^+ \pi^-$ decays only), and combinatorial background. The random π_s^+ component describes correctly reconstructed D^0 decays combined with a random slow pion. Misreconstructed D^0 events have one or more D^0 decay products either missing or reconstructed with the wrong particle hypothesis. We account for $K_S^0 K_S^0$ background events since they exhibit a characteristic DP distribution and a signal-like shape in the variables t and σ_t. Combinatorial background events are those not described by the above components. The functional forms of the m_{D^0}, Δm probability density functions (PDFs) for the signal and background components are chosen based on studies performed on large Monte Carlo (MC) samples. These account for the observed correlations between Δm and m_{D^0} for signal and misreconstructed D^0 events. The PDF parameters are determined from two-dimensional likelihood fits to data over the full m_{D^0} and Δm region, or over dedicated sideband data samples. We find $540\,800 \pm 800$ ($79\,900 \pm 300$) signal events in the $K_S^0 \pi^+ \pi^-$ ($K_S^0 K^+ K^-$) signal region, with purities of 98.5% (99.2%), and reconstruction efficiencies of 14.4% (14.6%). Random π_s^+, misreconstructed D^0, and combinatorial background events account for 23% (53%), 52% (23%), and 22% (24%) of the total background. Projections of these fits and the contributions from the different background components can be found in [18].

The amplitudes $\mathcal{A}(s_+, s_-)$ are described by a coherent sum of quasi-two-body amplitudes [14,19]. The dynamical properties of the P- and D-wave amplitudes are parametrized through intermediate resonances with mass-dependent relativistic Breit-Wigner (BW) or Gounaris-Sakurai (GS) propagators, Blatt-Weisskopf centrifugal barrier factors, and Zemach tensors for the angular distribution [19]. The $\pi\pi$ S-wave dynamics is described through a K-matrix formalism with the P-vector approximation and 5 poles [14,20]. For the $K\pi$ S wave we include a BW for the $K_0^*(1430)^\pm$ with a coherent nonresonant contribution parametrized by a scattering length and effective range similar to those used to describe $K\pi$ scattering data [18,21]. For the $K\bar{K}$ S wave, a coupled-channel BW is used for the $a_0(980)$ isovector with BWs for the $f_0(1370)$ and $a_0(1450)$ states.

We define PDFs to describe the dependence of the components in our event sample upon DP position (s_+, s_-) and upon decay time t. For signal, $|\mathcal{M}|^2$ or $|\overline{\mathcal{M}}|^2$ is convolved with a proper-time resolution function different for $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ events with parameters determined by the mixing fit to the data. The resolution function is a sum of three Gaussians with one of the means allowed to differ from zero (the offset, t_0), and two of the widths proportional to σ_t. While t_0 does not depend sensitively on the DP position, the ability to reconstruct t varies as a function of (s_+, s_-). Hence, the observed distributions of σ_t in a number of DP regions are included in the signal PDF. We apply corrections for efficiency variations and neglect the invariant mass resolution across the DP. The time-dependent PDF for the small random π_s background component is described by an equal combination of D^0 and $\bar{D}^0$ signal events assuming no mixing, since the slow pion is positive or negative with approximately equal probability, and carries little weight in the vertex fit. The PDFs for misreconstructed D^0 events and combinatorial background are determined from m_{D^0} sideband samples. A nonparametric approach is used to construct the DP distributions, while the proper-time distributions are described by a sum of two Gaussian functions, one of which has a power-law tail to account for a small long-lived component. The background components containing real and misreconstructed D^0 decays have different σ_t distributions, which are determined from the signal and m_{D^0} sideband regions.

Results for our nominal mixing fit, in which D^0 and $\bar{D}^0$ samples from $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ channels are combined, are reported in Table I. The proper-time distributions with their fit projections are shown in Fig. 2. Additional fit results and projections can be found in [18]. We evaluate the amplitude model fit to the DP distribution with a χ^2 test with two-dimensional adaptive binning, and obtain $\chi^2 = 10\,429.2$ (1511.2) for 8626–41 (1195–17) degrees of freedom (ndof), for $K_S^0 \pi^+ \pi^-$ ($K_S^0 K^+ K^-$). MC studies show that a significant contribution to these χ^2 values, $\Delta\chi^2/\mathrm{ndof} \approx 0.16$, arises from

TABLE I.   Results from the mixing fits. The first uncertainty is statistical, the second systematic and the third systematic from the amplitude model. For the nominal fit, the corresponding correlation coefficients between x and y are 3.5%, 16.0%, and -2.7%, respectively.

Fit type	$x/10^{-3}$	$y/10^{-3}$
Nominal	$1.6 \pm 2.3 \pm 1.2 \pm 0.8$	$5.7 \pm 2.0 \pm 1.3 \pm 0.7$
$K_S^0 \pi^+ \pi^-$	2.6 ± 2.4	6.0 ± 2.1
$K_S^0 K^+ K^-$	-13.6 ± 9.2	4.4 ± 5.7
D^0	0.0 ± 3.3	5.5 ± 2.8
$\bar{D}^0$	3.3 ± 3.3	5.9 ± 2.8

imperfections in modeling experimental effects, mostly the efficiency variations at the boundaries of the DP and the invariant mass resolution. The fitted average lifetime $\tau = 1/\Gamma$ is found to be consistent with the world average lifetime [17], while t_0 is found to be 5.1 ± 0.8 fs (5.1 ± 2.2 fs) for the $K_S^0 \pi^+ \pi^-$ ($K_S^0 K^+ K^-$) mode, consistent with expectations from small misalignments in the detector [10].

A variety of studies using large MC samples with both parametrized and full detector simulations and data have been performed to validate the analysis method and fitting procedure and to check the consistency of the results. These studies demonstrate that the analysis correctly determines the mixing parameters with insignificant biases and well-behaved Gaussian errors. No significant variations of the mixing parameters are observed as a function of momentum, polar, and azimuthal angles of the D^0 meson, and data taking period. Including the m_{D^0}, Δm PDFs in the mixing fit does not significantly change the values for x and y. The mixing fit has also been performed separately for the $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ data samples, and for D^0 and $\bar{D}^0$ decays, with the results listed in Table I. Fitting separately for D^0 and $\bar{D}^0$ provides a check against possible effects from CPV in mixing and in decay. Finally, if we fit the data forcing the decay amplitudes for D^0 and $\bar{D}^0$ to be the same (no direct CPV), but allowing their x and y values to differ, we find these values to be consistent (i.e., no evidence for CPV in mixing).

Systematic uncertainties arise from approximations in the modeling of experimental and selection criteria effects [18]. We account for variations in the signal and background yields, the efficiency variations across the DP, the modeling of the DP and proper-time distributions for events containing misreconstructed D^0 decays, the misidentification of the D^0 flavor for signal and random π_s^+ events, potential effects due to mixing in the random π_s^+ background component, and PDF normalization. We also consider variations of the resolution function and σ_t PDFs, including alternatives to describe the correlation between σ_t and the DP position (e.g., neglecting the dependence of the σ_t distributions on the DP position entirely). The dominant sources of experimental systematic uncertainty are the limited statistics of full detector simulations (used to study potential biases due to the event selection, invariant mass resolution, residual correlations between the fit variables, and fitting procedure) and instrumental effects arising from the small misalignment of the detector. Effects from our selection criteria are estimated by varying the m_{D^0}, Δm, t, and σ_t requirements.

Assumptions in the amplitude models are also a source of systematic uncertainty [14,18]. We use alternative models where the BW parameters are varied according to their uncertainties or changed to values measured by other experiments, the reference K-matrix solution [14] is replaced by other solutions [20], and the standard parametrizations are substituted by other related choices. These include replacing the GS by BW line shapes, removing the mass dependence in the P vector [22], changes in form factors such as variations in the Blatt-Weisskopf radius and the effect of evaluating the momentum of the spectator particle in the D^0 meson frame rather than in the resonance rest frame, and adopting a helicity formalism [19] to describe the angular dependence. Other models are built by removing or adding resonances with small or negligible fractions. The largest effect arises when the uncertainties in the amplitude model parameters obtained from the fit to the DP variables only [18] are propagated to the mixing fit. These uncertainties are dominated by the parameters related to the $K\pi$ S and P waves.

The mixing significance is evaluated by the variation of the negative log-likelihood ($-2\Delta \ln \mathcal{L}$) in the mixing parameter space. We account for the systematic uncertainties by approximating $\mathcal{L}$ as a two-dimensional Gaussian with covariance matrix resulting from the sum of the corresponding statistical, systematic, and amplitude model matrices. Figure 3 shows the confidence-level (C.L.) contours in two dimensions (x and y) with systematic uncertainties included. The variation in $-2\Delta \ln \mathcal{L}$ for the no-mixing point is 5.6 units which corresponds to a C.L. equivalent to 1.9 standard deviations, including the systematic uncertainties.
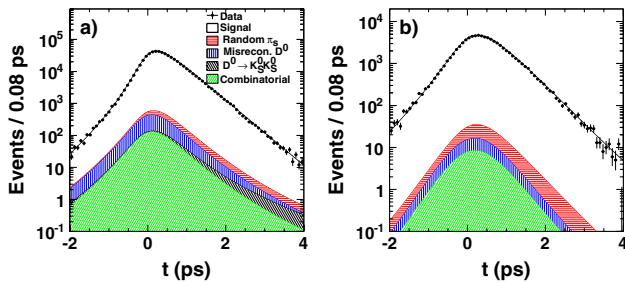

FIG. 2 (color online). Proper-time distributions for (a) $K_S^0 \pi^+ \pi^-$ and (b) $K_S^0 K^+ K^-$ data in the signal region with $-2 < t < 4$ ps (points). The curves show the fit projections for signal plus background (solid lines) and for different background components (shaded regions).
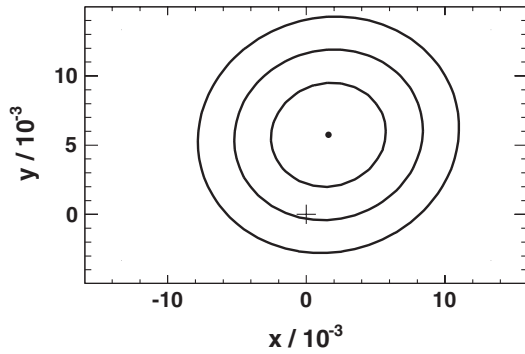
FIG. 3.   Central value (point) and C.L. contours (including statistical, systematic, and amplitude model uncertainties) in the x-y plane for C.L. = 68.3%, 95.4%, 99.7%. The no-mixing point is shown as a plus sign ($+$).

In summary, we have directly measured the mixing parameters $x = [1.6 \pm 2.3(\text{stat}) \pm 1.2(\text{syst}) \pm 0.8(\text{model})] \times 10^{-3}$, and $y = [5.7 \pm 2.0(\text{stat}) \pm 1.3(\text{syst}) \pm 0.7(\text{model})] \times 10^{-3}$, using, for the first time, a combined analysis of $D^0 \to K^0_S \pi^+ \pi^-$ and $D^0 \to K^0_S K^+ K^-$ decays. These results are consistent with the previous similar measurements of $K^0_S \pi^+ \pi^-$ alone [15,16] and have improved precision. They disfavor the no-mixing hypothesis with a C.L. equivalent to 1.9 standard deviations and are in agreement with the range of SM predictions [1–5]. Our measurements favor lower values for x than for y, and lower x and y values (but still consistent) than those obtained when combining results from other D^0 decays [6–11,23]. Adding our results to the combination of all previous analyses significantly improves our current knowledge of the mixing parameters x and y, whose average values change from $(9.8 \pm 2.5) \times 10^{-3}$ and $(8.3 \pm 1.6) \times 10^{-3}$ to $(5.9 \pm 2.0) \times 10^{-3}$ and $(8.0 \pm 1.3) \times 10^{-3}$, respectively [24].

*Present address: Temple University, Philadelphia, PA 19122, USA.

†Also at Università di Perugia, Dipartimento di Fisica, Perugia, Italy.

‡Also at Università di Roma La Sapienza, I-00185 Roma, Italy.

§Present address: University of South Alabama, Mobile, AL 36688, USA.

‖Also at Università di Sassari, Sassari, Italy.

¶Also at University of Oxford, Theoretical Physics Department, Oxford, OX1 3NP, United Kingdom.

[1] A. A. Petrov, Int. J. Mod. Phys. A **21**, 5686 (2006).

[2] A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, Phys. Rev. D **65**, 054034 (2002).

[3] I. I. Y. Bigi and N. G. Uraltsev, Nucl. Phys. **B592**, 92 (2001).

[4] L. Wolfenstein, Phys. Lett. B **164**, 170 (1985).

[5] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir, and A. A. Petrov, Phys. Rev. D **69**, 114021 (2004).

[6] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. **98**, 211802 (2007).

[7] M. Staric *et al.* (Belle Collaboration), Phys. Rev. Lett. **98**, 211803 (2007).

[8] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 121802 (2008).

[9] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. D **78**, 011105 (2008).

[10] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. D **80**, 071103 (2009).

[11] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. **103**, 211801 (2009).

[12] B. Aubert *et al.* (BABAR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **479**, 1 (2002).

[13] Reference to the charge-conjugate state is implied here and throughout the text unless otherwise stated.

[14] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. D **78**, 034023 (2008).

[15] D. M. Asner *et al.* (CLEO Collaboration), Phys. Rev. D **72**, 012001 (2005).

[16] L. M. Zhang *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 131803 (2007).

[17] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[18] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.105.081803 for additional mixing fit results and figures, and breakdown of systematic uncertainties.

[19] See review on Dalitz plot analysis formalism in [17].

[20] V. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A **16**, 229 (2003).

[21] D. Aston *et al.*, Nucl. Phys. **B296**, 493 (1988); W. Dunwoodie (private communication).

[22] I. J. R. Aitchison, Nucl. Phys. **A189**, 417 (1972).

[23] J. L. Rosner *et al.* (CLEO Collaboration), Phys. Rev. Lett. **100**, 221801 (2008).

[24] A. Schwartz *et al.* (Heavy Flavor Averaging Group), http://www.slac.stanford.edu/xorg/hfag/charm/index.html.