Oscillon Resonances and Creation of Kinks in Particle Collisions

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We present a numerical study of the process of production of kink-antikink pairs in the collision of particlelike states in the one-dimensional ϕ^4 model. It is shown that there are 3 steps in the process: The first step is to excite the oscillon intermediate state in the particle collision, the second step is a resonance excitation of the oscillon by the incoming perturbations, and, finally, the soliton-antisoliton pair can be created from the resonantly excited oscillon. It is shown that the process depends fractally on the amplitude of the perturbations and the number of perturbations. We also present the effective collective coordinate model for this process.

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Introduction.—Nonlinear field theories in the weak coupling regime usually contain two different mass scales associated with the perturbative particlelike states and with the soliton sector of the model, respectively. Conjecture about the role nonperturbative effects, related with the production of the solitonlike states in particle collisions, may play in high energy physics [1] has attracted a lot of attention recently. Over the past two decades, the problem of the transition between perturbative and nonperturbative sectors of the theory has been considered in several contexts.

The simplest example of the topological solitons in one dimension is the kink solution of the ϕ^4 model. Dynamical properties of kinks, the processes of their scattering, radiation, and annihilation have already been discussed in a number of papers; see, e.g., [2–9]. In integrable theories, like the sine-Gordon model, there is no energy loss to radiation and kinks do not annihilate antikinks. However, in the nonintegrable ϕ^4 model, the radiation effects in the process of kink-antikink ($K\tilde{K}$) collision become very important, and, depending on the impact velocity, the collision may produce various results; e.g., an oscillating bound state can be formed, and also the soliton and antisoliton may bounce and reflect from each other.

Although the process of annihilation of the $K\tilde{K}$ state of the ϕ^4 model has been investigated in detail [4,6,9], there is not much information about the inverse process, the creation of the $K\tilde{K}$ pairs by the collision of two identical bunches of particles. In the recent work [10], production of a $K\tilde{K}$ pair was considered in the assumption that two colliding wave trains are composed of the bunches of identical breathers, i.e., tightly coupled $K\tilde{K}$ states. Evidently, the kink-antikink production may proceed even in the case when there are no kinklike states in the initial configuration at all. Here we aim to elucidate the mechanism for this process.

It is known that the collision of a kink and an antikink is chaotic; i.e., for some values of the impact velocity the solitons bounce back, while for some different impact velocity, smaller or larger, they annihilate [9,11]. This behavior is related with a resonance effect between the oscillations of the $K\tilde{K}$ pair and excitation of the discrete vibrational mode of the kink.

So we might expect the opposite process of the production of the $K\tilde{K}$ pairs in the collision of particles will also have a similar fractal character due to the resonance effect between the oscillon created in the particle collision and the oscillation of the correlated $K\tilde{K}$ pair.

Here we investigate the oscillon resonance numerically. We find that this resonance excitation plays a crucial role during the process of creation of the $K\tilde{K}$ pairs in the collision of particles. We observe furthermore the fractal structure of this process.

The model.—We consider the standard one-dimensional ϕ^4 theory, with two vacua $\phi = \pm 1$, defined by the rescaled Lagrangian density

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}(\phi^2 - 1)^2. \quad (1)$$

The perturbative sector of the model consists of small linear perturbations around one of the vacua with the mass $m = 2$. The static kink solution for this model interpolates between the vacua $\phi_0 = -1$ and $\phi_0 = 1$ as x increases from $-\infty$ to ∞: $\phi_K(x, t) = \tanh x$.

This simple model arises in many contexts. It has a number of applications in condensed matter physics [12], and its statical limit appears as a phenomenological theory of second-order phase transitions [13]. It has been used as a model of the displacive phase transitions [14], especially in the case of uniaxial ferroelectrics [15], or as a phenomenological theory of the nonperturbative transition in a polyacetylene chain [16]. In condensed matter physics, it has been used to describe solitary waves in shape-memory alloys [17]. In a cosmological context, it is used to model dynamics of the domain walls [18]. Furthermore, this model has been applied in biophysics to describe soliton excitations in DNA double helices [19]. In quantum field theory, it is used as a model example to investigate transition between perturbative and nonperturbative sectors of the theory [1]; it is also a model of quantum mechanical instanton transitions in a double-well potential [20].

There is a lot of similarity between the nonintegrable model (1) and its integrable sine-Gordon counterpart. However, the states of the perturbative sector are different in these theories. Evidently, in both models there are zero translational modes in the spectrum of the linear perturbation about the kinks, but a single ϕ^4 kink has in addition a normalizable discrete vibrational mode, which oscillates harmonically with frequency $\omega_1 = \sqrt{3}$. The continuum modes on the kink background have higher frequencies $\omega > 2$. Evidently, if the amplitude of the oscillation is large enough, such a periodically expanding and contracting kink can be treated as a kink-antikink-kink bound state, and this excitation can be considered as an intermediate step in the process of creation of the $K\tilde{K}$ pair on the kink background [5,8].

Another situation is related to the possibility of production of the $K\tilde{K}$ pairs on the trivial background. Indeed, the linear excitation spectrum around the trivial vacuum contains the radiation modes, and, within the ϕ^4 model, the collision of these particlelike states may produce $K\tilde{K}$ pairs [21].

Note that nonlinear field theories usually contain several types of topological and nontopological excitations. Indeed, besides the solitonic configurations there is another spatially localized nonperturbative oscillon solution which, although unstable, is extremely long-lived [22–24]. The oscillon states naturally appear in various models [25–27].

In the ϕ^4 model the oscillon solutions are almost periodic. One can find the oscillon numerically by solving the field equation in the Fourier series in time:

$$\phi = 1 + \eta_0(x) + \eta_1(x)\cos(\Omega t) + \eta_2(x)\cos(2\Omega t) + \cdots. \tag{2}$$

If $\Omega < m = 2$, the oscillations are below the threshold and cannot propagate as modes of the continuum, so the oscillon remains relatively stable and the η_1 term dominates.

It was pointed out recently that an oscillation mode of the ϕ^4 model may decay into a $K\tilde{K}$ pair [10].

Numerical results.—The initial data used in our simulations represent two widely separated identical wave trains propagating from both sides on the trivial background towards a collision point:

$$\phi(x, t) = 1 + C[F(x + vt)\sin(\omega t + kx) + F(x - vt)\sin(\omega t - kx)], \tag{3}$$

where k is the wave number of the incoming wave, $\omega = \sqrt{k^2 + 4}$ is the frequency, and $v = k/\omega$ is the velocity of propagation of the wave train. We consider the envelope of the train $F(x) = [\tanh(x - a_1) - \tanh(x - a_2)]$; also the Gaussian envelope $F(x) = e^{-(x-a_3)^2}$ was used to prove that our results are independent of the particular choice of the initial state. The parameters a_1, a_2, and a_3 define the length of the train and the initial separation between the trains. Typically, we used the values $a_1 = 10$, $a_2 = 30$, and $a_3 = 20$. The amplitude C and the wave number k are the impact parameters, which can be changed freely. To find a numerical solution of the partial differential equation (PDE) describing the evolution of the system, we used the pseudospectral method on a discrete grid containing 1024 nodes with periodic boundary conditions. For the time-stepping function we used the symplectic (or geometric) integrator of fourth order to ensure that the energy is conserved. The time and the spatial steps are $\delta x = 0.05$ and $\delta t = 0.025$, respectively, so the numerical errors scale as $(\delta t)^4$. We have tested our method by changing δx and δt.

In our numerical analysis we found that after small amplitude collisions, the two wave trains separate and move in opposite directions, and the radiation is created due to the interaction between these trains. In the center of collision an oscillating lump remains. For small amplitudes, the frequency of the oscillation is just a bit above the mass threshold. This indicates that the lump could be identified with low wave-number linear excitation of the trivial vacuum. For large amplitude collisions, the remaining lump oscillates with frequency within the mass gap, so such a state can be identified as an oscillon.

Furthermore, for a certain range of values of the impact parameters C and k, we observed creation of the $K\tilde{K}$ pairs. During this process also an oscillon is created in the collision center (Fig. 1). The most important feature of this process is that, in the space of parameters, the regions of creation of the solitons and the regions where this process in not taking place are separated by a fractal-like boundary (Fig. 2). Indeed, this diagram is made of elementary plaquettes (boxes). In our calculations, we typically used the pixel resolution 600×600. If N is a number of boxes covering the boundary between the regions of creation and the trivial sector and l is the side length of the boxes, the box-counting (fractal) dimension is defined as the ratio $d = -\lim_{l\to 0} \log N/\log l$. For finite wave trains we do not expect this boundary would be a real fractal, but some properties of scaling are observed. We have mea-

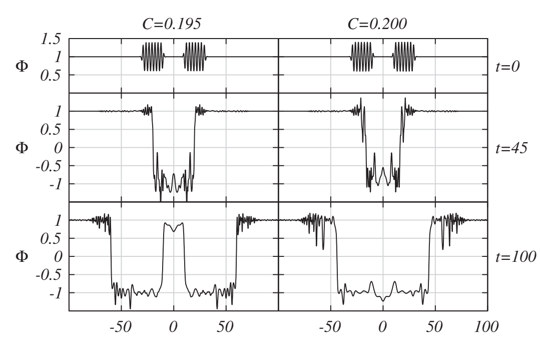

FIG. 1. Production of the kinks in the collision of two identical wave trains. The initial and final field configurations are plotted at $t = 0$, $t = 45$, and $t = 100$, respectively.

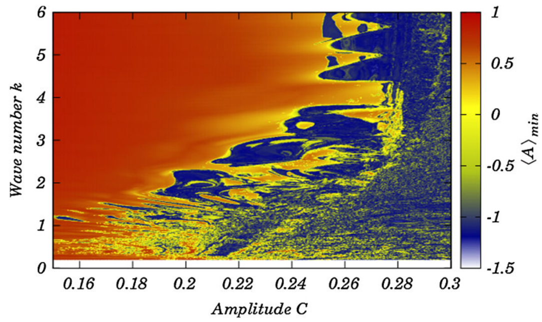

FIG. 2 (color online). Fractal structure in the C, k plane. Shading (or color) represent the measured minimum of average of the field $\langle A\rangle = \frac{1}{20}\int_{-10}^{10} dx\phi(x,t)$. The dark regions (blue in color) where $\langle A\rangle < -1$ indicate creation of the $K\tilde{K}$ pairs.

sured the fractal dimension to be $d = 1.770 \pm 0.011$, which is much more than 1. In the case of the Gaussian envelope we found $d = 1.865 \pm 0.007$. The interesting peculiarity of the latter case is that the $K\tilde{K}$ pairs can be created even if $k = 0$ (standing wave perturbation).

For certain values of impact parameters, an oscillon remaining in the collision center decays into the second $K\tilde{K}$ pair. Sometimes the second pair moves even faster than the first pair, and it may annihilate with the first one, creating two moving oscillons. We know that the process of a collision $K\tilde{K}$ pair also leads to fractal structure in the velocity space [4,9]. In our process, instead of creation on a $K\tilde{K}$ pair, two oscillons could also be ejected from the collision center, and after a while they could decay into two pairs of $K\tilde{K}$. We observed some evidence that these processes also yield the fractal dependency of impact parameters. This study will be reported elsewhere.

Effective collective coordinate model.—In order to capture the most important steps in the process of the creation of a $K\tilde{K}$ pair, in the collision of two identical bunches of particles, we use the collective coordinate method, which allows us to identify the physical degrees of freedom of the system under consideration. This approach has been applied to describe the dynamics of the kink-antikink system [28].

First, we describe the process of creation of the oscillon in the collision of the incoming wave trains. We assume an initial field configuration on the trivial background

$$\phi(x,t) = 1 + \frac{A(t)}{\cosh(x/x_0)} + \phi_K[X(t)] + \phi_{\tilde{K}}[X(t)] + \xi(x,t), \quad (4)$$

where $X(t)$ is the usual translational collective coordinate of the kink [28] and the variable $A(t)$ is introduced as the collective coordinate of the oscillon [6,23] with the parameter x_0 representing the oscillon width. The perturbation ξ should represent two wave trains coming from $\pm\infty$. For the sake of simplicity, we take the perturbation of the form (3). The Gaussian approximation to the oscillon configuration [24] was also used to check the results. From the expansion (2) we know that when $\xi = 0$ the oscillon should, in the first approximation, oscillate as $A(t) = A_0\cos(\Omega t)$, where A_0 is the amplitude of the oscillations, $\Omega < 2$, and the value of the parameter x_0 depends on the amplitude A_0. In the presence of the external field ξ, the amplitude of the oscillon changes. However, for the sake of simplicity we set $x_0 = 1.5$ as it is the width of the oscillon oscillating with amplitude $A_0 = 0.4$. Substituting the explicit form of the kink solution ϕ_K and perturbation (4) into (1) and after integration over all space x gives the effective Lagrangian, which can be split into six parts which describe the dynamics of free kinks, the free oscillon, the perturbations ξ, and the corresponding interactions between the kinks and the oscillon, between the kinks and the perturbation, and the interaction between the oscillon and the perturbation, respectively. Truncating this model with respect to kink collective coordinates yields the effective Lagrangian of the oscillon-perturbation system

$$L(A,\dot{A}) = L_A + L_\xi + L_{\rm int}. \quad (5)$$

The Lagrangian of the free oscillon has the form

$$L_A/x_0 = (\dot{A})^2 - \frac{2}{3}A^4 - \pi A^3 - \left(4 + \frac{1}{3x_0^2}\right)A^2. \quad (6)$$

This is the Lagrangian of an anharmonic oscillator with frequency $\Omega_0 = \sqrt{4 + (1/3x_0^2)} > 2$. Since the frequency of the oscillon must be smaller than $m = 2$, the amplitude of the oscillations must be large enough to decrease the oscillation frequency below the mass threshold [24,29], so the nonlinearities are crucial for the existence of the oscillon. We assume that the field ξ is a solution to the equation of motion of the Lagrangian L_ξ. In the last part of the Lagrangian we take only the linear term in ξ:

$$L_{\rm int} = \alpha(t)A(t) + \beta(t)A^2 + \gamma(t)\dot{A} + \mathcal{O}(A^3) + \mathcal{O}(\xi^2), \quad (7)$$

where $\alpha(t) = \int dx \frac{4\xi}{\cosh(x/x_0)} + \frac{\xi_x \sinh(x/x_0)}{x_0\cosh^2(x/x_0)}$, $\beta(t) = -\int \frac{6\xi dx}{\cosh^2(x/x_0)}$, and $\gamma(t) = \int \frac{\xi_t dx}{\cosh(x/x_0)}$, although we have also investigated the effect of the higher order terms. Because ξ is an oscillating function over a compact support, the above Lagrangian along with L_A describes dynamics which is similar to the dynamics governed by Mathieu's equation [especially the term proportional to $\beta(t)$] but with additional nonlinear terms and a source term [proportional to $\alpha(t)$ and $\gamma(t)$]. We have studied the dynamics of the system numerically. Unfortunately, we could not find the analytic form of the integrals $\alpha(t)$, $\beta(t)$, and $\gamma(t)$, so we used numerical methods which are on the same level of complexity as the explicit solution of the underlying PDE. However, within the collective coordinate approach we can separate the most important degrees of freedom and show that the corresponding nonlinear interaction is responsible for generation of the fractal structure.

The initial condition is that $A(0) = 0$. As the wave train approaches the point of the collision, the oscillon mode is

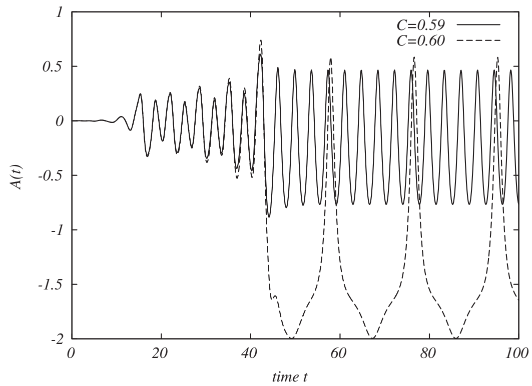

FIG. 3. Resonance of the amplitude of the oscillon (dashed line) within the effective model.

excited. If the amplitude of the perturbation is relatively small, then the oscillon, created in the collision, oscillates with a constant amplitude around $A = 0$. However, if the amplitude is large enough, or the incoming perturbations are close to one of the (Mathieu) resonances, the amplitude of the oscillon rapidly increases and it starts to oscillate around $A = -1$ (or, in other words, around $\phi = 0$) with amplitude of order 1, as illustrated in Fig. 3. This clearly breaks our effective approach, but it also means that the system has changed the ground state. Such a resonant oscillation with a large amplitude, on the other hand, shifts the center of the oscillation. Evidently, coupling of the oscillon collective variable $A(t)$ with the kink collective coordinate $X(t)$ in the complete effective Lagrangian, which is given by the terms $F[X(t)]A(t) + G[X(t)]\dot{A}\dot{X}$ with some functions $F(X)$ and $G(X)$, in the resonance case is much stronger than other linear perturbations, so this transition can be related with excitation of the collective coordinate $X(t)$ related with creation of the $K\tilde{K}$ pairs.

Again, when we examined this effective model, we found a fractal structure on the plane A, k. This fractal structure is less complicated and more localized than in the case of the full PDE. That means that, although our effective model works and captures qualitatively the most important features of the full system, it also fails to reproduce some of the details, which is not a surprise for such a complicated dynamical process. We have also introduced an approximation for $\alpha(t)$, $\beta(t)$, and $\gamma(t)$, and again, we could reproduce both the resonance excitation of the oscillon and the generation of the fractal structure.

This result shows that even after performing so many simplifications we could reproduce (at least qualitatively) the most important features of the evolution of the system. This result confirms our conjecture about the mechanism of the creation of the $K\tilde{K}$ pair in a 3-stage process (i.e., excitation of the oscillon, resonance, and oscillon decay into the $K\tilde{K}$ pair). Second, we conclude that the interaction between the incoming wave trains and the oscillon is the underlying reason for the generation of the fractal structure. Third, given the generality of our approach, we expect that the effective nonlinear interactions of the same type can be found in many different models, so the fractal structure should not be limited only to the case of the ϕ^4 model. We expect that other topological defects, like vortices, hopfions, or monopoles, can be created in the process of the same type, i.e., via resonance excitation of an oscillon in the particle collision.

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