Percolation Simulation of Laser-Guided Electrical Discharges

Akira Sasaki
Quantum Beam Science Directorate, Japan Atomic Energy Agency, 8-1 Umemidai, Kizukawa-shi, Kyoto 619-0215, Japan

Yasuaki Kishimoto
Faculty of Energy Science, Kyoto University, Yoshidahonnmachi, Sakyo-ku, Kyoto, 606-8501, Japan

Eiichi Takahashi and Susumu Kato
National Institute of Advanced Industrial Science and Technology (AIST), 1-1-1 Umezono, Tsukuba, Ibaraki, 305-8568, Japan

Takashi Fujii
Electric Power Engineering Research Laboratory, Central Research Institute of Electric Power Industry,
2-6-1 Nagasaka, Yokosuka, Kanagawa 240-0196, Japan

Seiji Kanazawa
Faculty of Engineering, Oita University, 700 Dannoharu,Oita-shi, 870-1192, Japan

(Received 3 March 2010; published 12 August 2010)

A three-dimensional simulation of laser-guided discharges based on percolation is presented. The model includes both local growth of a streamer due to the enhanced electric field at the streamer's tip and propagation of a leader by remote ionization such as that caused by runaway electrons. The stochastic behavior of the discharge through a preformed plasma channel is reproduced by the calculation, which shows complex path with detouring and bifurcation. The probability of guiding is investigated with respect to the ionized, conductive fraction along the channel.

DOI: 10.1103/PhysRevLett.105.075004 PACS numbers: 52.80.Mg, 47.54.−r, 51.50.+v

Triggering and guiding discharge using an ultrashort-pulse laser has been attracting attention [1,2]. A laser pulse can propagate over long distances forming a plasma channel so that discharges over several meters have been demonstrated [3,4]. For example, Fujii *et al.* performed an experiment in which a laser channel was created between a rod and sphere electrodes [5]. Discharge along the channel was observed when voltages up to 440 kV were applied across a 1 m gap. Modeling such laser-guided discharges has essential importance for the development of applications such as lightning protection and control.

Stochastic models of the discharge including those based on percolation [6] have been studied, and the growth of streamers [7–9] has been investigated. Recently, hybrid particle and hydrodynamics simulations [10,11] have reproduced the structure of streamers and identified that their growth is caused by the local acceleration of electrons and ionization of air at the tip of the streamers [12].

We present a simulation model of discharge based on the percolation. We show that the model reproduces stochastic behaviors of the laser-guided discharge, by including both local ionization at the tip of streamers and random remote ionization such as that caused by runaway electrons.

Propagation of an ultrashort-pulse laser forms a plasma channel through the nonlinear Kerr self-focusing effect [2]. However, due to refraction caused by the radial electron density profile of the channel, the laser filament repeats focusing and expansion along its propagation path. Therefore, a plasma channel with an intermittent structure along the laser filament is produced.

It is shown that discharge through the laser channel requires a reduced albeit still large applied electric field. The voltage also has a considerable statistical variation. The propagation speed of the discharge is only of the order of 10^6 m/s [5,13]. These observations indicate that a single continuous channel is not always formed, even though breakdown of the gaps is expected to occur immediately as the electric field is significantly enhanced in the gaps.

On the other hand, the discharge sometimes occurs through a detouring path away from the laser channel. Such stochastic behaviors have been observed in atmospheric discharges over long distances where a leader channel is formed [14,15].

Recent studies suggest that the behaviors of leaders arise from an effect of runaway electrons [16]. Gamma and x rays have been detected, originating from runaway electrons accelerated by the electric field in thunderclouds [17]. Besides those originating from cosmic rays, the runaway electrons may be produced at the tip of streamers [18]. In the case of laser-guided discharge, the electric field is significantly enhanced in the short gap between filaments, which could accelerate electrons to sufficiently high energy to travel long distances (>10 cm), to cause ionization of air away from the channel. As shown in a Monte Carlo simulation [19], in the case of ionization caused by runaway electrons, there is no correlation between successive spatial points of ionization. Thus, we develop a model of discharge taking the random remote ionization into account.

The effect of remote ionization has also been demonstrated by the particle-in-cell simulations. Kishimoto and Masaki [20] showed discharge in Ne gas under a strong applied electric field with a small spot of initial ionized region. Initially, a streamerlike structure grew gradually from the spot. Eventually a flashover occurs with a large current over the entire region creating a netlike structure. It has been suggested that avalanches occur randomly in the air due to runaway electrons, and that the breakdown occurs when there is a sufficiently large number of spots where adjacent ionized regions become connected to each other.

In this study, we apply the percolation model [21] to an air gap between a pair of planar electrodes, as illustrated in Fig. 1. Three-dimensional meshes are used to partition the gap into cells. Whether the air in each cell is ionized or not is determined according to its resistivity. To calculate the discharge current, a network of resistors is considered, which connects to the center of the cell [22]. The current for each resistor and the potential for each cell are calculated using Kirchhoff's current law. If two adjacent cells are determined to be ionized, the resistor between them is assigned a resistivity that is 10^{-6} times smaller than the resistivity of the surrounding air. We note that calculation results are insensitive to the resistivity of the ionized region, provided it is sufficiently lower than that of air.

At each step, one cell is ionized according to the relative probabilities of the cells. Once ionized, the cells are assumed to maintain their state. The current, potential, and electric field strength inside each cell are updated after each ionization event.

We modify the probability [23] of the ionization p with the electric field strength E according to $p \propto E^{\alpha}$, as in the percolation model of conduction [6] and discharge simulation [9]. This allows one to take both the local and remote ionization into account in the present model.

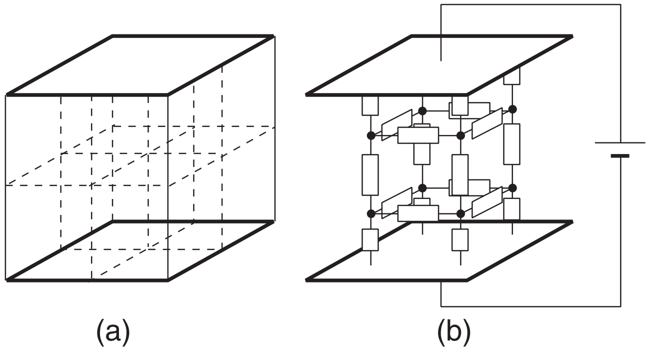

FIG. 1. Models of (a) a $2 \times 2 \times 2$ spatial cell and (b) the corresponding resistor network.

An example of the development of a discharge is shown in Fig. 2. This simulation was performed using $21 \times 21 \times 22$ cells. In this case the probability of ionization is assumed to be uniform by setting $\alpha = 0$. One sees that a current appears suddenly when the ionized fraction exceeds some threshold value. Because of the small number of cells, the threshold value has a statistical variation. But the averaged value for several calculations approaches 0.31, in agreement with the result of a detailed calculation [21].

Figure 2 clearly shows that the discharge path has a complex structure. It is also found that a new path appears one after another as the ionized fraction increases. For example, at an ionized fraction of 0.32, a new path merges with the original path to create a treelike structure. This suggests that bifurcation of streamers occurs not only by splitting of their tips but also by the reconnection of existing paths [24]. As the ionized fraction increases further, the number of paths increases rapidly and the whole region is filled with the channels. This filling may be partly due to a dc voltage that is applied, and additional heating and subsequent change of conductivity is not taken into account in the present calculation.

Next, we investigate the properties of laser-guided discharge. Calculations are carried out with initial ionized cells placed along the preformed channel, connecting centers of left and right electrodes. Experiments [25] and analyses of the Kerr self-focusing [5] show that the typical

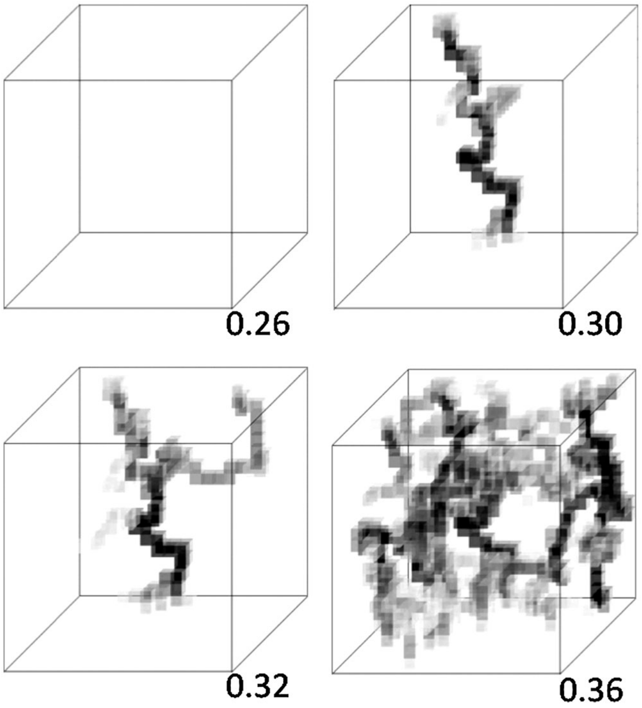

FIG. 2. Development of discharge due to random ionization. The number next to each graph is ionized fraction, the ratio of the ionized region of the air. The degree of darkening in the plots qualitatively indicates the amount of current.

length of a filament is of the order of 10 cm. We model those experimental conditions by placing a sequence of short ionized regions with one-cell spaces between them, such that the initial ionized fraction along the channel is 75%, as shown in Fig. 3(a). In the present calculation, having the edge of the cell at 10 cm, the calculation corresponds to the laser-guided discharge over a 2.2 m long gap with six of 30 cm long filaments.

In this calculation, the probability of ionization is scaled by the local electric field using $\alpha = 2$, because preliminary calculations have shown similar discharge patterns to those observed experimentally. Using $\alpha = 0$, ionization occurs randomly; thus, no noticeable effect of the preformed channel is seen. On the other hand, for large values of α ionization at the tip of streamers dominates. In the case of $\alpha \geq 2.5$, the discharge almost always occurs along the channel, because ionization immediately closes the gap between the filaments.

Figures 3(b)–3(d) show examples of the calculated discharge paths chosen from 150 runs. Guiding is found to be successful at a probability of 84%, where the discharge along the initial channel is obtained. The percolation threshold decreases by 50%, when a channel is added, from 0.18 with no channel to 0.09 with a channel, as also shown in Fig. 4(a). This decrease may correspond to the experimentally observed 50% reduction in the breakdown voltage [3]. Calculations also reproduce partial guiding [13] and transitions between filaments [5]. The latter is obtained by placing two parallel channels as the initial condition.

Using a fixed value of the resistance of ionized air, we can estimate the length L of the discharge path normalized by the separation of electrodes, $L = V/(InR)$, where V, I, R, and n are the applied voltage, the current, the resistance of the cell, and the number of cells across the gap between the electrodes, respectively. We assume the channel width is equal to one cell. A histogram of the resulting length of the path is shown in Fig. 4(c). Even in the case of successful guiding, although most discharges occur through the preformed channel, some discharge paths show considerable detouring. When the guiding fails, it is shown that the discharge can occur through a path more than twice as long as the separation of the electrodes. Because the electric field in the cell at the gap is 4 times greater than in the other

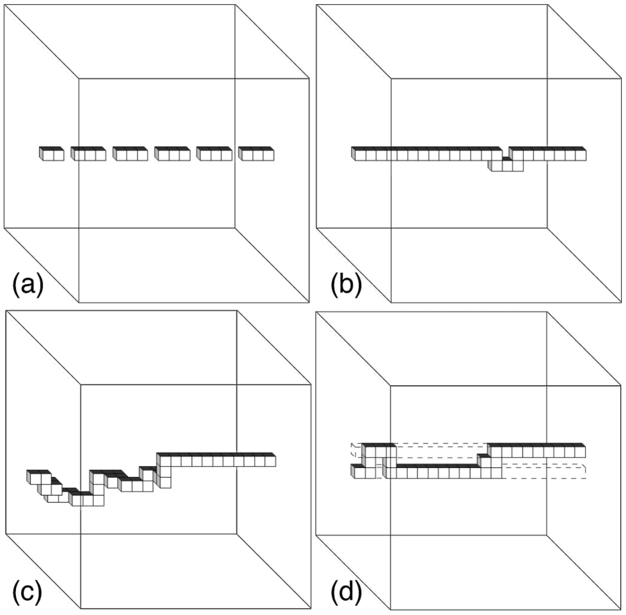

FIG. 3. Initial ionized channel (a) and calculated path (b)–(d) from the simulations corresponding to laser-guided discharge. An electric field is applied between the left and right boundaries. Panel (b) shows a case of successful guiding, where a nearly direct path between the centers of the electrodes is obtained. Panel (c) shows the case of failed guiding, or for partial guiding, where the discharge path deviates from a direct route. Finally, panel (d), for two initially parallel channels indicated by dotted lines, shows transitions between channels.

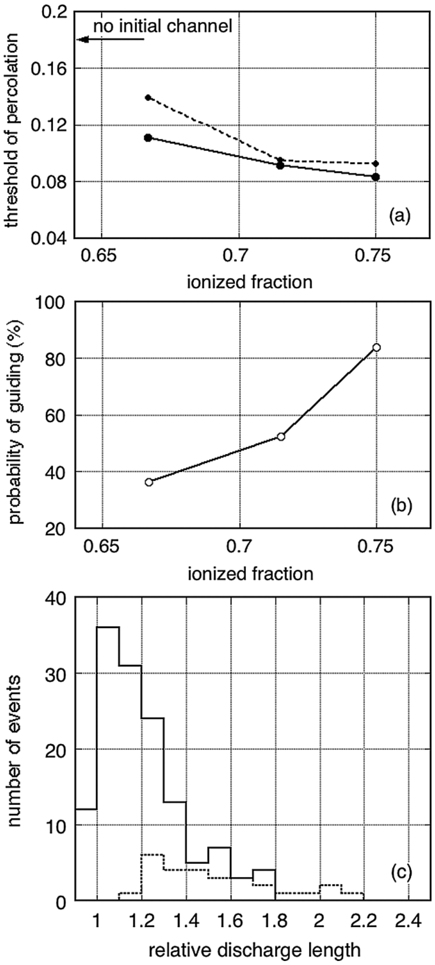

FIG. 4. (a) Threshold of percolation and (b) probability of successful guiding, as a function of the ionization fraction along the initial channel. (c) Histogram of the length of the discharge path in the case of ionization fraction of 0.75. In (a) and (c), the results for when the guiding is successful or failed is shown using the solid or dotted line, respectively.

cells, it is 16 times easier to ionize. Nevertheless, low probability discharge through alternate paths can be established before the breakdown across the gap occurs.

The probability of guiding is found to increase by additional heating of the plasma channel using a double pulse laser irradiation [4]. To simulate that case, calculations with an initial channel with different ionized fractions are carried out.

Preformed channels used in the calculations consist of two ionized cells with a one-cell gap (2:1), as well as alternating two and three ionized cells with a one-cell gap (2:1 + 3:1), for which the averaged fraction of the ionized region is determined to be 67% and 71.4%, respectively. As can be seen in Fig. 4(b), the probability of guiding increases as the ionized fraction of the initial channel increases. Use of two laser pulses increases the probability of discharge from 10% to 70%, whereas the present calculation shows an increase of probability of 38% to 83% by increasing the ionized fraction along the channel from 67% to 75%. It is also shown that the discharge occurs more easily because the percolation threshold decreases. These results imply that the increase of the probability of guiding in the experiment also arises from an increase of the conductive fraction along the channel.

In summary, we have presented simulation results for the development of a discharge path based on the combined percolation and resistor network model. The results reproduce properties of the laser-guided discharge. In the present simulation, the ionization of air, caused by electrons from cosmic rays and those resulting from enhanced electric field at the tip of the streamer, and also possible ionization by photons emitted from excited molecules at the streamer head, are taken into account using a simple power-law model in the local electric field. By including the ionization effects consistent with elementary atomic processes [26], the present method will be useful for modeling a variety of discharge devices by quantitatively predicting spatial and temporal features of discharges.

A part of this study was carried out under the auspices of the Grant-in-Aid for Scientific Research (B) No. 20340166 and No. 21340171 from the Japan Society for the Promotion of Science (JSPS).

[1] M. Miki, T. Sindo, and Y. Aihara, J. Phys. D **29**, 1984 (1996).
[2] J. Kasparian and J.-P. Wolf, Opt. Express **16**, 466 (2008).
[3] H. Pepin, D. Comtois, F. Vidal, C. Y. Chien, A. Desparios, T. W. Johnston, J. C. Kieer, B. La Fontaine, F. A. M. Martin, F. Rizk, C. Potvin, P. Couture, H. P. Mercure, A. Bondiou-Ciegerie, P. Lalande, and L. Gallimberti, Phys. Plasmas **8**, 2532 (2001).
[4] G. Mejean, R. Ackermann, J. Kasparian, E. Salmon, J. Yu, and J.-P. Wolf, Appl. Phys. Lett. **88**, 021101 (2006).
[5] T. Fujii, M. Miki, N. Goto, A. Zhidkov, T. Fukuchi, Y. Oishi, and K. Nemoto, Phys. Plasmas **15**, 013107 (2008).
[6] S. Kirkpatrick, Rev. Mod. Phys. **45**, 574 (1973).
[7] L. Niemeyer, L. Pietronero, and H. J. Wiesmann, Phys. Rev. Lett. **52**, 1033 (1984).
[8] L. Niemeyer, L. Ullrich, and N. Wiegart, IEEE Trans. Electr. Insul. **24**, 309 (1989).
[9] L. Pietronero and H. J. Wiesmann, J. Stat. Phys. **36**, 909 (1984).
[10] M. Arrayas, U. Ebert, and W. Hundsdorfer, Phys. Rev. Lett. **88**, 174502 (2002).
[11] L. Chao, U. Ebert, and W. Hundsdorfer, J. Comput. Phys. **229**, 200 (2010).
[12] U. Ebert, C. Montijn, T. M. P. Briels, W. Hundsdorfer, B. Mulenbroek, A. Rocco, and E. M. van Veldhuizen, Plasma Sources Sci. Technol. **15**, S118 (2006).
[13] M. Rodriguez, R. Sauerbrey, H. Wille, L. Woste, T. Fujii, Y.-B. Andre, A. Mysyrowicz, L. Klingbeil, K. Rethmeier, W. Kalkner, J. Kasparian, E. Salmon, J. Yu, and P.-J. Wolf, Opt. Lett. **27**, 772 (2002).
[14] V. A. Rakov and M. A. Uman, *Lightning Physics and Effects* (Cambridge University Press, Cambridge, England, 2003).
[15] N. Goelian, P. Lalande, A. Gondiou-Clergerie, G. L. Bacchiega, A. Gazzani, and I. Gallimberti, J. Phys. D **30**, 2441 (1997).
[16] A. V. Gurevich, G. M. Milikh, and R. Roussel-Dupre, Phys. Lett. A **165**, 463 (1992).
[17] H. Tsuchiya, T. Enoto, S. Yamada, T. Yuasa, M. Kawaharada, T. Kitaguchi, M. Kokubun, H. Kato, M. Okano, S. Nakamura, and K. Makishima, Phys. Rev. Lett. **99**, 165002 (2007).
[18] J. R. Dwyer, Geophys. Res. Lett. **32**, L20808 (2005).
[19] T. Torii, T. Nishijima, Z.-I. Kawasaki, and T. Sugita, Geophys. Res. Lett. **31**, L05113 (2004).
[20] Y. Kishimoto and T. Masaki, J. Plasma Phys. **72**, 971 (2006).
[21] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, Philadelphia, 1994), 2nd ed.
[22] H. Takayasu, Phys. Rev. Lett. **54**, 1099 (1985).
[23] L. J. Duckers and R. G. Ross, Phys. Lett. A **49**, 361 (1974).
[24] S. Nijdam, C. G. C. Geurts, E. M. van Veldhuizen, and U. Ebert, J. Phys. D **42**, 045201 (2009).
[25] S. Eisenmann, A. Pukhov, and A. Zigler, Phys. Rev. Lett. **98**, 155002 (2007).
[26] T. B. Petrova, H. D. Ladouceur, and A. P. Baronavski, Phys. Rev. E **76**, 066405 (2007).