

## Effect of the Tensor Force on the Charge Exchange Spin-Dipole Excitations of $^{208}\text{Pb}$

C. L. Bai,<sup>1,2</sup> H. Q. Zhang,<sup>1,2</sup> H. Sagawa,<sup>3</sup> X. Z. Zhang,<sup>1</sup> G. Colò,<sup>4</sup> and F. R. Xu<sup>2</sup>

<sup>1</sup>*China Institute of Atomic Energy, 102413 Beijing, China*

<sup>2</sup>*School of Physics, Peking University, 100871 Beijing, China*

<sup>3</sup>*Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan*

<sup>4</sup>*Dipartimento di Fisica, Università degli Studi di Milano and INFN, Sezione di Milano, 20133 Milano, Italy*

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The charge exchange spin-dipole (SD) excitations of  $^{208}\text{Pb}$  are studied by using a fully self-consistent Skyrme Hartree-Fock plus random phase approximation formalism which includes the tensor interaction. It is found, for the first time, that the tensor correlations have a unique, multipole-dependent effect on the SD excitations; that is, they produce a softening of  $1^-$  states, but a hardening of  $0^-$  and  $2^-$  states. This paves the way to a clear assessment of the strength of the tensor terms. We compare our results with a recent measurement, showing that our choice of tensor terms improves the agreement with experiment. The robustness of our results is supported by the analytic form of the tensor matrix elements.

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The nuclear effective interactions such as the zero-range Skyrme forces have been quite successful to in describing many nuclear properties. These forces are fitted using empirical properties of uniform nuclear matter, together with the masses and charge radii of selected reference nuclei. They describe in a reasonable way the global trends of the ground-state properties along the nuclear chart (binding energies, radii, and deformations). Properties of excited states such as vibrations and rotations have been studied successfully as well, allowing a large amount of physical insight [1,2]. In the quest for a universal local energy density functional (EDF) for nuclei, the Skyrme framework is often used as a starting point.

While most of the Skyrme parameter sets which have been widely used are purely central, many groups have recently devoted attention to the role played by the zero-range tensor terms that can be added (see Refs. [3–10]). This blooming of theoretical studies has followed the claim by the authors of Ref. [11] that the tensor force is crucial for the understanding of the evolution of the single-particle energies in exotic nuclei.

There exist different strategies to fix the tensor part of the interaction. One can be inspired by a bare or a  $G$ -matrix interaction [3,12]. Since the tensor force affects the spin-orbit splittings as described below, another possibility is to add it to an existing Skyrme set and try to reproduce at best the evolution of single-particle states along isotopic or isotonic chains [5,6]. At present, the most accurate and systematic way to produce effective interactions with the tensor has been a full variational procedure to fit the tensor and the central terms on equal footing [7]. All these attempts have produced results which are not at all conclusive.

The studies based on a refit of all Skyrme parameters suffer from the drawback that the tensor force has a

moderate effect on the ground-state quantities such as the total binding energies. The single-particle energies do not lie within the EDF framework and can be affected by correlations such as particle-vibration coupling. In this Letter we follow the idea that collective excitations (especially the spin-dependent ones) may be better candidates to constrain the tensor force.

Recently, self-consistent Hartree-Fock plus random phase approximation (HF + RPA) schemes with tensor interactions have been developed [13–15]. The Gamow-Teller (GT) and charge exchange  $1^+$  spin-quadrupole transitions in  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$  have been studied in Refs. [13,14], whereas the non-charge-exchange multipole responses of several magic nuclei have been calculated in Ref. [15]. In this Letter, we study the charge exchange spin-dipole (SD) excitations of  $^{208}\text{Pb}$ , inspired by recent accurate measurements [16]. Our specific goal is to find a clear, unambiguous signature of the effective tensor force. Spin-isospin collective modes have been instrumental for the understanding of nuclear structure for almost three decades [17,18]. In particular, the total SD strength distribution in  $^{90}\text{Zr}$  has been measured [19,20] and this has allowed us to extract conclusions on the neutron radius [21]. Neutron radii have been shown to be strongly correlated with the features of the neutron matter equation of state which, in turn, has relevant implications for neutron stars [22–24]. However, for the determination of the effect of the tensor force which is strongly spin dependent, one needs to know separately the strength distributions of the  $J^\pi = 0^-, 1^-,$  and  $2^-$  components.

The triplet-even (TE) and triplet-odd (TO) zero-range tensor terms of the Skyrme force are expressed as [12,25]

$$\begin{aligned}
V^T = & \frac{T}{2} \left[ \left[ (\boldsymbol{\sigma}_1 \cdot \mathbf{k}') (\boldsymbol{\sigma}_2 \cdot \mathbf{k}') - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}'^2 \right] \delta(\mathbf{r}) \right. \\
& + \delta(\mathbf{r}) \left[ (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) - \frac{1}{3} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}^2 \right] \\
& + \frac{U}{2} \left\{ (\boldsymbol{\sigma}_1 \cdot \mathbf{k}') \delta(\mathbf{r}) (\boldsymbol{\sigma}_2 \cdot \mathbf{k}) + (\boldsymbol{\sigma}_2 \cdot \mathbf{k}') \delta(\mathbf{r}) (\boldsymbol{\sigma}_1 \cdot \mathbf{k}) \right. \\
& \left. \left. - \frac{2}{3} [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \mathbf{k}' \cdot \delta(\mathbf{r}) \mathbf{k}] \right\}. \quad (1)
\end{aligned}$$

In the above expression, the operator  $\mathbf{k} = (\nabla_1 - \nabla_2)/2i$  acts on the right and  $\mathbf{k}' = -(\nabla_1' - \nabla_2')/2i$  acts on the left. The coupling constants  $T$  and  $U$  denote the strengths of the triplet-even and triplet-odd tensor interactions, respectively.

The main effect of the tensor terms on the HF results is a modification of the spin-orbit potential, which reads

$$U_{\text{SO}}^{(q)} = \frac{W_0}{2r} \left( 2 \frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr} \right) + \left( \alpha \frac{J_q}{r} + \beta \frac{J_{q'}}{r} \right). \quad (2)$$

In this expression,  $q = 0$  (1) labels neutrons (protons).  $\rho_q$  are the densities while  $J_q$  are the so-called spin-orbit densities. Their definitions can be found in Refs. [3–10,12]. The first term in the right-hand side of Eq. (2) comes from the Skyrme two-body spin-orbit interaction, whereas the second term includes both a central exchange and a tensor contribution; that is,  $\alpha = \alpha_C + \alpha_T$  and  $\beta = \beta_C + \beta_T$ : their complete expressions can be found, e.g., in Refs. [5,9]. In this Letter, we employ different Skyrme parameter sets which include the tensor terms. The sets *TIJ* have been introduced in Ref. [7]. The set *SLy5 + T<sub>w</sub>* is a set in which the tensor terms are added perturbatively on top of the existing force *SLy5*: the tensor parameters  $T$  and  $U$  are obtained by the low- $q$  limit of the  $G$ -matrix calculations [12]. One should notice that the tensor part of *SLy5 + T<sub>w</sub>* is different from that of *SLy5 + T*, which was adopted in Ref. [5]. We tried to use the set *SLy5 + T*, but its effect on SD states is larger and does not match the SD experimental data in  $^{208}\text{Pb}$  [16]. The values of  $T$ ,  $U$ ,  $\alpha$ , and  $\beta$  for the adopted interactions are listed in Table I.

In our model, we first solve the HF equations in coordinate space. The unoccupied levels are found by diagonalizing the mean field in a harmonic oscillator basis (up to a maximum value of the major quantum number  $N_{\text{max}} = 12$ ). We then perform fully self-consistent RPA by including both the two-body spin-orbit and tensor interactions. It has been checked that the adopted basis is large enough to make the results stable.

The charge exchange SD operator is defined as

$$\hat{O}_{\pm}^{\lambda} = \sum_i t_{\pm}^i r_i (Y_1^i \sigma^i)_{\lambda}. \quad (3)$$

The  $n$ th energy weighted sum rules  $m_n$  for the  $\lambda$ -pole SD operator are defined as

$$m_n^{\lambda}(t_{\pm}) = \sum_i E_i^n | \langle i | \hat{O}_{\pm}^{\lambda} | 0 \rangle |^2. \quad (4)$$

TABLE I. The tensor strength parameters  $T$  and  $U$  of Eq. (1) as well as the  $\alpha$  and  $\beta$  values of Eq. (2). All values are in  $\text{MeV} \cdot \text{fm}^5$ .

Force	$T$	$U$	$\alpha_T$	$\beta_T$	$\alpha_c$	$\beta_c$
<i>SLy5 + T<sub>w</sub></i>	820.0	323.4	134.76	238.2	80.2	-48.9
<i>T11</i>	259.0	-342.8	-142.8	-17.5	82.8	-42.5
<i>T22</i>	356.0	-217.5	-90.6	28.9	90.6	-28.9
<i>T32</i>	613.1	-2.3	-96.5	79.5	96.5	-19.5
<i>T43</i>	590.6	-147.5	-61.5	92.3	121.5	27.7
<i>T44</i>	520.1	21.5	9.0	112.8	111.0	7.1

The model independent sum rule which is known to hold is  $m_0^{\lambda}(t_-) - m_0^{\lambda}(t_+) = \frac{2\lambda+1}{4\pi} (N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p)$ . This sum rule has been shown to be fulfilled with 1% accuracy by the numerical calculations.

We performed two kinds of calculations. In the first one, the tensor terms are neither included in HF nor in RPA. In the second one, the tensor terms are included both in HF and in RPA. The terms containing the spin-orbit densities  $J_q$  which arise from the central momentum-dependent part of the Skyrme interaction are included in all the following HF and RPA calculations. Previously, it had been found that the effect of tensor correlations in HF is large for the Gamow-Teller mode and also, to some extent, for some low-lying non-charge-exchange excitations like the first  $2^+$ . This is largely due to the fact that unperturbed GT transitions are exactly those among spin-orbit partners. The unperturbed  $p$ - $h$  energy of the lowest  $2^+$  is also determined by the spin-orbit splitting. On the other hand, this is not the case for the SD excitations: the average unperturbed energies are not much affected by the spin-orbit splittings since they are the  $1\hbar\omega$ -type excitations.

The numerical results of the HF + RPA calculations with the forces *T43* and *SLy5 + T<sub>w</sub>* are shown in Fig. 1. They are compared with experimental data obtained by distorted wave impulse approximation and multipole decomposition analysis of the  $(p, n)$  reaction [16]. From Figs. 1(a) and 1(b) one can see that in the case of the *T43* interaction the main peaks of the  $0^-$  and  $1^-$  strength distributions are shifted upwards by about 7.5 MeV and downwards by about 5 MeV, respectively, due to the tensor correlations. There are several  $2^-$  peaks [cf. Fig. 1(c)]. The peak at excitation energy  $E_x \approx 17.7$  MeV is moved upward by about 2 MeV by including tensor, and comes close to an experimental peak, while another peak at  $E_x \approx 3.9$  MeV is shifted downwards by about 0.6 MeV and is also eventually close to the observed low energy peak. For the total SD strength in Fig. 1(d), it is remarkable that the main peak at 26 MeV is shifted to 21 MeV when tensor is included, and this provides good agreement with the experimental data.

The SD strength distributions in  $^{208}\text{Pb}$  calculated by using the set *SLy5 + T<sub>w</sub>* are also shown in Figs. 1(e)–1(h). From Fig. 1(e), we see that the calculated  $0^-$  strength is concentrated in one peak which is shifted upwards by

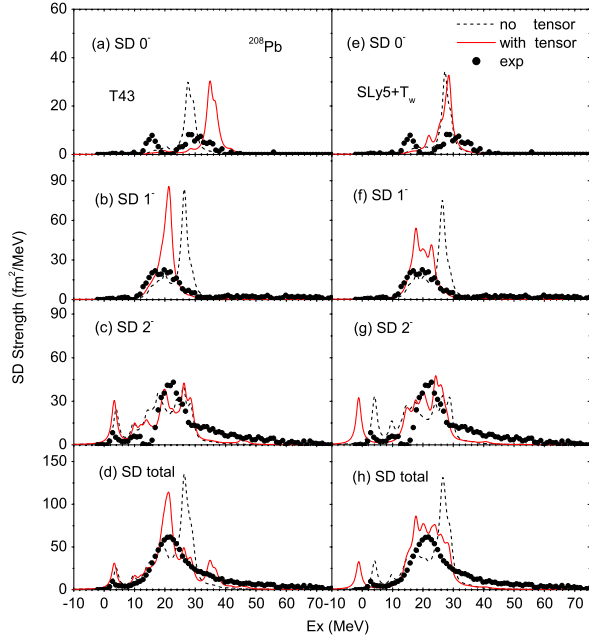


FIG. 1 (color online). Charge exchange  $SD_-$  strength distributions in  $^{208}\text{Pb}$ . In (a)–(c) the RPA results obtained by employing the interaction  $\text{SLy5} + T_w$  for the multipoles  $0^-$ ,  $1^-$ ,  $2^-$  are displayed. In (d) we show the total strength distribution. Panels (e)–(h) correspond to similar results when the parameter set  $T43$  is employed. All these discrete RPA results have been smoothed by using a Lorentzian averaging with a width of 2 MeV and compared with experimental findings. The excitation energy is with respect to the ground state of  $^{208}\text{Bi}$ . The experimental data are taken from Ref. [16]. See the text for details and discussion.

about 1.3 MeV by the tensor correlations. In Fig. 1(f), the RPA tensor correlations move the  $1^-$  peak downwards and split it into three peaks, in qualitative agreement with the bumplike experimental strength. In the case of the  $2^-$  [Fig. 1(g)], the main peaks in the high energy region are rather near to the experimental main peak. Therefore, as shown in Fig. 1(h), the inclusion of the tensor terms in HF + RPA can make the calculated main peak of the total SD strength coincide with the main measured peak. However, in the low energy region the agreement is not good compared with the experimental result in the case of  $\text{SLy5} + T_w$  (due also to the fact that with tensor a peak appears below the g.s. of the daughter nucleus). The spin-orbit density gives contributions to the  $p$ - $h$  matrix elements of spin-dependent excitations [26]. We find out in the case of the  $T22$  parameter set that the two-body interaction generated by the terms containing  $J_q$  from the central exchange interaction have a much smaller effect on the SD excitations than the two-body tensor interaction [27].

We would like at this stage to obtain a better understanding of this peculiar role of tensor interactions. The diagonal matrix element of the TE term on a state with multipolarity  $\lambda$  can be expressed as [15]

$$V_{\text{TE}}^{(\lambda)} = \frac{5T}{4} \sum_{\ell, k, k'} \frac{(-)^{k+k'+\lambda+\ell+1} \hat{k} \hat{k}'}{2\lambda+1} \times \left\{ \begin{matrix} k & k' & 2 \\ 1 & 1 & \ell \end{matrix} \right\} \left\{ \begin{matrix} 1 & 1 & 2 \\ k' & k & \lambda \end{matrix} \right\} \langle p || \hat{O}_{k',\lambda} || h \rangle \langle p || \hat{O}_{k,\lambda} || h \rangle^*, \quad (5)$$

in terms of the reduced matrix elements of the operator  $\hat{O}_{k,\lambda} = \sum_i \{ \sigma_i \otimes [\nabla_i \otimes Y_\ell(i)]^{(k)(\lambda)} \}$  and  $6j$  symbols. In Eq. (5), the notation  $\hat{k} \equiv \sqrt{2k+1}$  is used. For the SD excitations, taking  $\ell = 0$  and  $k = k' = 1$ , Eq. (5) gives

$$V_{\text{TE}}^{(\lambda)} = -\frac{5}{12} T \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} |\langle p || \hat{O}_{1,\lambda} || h \rangle|^2 \quad (6)$$

$$\text{for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}.$$

The TO tensor part is also expressed in a similar way with the same multipole-dependent coefficients. We can see in Eq. (6) that the diagonal  $p$ - $h$  matrix element in the  $0^-$  case is the largest and that the matrix element for  $1^-$  is the next largest. The effect on  $2^-$  is rather small. It should be noticed that these relative strengths of the Skyrme tensor interactions on each multipole are similar to those obtained from the finite-range tensor interactions both in magnitude and in sign [28]. We can sum the TE and TO direct matrix elements as

$$V_T^{(\lambda)} = V_{\text{TE}}^{(\lambda)} + V_{\text{TO}}^{(\lambda)} \equiv a_\lambda T + b_\lambda U. \quad (7)$$

The proper antisymmetrization is easy to obtain for contact interactions and gives, in the isovector channel,

$$V_{T,\text{AS}}^{(\lambda)} = \left[ -\frac{1}{2} a_\lambda T + \frac{1}{2} b_\lambda U \right] \langle \tau_1 \cdot \tau_2 \rangle. \quad (8)$$

Since the coupling constant  $T$  is positive for the interactions we considered,  $V_{\text{TE}}^{(\lambda)}$  is repulsive for the  $0^-$  and  $2^-$  case, while it is attractive for  $1^-$ . The  $V_{\text{TO}}^{(\lambda)}$  part may contribute with the same sign as the  $V_{\text{TE}}^{(\lambda)}$  part if the value of  $U$  is negative. For the  $TIJ$  family, the value of  $U$  is negative or small positive, so that the  $V_{\text{TO}}^{(\lambda)}$  contributions have the same multipole dependence or almost negligible. All together, the tensor correlations are strongly repulsive for  $0^-$  and weakly repulsive for  $2^-$  in general. For  $1^-$ , the net effect will be attractive. For  $\text{SLy5} + T_w$ , the value of  $U$  is positive and will give opposite contributions to those of  $T$ . However, the  $T$  value is much larger than the value of  $U$  so that the same argument given for the  $TIJ$  family will hold. One can see from Table II that Eq. (8) provides a very effective guideline for interpreting the numerical results of microscopic RPA. In Table II we also provide values of the sum rules  $m_0$  and  $m_1$  for the different multipoles and effective forces.

In the parameter sets  $T43$  and  $T44$  used for Table I, the spin-orbit strengths are larger than in the other parameter

TABLE II. The SD sum rules  $m_0$  and  $m_1$  for  $^{208}\text{Pb}$  with and without the tensor terms.  $\Delta E$  is the difference between  $m_1/m_0$  calculated with and without tensor.

Force	$\lambda^\pi$	Without tensor			With tensor			$\Delta E$
		$m_0$	$m_1$	$m_1/m_0$	$m_0$	$m_1$	$m_1/m_0$	
SLy5 + $T_w$	$0^-$	158.6	4718	29.7	171.9	5168	29.9	0.2
	$1^-$	432.0	11746	27.2	440.0	10111	23.0	-4.2
	$2^-$	646.0	13742	21.3	657.4	14408	21.3	0
	Sum	1236.4	30206	24.4	1269.0	29256	23.0	-1.4
T11	$0^-$	158.6	4559	28.8	163.4	6074	37.2	8.4
	$1^-$	434.5	11581	26.7	431.0	10376	24.1	-2.6
	$2^-$	657.3	13710	21.2	646.2	14296	22.1	1.0
	Sum	1240.0	29850	24.1	1241.0	30746	24.8	0.7
T22	$0^-$	157.6	4650	29.5	163.7	5943	36.3	6.8
	$1^-$	434.5	11771	27.1	433.9	10504	24.2	-2.9
	$2^-$	645.9	13812	21.4	647.1	14327	22.1	0.7
	Sum	1238.0	30233	24.4	1244.7	30774	24.7	0.3
T32	$0^-$	157.2	4698	29.9	166.9	6479	38.8	8.9
	$1^-$	435.3	11886	27.3	435.3	9928	22.7	-4.6
	$2^-$	646.1	13897	21.5	649.6	14619	22.5	1.0
	Sum	1238.6	30481	24.6	1253.2	31026	24.8	0.2
T43	$0^-$	154.8	4693	30.3	164.0	6170	37.5	7.2
	$1^-$	440.3	12138	27.6	444.1	10366	23.3	-4.3
	$2^-$	645.5	14067	21.8	649.4	14675	22.6	0.8
	Sum	1241.0	30898	24.9	1258.0	31211	24.8	-0.1
T44	$0^-$	155.6	4811	30.9	163.2	5637	34.5	3.6
	$1^-$	436.3	12174	27.9	440.6	10854	24.6	-3.3
	$2^-$	664.2	14059	21.8	649	14406	22.2	0.4
	Sum	1256.0	31044	24.7	1253.0	30897	24.7	0.0

sets. However, these large spin-orbit strengths cancel substantially with the effect of the terms  $\alpha$  and  $\beta$  of Eq. (2), and the net results are quite similar for the single-particle spectra of the parameter sets in Table I.

In summary, we have studied the effect of tensor interactions on the charge exchange  $t_-$  SD excitations of  $^{208}\text{Pb}$  by using the self-consistent HF + RPA model. We have demonstrated clearly for the first time that tensor correlations have a specific multipole dependence; that is, they produce a strong hardening effect on the  $0^-$  mode and a softening effect on the  $1^-$  mode. A weak hardening effect is also observed on the  $2^-$  mode. The characteristic effect of the tensor force was further examined by using the analytic formulas based on the multipole expansion of the contact tensor interaction. It is found that both the TE and TO tensor interactions do provide the characteristic multipole-dependent effect on the SD excitations. These effects are shown to be robust irrespective of the adopted Skyrme force with tensor interactions. Our calculated results are compared with recent SD excitation spectra obtained in the  $(p, n)$  reaction on the  $^{208}\text{Pb}$  target. The softening and the hardening effects produced by tensor correlations on the  $1^-$  and  $2^-$  modes are confirmed by comparing the experimental data and the calculated results with and without tensor correlations. Consequently, from the study of SD excitations, we are able to give a clear constraint on the effective tensor force which has been missing until now.

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