Nonuniversality of Transverse Momentum Dependent Parton Distributions at Small x

Bo-Wen Xiao[1] and Feng Yuan[1,2]

[1]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[2]*RIKEN BNL Research Center, Building 510A, Brookhaven National Laboratory, Upton, New York 11973, USA*

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We study the universality issue of the transverse momentum dependent parton distributions at small x, by comparing the initial and final state interaction effects in di-jet-correlations in pA collisions with those in deep inelastic lepton-nucleus scattering. We demonstrate the nonuniversality by performing an explicit calculation in a particular model where the multiple gauge boson exchange contributions are summed up to all orders. We comment on the implications of our results on the theoretical interpretation of dihadron correlation in dA collisions in terms of the saturation phenomena in deep inelastic lepton-nucleus scattering.

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Introduction.—The Feynman parton distribution functions describe the internal structure of hadrons in terms of the momentum distributions of partons in the infinite momentum frame. These distributions depend only on the longitudinal momentum fractions of the target hadrons carried by the partons. The measurements of high energy hadronic processes depending on the Feynman parton distributions have been made possible by proving the associated factorization theorem, which guarantees that the parton distributions studied in different processes are universal [1].

In recent years, hadronic physicists have become more interested in semi-inclusive high energy processes, where one can study the intrinsic transverse momentum of partons inside hadrons. The additional transverse momentum dependence helps in picturing the parton distribution in a three-dimensional fashion and builds the hadron tomographic image through the partonic structure [2]. A number of novel hadronic physics phenomena are also closely associated with the transverse momentum dependent (TMD) parton distributions, such as the single transverse spin asymmetries [3–6] and small-x saturation phenomena [7,8]. In the last few years, great progress has been made in understanding the fundamental questions associated with these TMD parton distributions, such as the gauge invariance and the QCD factorization [4,5,9,10]. In particular, the nonuniversality of these distribution functions due to initial and final state interaction effects has attracted intense investigation. It has been found that the difference between the final state interaction in deep inelastic scattering (DIS) and the initial state interaction in Drell-Yan lepton pair production in pp collisions leads to an opposite sign for the single spin asymmetries in these two processes [3,4]. More complicated relation was found for the single spin asymmetry in dijet correlation in pp collisions as compared to those in DIS and Drell-Yan processes [11–16]. This eventually leads to the conclusion that a standard TMD factorization breaks down for this process [14].

In this Letter, we extend the universality discussion of the TMD parton distributions to the small-x domain, where the k_t-dependent distributions have been commonly used to describe the relevant physics phenomena [8]. We expect nonuniversality for these objects as well. However, because different approximations have been made in the small-x region, the general arguments of Refs. [14,15] on the nonuniversality may not apply. As far as we know, there has been no explicit discussion of this issue in the literature, although similar studies been performed [8,17]. The objective of this Letter is to study this in detail. We will carry out an explicit calculation in a model where both small-x and low transverse momentum approximation are valid. Furthermore, we will resum the initial and final state interactions to all orders in perturbation to study the associated universality property.

In particular, we investigate the universality of the small-x transverse momentum dependent parton distributions probed in hadronic dijet correlation in nucleon-nucleus collisions, as compared to that in the deep elastic lepton-nucleus (nucleon) scattering. There have been interesting experimental results on dihadron correlation in deuteron-gold collisions at RHIC reported by the STAR collaboration, where a strong back-to-back decorrelation was found in the forward rapidity region of the deuteron as compared to the narrow back-to-back peaks observed in the central rapidity region [18]. However, the theoretical interpretation is not yet clear at this moment [19–22]. As schematically shown in Fig. 1(a), two partons from the nucleon projectile and nucleus target collide with each other, and produce two jets in the final state,

$$p + A \to \mathrm{Jet1} + \mathrm{Jet2} + X, \tag{1}$$

where the transverse momenta of these two jets are similar in size but opposite to each other in direction. In the ideal case, these two jets are produced back to back. However, the gluon radiation and intrinsic transverse momenta of the initial partons induce an imbalance between them. We are particularly interested in the kinematic region that the

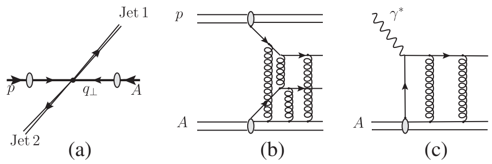

FIG. 1. (a) Schematic diagram showing that two partons from the nucleon projectile and the nucleus target collide and produce two jets in the final state, where the intrinsic transverse momentum $q_\perp$ from the nucleus dominates the imbalance between the two jets; (b) illustration of initial and final state interactions which will affect the TMD quark distribution from the nucleus in this process; (c) as a comparison, only the final state interaction effect is present in the deep inelastic lepton-nucleus (nucleon) scattering.

imbalance $\vec{q}_\perp = \vec{P}_{1\perp} + \vec{P}_{2\perp}$ is much smaller than the transverse momentum of the individual jet, namely, $|\vec{q}_\perp| \ll |\vec{P}_{1\perp}| \sim |\vec{P}_{2\perp}|$, which also corresponds to the kinematics in the STAR measurements. Only in this region, can the intrinsic transverse momentum have significant effects. Since there are two incoming partons, both intrinsic transverse momenta can affect the imbalance between the two jets. For large nucleus and small x, the dominant contribution should come from the intrinsic transverse momentum of the parton from the nucleus, which we label as $q_\perp$ in Fig. 1(a).

To understand the universality property of the TMD parton distribution, we study the multi-gluon exchange between the hard scattering part with the nucleus target [5,7,14,15]. We illustrate the generic diagrams of these interactions in Fig. 1(b), for the particular partonic channel $qq' \to qq'$. All other channels shall follow accordingly. Since the incoming and outgoing partons are all colored objects, there exist initial state interaction with the initial parton from the nucleon projectile, and final state interactions with the outgoing two partons. For comparison, we also plot in Fig. 1(c) a similar diagram for the deep inelastic lepton scattering on a nucleus target, where there are only final state interactions on the struck quark. Clearly, if these interactions affect the transverse momentum dependence, we would conclude that they are not universal between these processes.

In this Letter, we will only discuss the initial and final state interactions with the nucleus target. Including those with the incoming nucleon may break down the generalized TMD factorization [16]. We assume, however, our results are the dominant contribution at small x in the high gluon density in nucleus. Effectively, we will neglect the intrinsic transverse momentum effects from the nucleon. We will show that with the resummation of all order initial and final state interactions effects from the nucleus target, the parton distributions will be different from that in DIS and Drell-Yan lepton pair production processes, but the effects are calculable.

Initial and final state interaction effects.—We take the partonic channel $qq' \to qq'$ as an example to show the initial and final state interaction effects and calculate the quark distribution in dijet correlation $pA \to \mathrm{Jet1} + \mathrm{Jet2} + X$, and compare with that in the DIS. At small x, quark distributions are dominated by gluon splitting, and can be calculated from the relevant Feynman diagrams [23–25]. For the purpose of our calculation, we employ the Abelian model of Refs. [7,14,15]. Our results can be easily extended to the real QCD calculations for this particular channel. For all other channel, we expect similar results. It is a scalar QED model with Abelian massive gluons of mass λ. We construct the model in such a way that the large nucleus is represented by a heavy scalar target with mass M_A. The scalar quarks are generated by the Abelian gluon splitting which is the dominant contribution at small-x. The associated quark distribution in DIS has been calculated in this model in [5,7].

Since we are interested in studying the final state interaction effects on the parton distribution of the nucleus, for convenience, we choose the projectile as a single scalar quark with charge g_2, which differs from the charge of the scalar quark from the target nucleus, g_1. In addition, we assume that the Abelian gluon attaches to the target nucleus with an effective coupling g. All the partons in this calculation are set to be scalars with a mass m. The coupling g_2 being different from g_1 is to show the dependence of the parton distribution on the initial and final state interactions associated with the incoming parton. If the dependence on g_2 remains for the nucleus parton distributions, they are not universal [14,15].

We perform our calculations in the covariant gauge. The final result does not depend on the gauge choice. We organize the calculations in terms of orders of the coupling g. At each order, an additional gluon attaches the scalar quarks in the partonic scattering part to the nucleus target [7,14,15]. As shown in Fig. 2, the lowest-order graphs contain one soft gluon exchange with the momentum k. We calculate the scattering amplitude in the infinite momentum frame of the nucleus, i.e., $P_A^+ \to \infty$, where the plus component of a momentum p is defined as $p^+ = (p^0 + p^z)/\sqrt{2}$ and the nucleus is moving in $+\hat{z}$ direction.

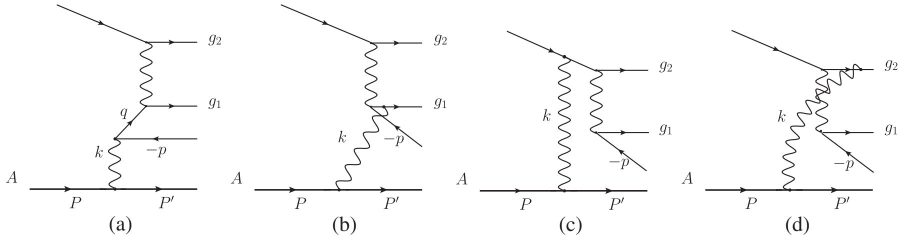

FIG. 2. Lowest-order graphs for dijet production in a hadron-hadron collision in the small-x limit. In these graphs, there is one soft gluon exchange with momentum k in addition to the hard gluon exchange.

One can also perform the calculations in the target rest frame and take the limit of $M_A \to \infty$, which will lead to the same result [7]. The small-x approximation ($q^+ \sim k^+ \ll P_A^+$) will be taken throughout the following calculations. In addition, we also follow the low transverse momentum approximation in terms of $q_\perp/P_{1\perp}$ ($q_\perp/P_{2\perp}$) by applying the power counting method [13]. An important simplification is the eikonal approximation, which replaces the gluon attachment to the initial and final state partons with the eikonal propagator and vertex. After taking the leading order contributions, we find that the $q_\perp$ dependence of these diagrams can be cast into an effective quark distribution [13], which takes the following form

$$\tilde{q}(x, q_\perp) = \frac{x}{32\pi^2}\int\frac{dp^-}{p^-}\frac{d^2k_\perp}{(2\pi)^4}(4P^+p^-)^2|A^{(\mathrm{tot})}(k,p)|^2, \tag{2}$$

with $p_\perp = k_\perp - q_\perp$. Here, the hard partonic part depending on the hard momentum scale $P_{i\perp}$ has been separated from the above quark distribution in the differential cross section [13]. This separation is only possible at the leading power contribution of $q_\perp/P_{i\perp}$. The contributions from Fig. 2 can be written as

$$A^{(1)}(k,p) = gg_1\frac{1}{k_\perp^2+\lambda^2}\left[\frac{1}{D_1}-\frac{1}{D_2}\right], \tag{3}$$

where we have defined $D(p_\perp) = 2xP^+p^- + p_\perp^2 + m^2$ and $D_1 = D(q_\perp)$ and $D_2 = D(p_\perp)$. In Eq. (3), the first and second terms in the square bracket correspond to Fig. 2(a) and 2(b), respectively. The contribution from Fig. 2(c) and 2(d) simply cancels. This means that at the leading order in the coupling constant the dependence on g_2 drops out, which will however change at higher orders.

At the next-to-leading order, there are 20 graphs in total in covariant gauge. We show one of these graphs as an example in Fig. 3(a), and additional diagrams can be obtained by attaching the gluons to all incoming and outgoing scalar quarks. The total contributions from these diagrams are

$$A^{(2)}(k,p) = \frac{i}{2}g^2\int d[1]d[2]\left\{g_1^2\left[\frac{1}{D_1}+\frac{1}{D_2}-\frac{1}{D_{21}}-\frac{1}{D_{22}}\right]\right. \\ \left.+\,g_1g_2\left[\frac{2}{D_2}-\frac{2}{D_{21}}\right]\right\}, \tag{4}$$

where $\int d[1]d[2]$ stands for $\int\frac{d^2k_{1\perp}d^2k_{2\perp}}{(2\pi)^4}\frac{1}{k_{1\perp}^2+\lambda^2}\frac{1}{k_{2\perp}^2+\lambda^2}\times(2\pi)^2\delta^{(2)}(k_\perp - k_{1\perp} - k_{2\perp})$, $D_{1i} = D(q_\perp - k_{i\perp})$ and $D_{2i} = D(p_\perp - k_{i\perp})$. Clearly, this result shows a dependence on g_2. However, in the amplitude squared calculation for the quark distribution Eq. (2), the g_2 dependence from $A^{(1)}A^{(2)*}$ is canceled out by its complex conjugate. Therefore, to see the residual dependence on g_2, we need to carry out the calculation of the amplitude up to order g^3.

At the g^3 order, there are 120 diagrams in total with three soft gluon-exchange [see, e.g., Fig. 3(b)], including all possible permutations of the attachments of these three gluons to the target nucleus. Summing up all these graphs, we obtain the three gluon-exchange amplitude,

$$A^{(3)}(k,p) = \frac{1}{3!}g^3\int d[1]d[2]d[3]\left\{g_1^3\left[\frac{1}{D_2}-\frac{1}{D_1}+\frac{3}{D_{13}}\right.\right. \\ \left.-\frac{3}{D_{21}}\right]+g_1^2g_2\left[\frac{3}{D_2}+\frac{3}{D_{13}}-\frac{3}{D_{21}}-\frac{3}{D_{22}}\right] \\ \left.+\,g_1g_2^2\left[\frac{3}{D_2}-\frac{3}{D_{21}}\right]\right\}, \tag{5}$$

where $\int d[1]d[2]d[3]$ follows a similar definition as in Eq. (4). Again, we see the dependence on g_2 in the second and third terms. An important cross-check of these results is that, if we set $g_2 = -g_1$, there is effectively no charge flow in the final state, and the quark distribution is identical to that in the Drell-Yan process in the same model. Applying $g_2 = -g_1$, we can easily see that indeed Eqs. (4) and (5) reproduce those calculated in Ref. [26].

With the amplitude calculated up to $\mathcal{O}(g^3)$, we are able to check the dependence on g_2 for the parton distribution in Eq. (2). Substituting the results in Eqs. (3)–(5) into Eq. (2), we find that the g_2 dependence still remains up to order g^4. If we drop all g_2 terms in these results, we obtain the quark distribution in DIS in the same model [5,7]. This clearly shows that the TMD quark distribution $\tilde{q}(x, q_\perp)$ is not universal.

This nonuniversality is better illustrated when we sum up all order multi-gluon-exchange contributions. To do that, we introduce the following Fourier transform [7],

$$A(R,r) = \int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2p_\perp}{(2\pi)^2}e^{-ik_\perp\cdot R_\perp - ip_\perp\cdot r_\perp}A(k,p). \tag{6}$$

From the Fourier transforms of $A^{(1,2,3)}(k,p)$, we can easily see that they follow the expansion of an exponential form,

$$A^{(\mathrm{tot})}(R,r) = \sum_{n=1}^{\infty}A^{(n)}(R,r) \\ = iV(r_\perp)\{1 - e^{igg_1[G(R_\perp+r_\perp)-G(R_\perp)]}\}e^{-igg_2G(R_\perp)}, \tag{7}$$

where $G(R_\perp) = K_0(\lambda R_\perp)/2\pi$ and $V(r_\perp) = K_0(Mr_\perp)/2\pi$ with $M^2 = 2xP^+p^- + m^2$. Therefore, the all order result

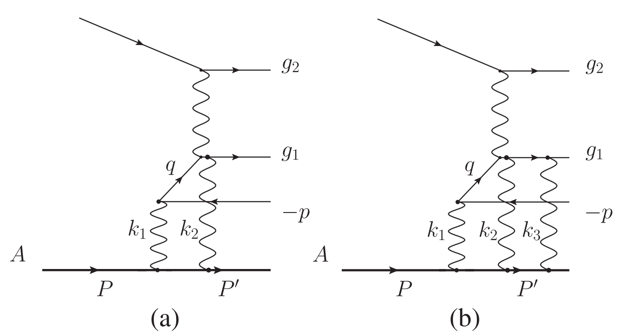

FIG. 3. Example diagrams for two (a) and three (b) gluon exchanges, where the gluons can attach all charged particles in the upper part of the diagrams to the nucleus target.

reads,

$$\tilde{q}(x, q_\perp) = \frac{xP^{+2}}{8\pi^4} \int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp \cdot (R_\perp - R'_\perp)} \times e^{-igg_2(G(R_\perp) - G(R'_\perp))} V(r_\perp) V(r'_\perp) \times \{1 - e^{igg_1[G(R_\perp + r_\perp) - G(R_\perp)]}\} \times \{1 - e^{-igg_1[G(R'_\perp + r'_\perp) - G(R'_\perp)]}\}, \tag{8}$$

where $r'_\perp = R_\perp + r_\perp - R'_\perp$. This TMD quark distribution is clearly different from that calculated in DIS in the same model [5,7]. In other words, this distribution is not universal. It is interesting to notice that the g_2 dependence disappears after the integration over the transverse momentum. This is consistent with the universality for the integrated parton distributions [5,14,15].

Summary and discussions.—In this Letter, we have demonstrated the nonuniversality for the small-x parton distributions in dijet correlation, by performing an explicit scalar calculation of the initial and final state interaction effects, and comparing to those in DIS on a nucleus target. After summing up to all orders, we find that the net effects are summarized into a phase which leads to a nonvanishing contribution to the quark distribution and breaks the universality. We have shown this through a particular partonic channel $qq' \to qq'$, and we expect that the results can be extended to all other channels.

Because the initial and final state interaction physics discussed in this Letter are quite general in any gauge theory, we expect that the nonuniversality of the TMD parton distributions hold for the real QCD calculations as well. In particular, we have checked the universality issue within the specific small-x formalisms, such as the color-dipole or color-glass condensate models [8], and found the same conclusion [27].

The nonuniversality for the TMD parton distributions at small x clearly imposes a challenge in explaining the dijet correlation data in dA collisions at RHIC with the parton distributions extracted *directly* from the DIS data. The nonuniversality, on the other hand, provides an opportunity to study QCD dynamics associated with the initial and final state interaction effects, which are calculable at small x (high gluon density limit) according to our results. These effects should be taken into account to understand the experimental data. We plan to address this issue in the saturation models [8,24] in a future publication, together with a detailed derivation of this Letter.

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