Non-Pauli Transitions from Spacetime Noncommutativity

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The consideration of noncommutative spacetimes in quantum theory can be plausibly advocated from physics at the Planck scale. Typically, this noncommutativity is controlled by fixed "vectors" or "tensors" with numerical entries like $\theta_{\mu\nu}$ for the Moyal spacetime. In approaches enforcing Poincaré invariance, these deform or twist the method of (anti)symmetrization of identical particle state vectors. We argue that the Earth's rotation and movements in the cosmos are "sudden" events to Pauli-forbidden processes. This induces (twisted) bosonic components in state vectors of identical spinorial particles. These components induce non-Pauli transitions. From known limits on such transitions, we infer that the energy scale for noncommutativity is $\gtrsim 10^{24}$ TeV. This suggests a new energy scale beyond the Planck scale.

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INTRODUCTION.—The nature of a spacetime manifold can be studied by using the algebra of functions on that manifold. This algebra with its pointwise product is commutative as it comes from a manifold.

There are physical arguments [1] suggesting that spacetime at Planck scales is noncommutative. We can account for this proposal by deforming the above commutative, pointwise product to a noncommutative, star product.

In this Letter, we work with such an algebra $\mathcal{B}_{\chi\hat{n}}$ and explore one of its important physical consequences, namely, the violation of the Pauli principle. If $\hat{x}_\mu$ are coordinate functions on spacetime, $\hat{x}_\mu(x) = x_\mu$, then $\mathcal{B}_{\chi\hat{n}}$ is defined by the relation

$$[\hat{x}_0, \hat{x}_j] = i\chi\epsilon_{ijk}n_i\hat{x}_k, \tag{1}$$

where $i \in [1, 2, 3]$, $\chi \in \mathbb{R}$ is a constant, and $\vec{n}$ is a fixed unit vector. There is no noncommutativity between spatial coordinates.

When we consider multiparticle states, we need the notion of a coproduct from Hopf algebra theory to determine how the symmetry group acts on such states. A change in its definition can lead to interesting new physics. To implement the Poincaré group $\mathcal{P}$ on $\mathcal{B}_{\chi\hat{n}}$, the canonical coproduct Δ_0 of the group algebra $\mathbb{C}\mathcal{P}$, $\Delta_0(g) = g \otimes g$, $g \in \mathcal{P}$, is deformed to $\Delta_{\chi\hat{n}}$ by a Drinfel'd twist $\mathcal{G}_{\chi\hat{n}}$:

$$\Delta_{\chi\hat{n}}(g) = \mathcal{G}_{\chi\hat{n}}^{-1}\Delta_0(g)\mathcal{G}_{\chi\hat{n}}, \tag{2}$$

$$\mathcal{G}_{\chi\hat{n}} = e^{-i(\chi/2)(P_0\otimes\hat{n}\cdot\vec{J} - \hat{n}\cdot\vec{J}\otimes P_0)}. \tag{3}$$

For comprehensive reviews of noncommutative spacetimes and its symmetries, see [2]. Here P_0 and $\vec{J}$ are time translation and rotation generators of $\mathcal{P}$, respectively.

Let $\mathcal{H}$ be a representation space of $\mathbb{C}\mathcal{P}$. Then we can define the flip operator τ_0 on $\mathcal{H} \otimes \mathcal{H}$:

$$\tau_0(v \otimes w) = (w \otimes v), \qquad v, w \in \mathcal{H}. \tag{4}$$

Since τ_0 commutes with the action of $\Delta_0(g)$ on $\mathcal{H} \otimes \mathcal{H}$, we can symmetrize and antisymmetrize $\mathcal{H} \otimes \mathcal{H}$ for $\chi = 0$ by using the projectors $\frac{1}{2}(1 \pm \tau_0)$ to get untwisted bosons and fermions. But τ_0 does not commute with the action of $\Delta_{\chi\hat{n}}$ if $\chi \neq 0$. Instead, the twisted symmetrizer

$$\tau_{\chi\hat{n}} = \mathcal{G}_{\chi\hat{n}}^{-1}\tau_0\mathcal{G}_{\chi\hat{n}} = \mathcal{G}_{\chi\hat{n}}^{-2}\tau_0 \tag{5}$$

does. The projectors $\frac{1}{2}(1 \pm \tau_{\chi\hat{n}})$ on $\mathcal{H} \otimes \mathcal{H}$ then give the twisted bosons and fermions. Such twisted (anti)symmetrization can be extended to $\mathcal{H}^{\otimes k}$ for higher values of k.

The twisted flip operator in Eq. (5) depends on the vector $\chi\vec{n}$, which effectively changes with time (explained below) causing the projectors and hence what is meant by bosons and fermions to also change with time. This is exactly the reason leading to both "primary" violations of the Pauli principle and the spin-statistics theorem. There are compelling arguments for both when $\chi = 0$ and when the Hamiltonian is essentially independent of time [3].

Twisted antisymmetrization induces $\chi\hat{n}$ dependence in energy eigenstates of electrons (nucleons) in atoms (nuclei) and corrects lifetimes in atomic (nuclear) processes. These corrections are expected to have very long time scales, χ being of the order of the Planck length. [The corrections to rates from χ are $O(\chi^2E)$, and the corresponding times are $O(\chi^{-2}E^{-1})$. E is the typical energy involved in such transitions.] They are expected to be much longer than terrestrial times like 24 hours or 1 yr. Thus the Earth's motions are sudden for noncommutative effects. But the Earth, for example, is a rotating frame and not an inertial frame. J_i changes in that frame to $R_{ik}(t)J_k$, where we can permit $R(t) \in SO(3)$ to have dependence on time t. That changes $\hat{n}\cdot\vec{J}$ to $\hat{m}\cdot\vec{J}$, $m_i := n_kR_{ki}(t)$, and hence the twisted flip operator to $\tau_{\chi\hat{m}}$. Thus, *effectively*, the nondynamical $\vec{n}$ gets rotated to $\vec{m}$. An effect of this sort has been noticed before by the authors of Ref. [4]. The swift change of $\tau_{\chi\hat{n}}$ to $\tau_{\chi\hat{m}}$ induces (twisted) bosonic components in multifermion state vectors and leads to non-Pauli effects.

In other words, the energy eigenstates for the twisted flip $\tau_{\chi\hat{n}}$ depend on $\hat{n}$. When $\hat{n}$ suddenly changes to $\hat{m}$ due to the Earth's motions, these states do not change in the sudden approximation. But they are not eigenstates for the flip $\tau_{\chi\hat{m}}$. When expanded in $\tau_{\chi\hat{m}}$ eigenstates, they are found to have $\tau_{\chi\hat{m}} = 1$ (twisted boson) components as well. These cause the non-Pauli effects. They are due to transitions between these twisted boson components. We emphasize that matrix elements of observables between twisted boson and fermion components for fixed $\vec{m}$ are strictly zero, so that in this sense no superselection rule is violated in our case where $\vec{m}$ effectively changes with time.

THE DETAILS.—We focus on the neutral Be atom with its 4 electrons for specificity. Let $\vec{X}^{(\alpha)}$ $(\alpha = 1, 2)$ be the coordinate functions of the electrons in Be and $\vec{X}$ that of the nucleus. (We drop the hat on $\hat{X}$'s.) Each of them, and hence also their differences, fulfill Eq. (1). In particular, the relative coordinates $\vec{x}^{(\alpha)} = \vec{X}^{(\alpha)} - \vec{X}$ fulfill Eq. (1).

Let P_0 be the single electron Hamiltonian $-\frac{\vec{\nabla}^{2(\alpha)}}{2\mu} - \frac{4e^2}{|\vec{x}^{(\alpha)}|}$, where μ is the reduced mass. It represents $i\partial_t$ on single electron wave functions. On two-electron states, it acts as

$$\Delta_{\chi\hat{n}}(P_0) = P_0 \otimes \mathbb{I} + \mathbb{I} \otimes P_0, \tag{6}$$

P_0 commuting with J_i. As for coproducts of angular momentum $\vec{J}$, let $\hat{n}$, $\hat{n}^{(1)}$, and $\hat{n}^{(2)}$ form an orthonormal frame with $\hat{n}^{(1)} \wedge \hat{n}^{(2)} = \hat{n}$ and let $\vec{n}^{(\pm)} = \hat{n}^{(1)} \pm i\hat{n}^{(2)}$. Then, using $[\hat{n} \cdot \vec{J}, \vec{n}^{(\pm)} \cdot \vec{J}] = \pm\vec{n}^{(\pm)} \cdot \vec{J}$, we find

$$\Delta_{\chi\hat{n}}(\hat{n} \cdot \vec{J}) = \hat{n} \cdot \vec{J} \otimes \mathbb{I} + \mathbb{I} \otimes \hat{n} \cdot \vec{J}, \tag{7}$$

$$\Delta_{\chi\hat{n}}(\vec{n}^{(\pm)} \cdot \vec{J}) = \vec{n}^{(\pm)} \cdot \vec{J} \otimes e^{\mp i(\chi/2)P_0} + e^{\pm i(\chi/2)P_0} \otimes \vec{n}^{(\pm)} \cdot \vec{J}. \tag{8}$$

Our basic Pauli-forbidden process is that of two electrons in an excited state transiting to the ground two-electron state already occupied by two electrons. This transition can be caused by a generic perturbation $V_{\chi\vec{n}}$ of the two-electron Hamiltonian. For $\chi = 0$, for simplicity, we take V_0 to be spin-independent, like the Coulomb repulsion $\frac{e^2}{|\vec{x}^{(1)} - \vec{x}^{(2)}|}$, between the two electrons. As V_0 must commute with τ_0, it is also symmetric in the electron coordinates. In the presence of the twist, the perturbation, just as $\Delta_{\chi\vec{n}}(P_0)$ and all observables, must commute with $\tau_{\chi\vec{n}}$, making us modify V_0 to

$$V_{\chi\vec{n}} = \tfrac{1}{2}[V_0 + \tau_{\chi\vec{n}} V_0 \tau_{\chi\vec{n}}]. \tag{9}$$

So the two-electron Hamiltonian

$$H^{(2)} = \Delta_{\chi\vec{n}}(P_0) + V_{\chi\vec{n}} \tag{10}$$

is $\chi\vec{n}$-dependent.

Remark.—Hopf symmetry, like any other symmetry, can be broken. Since $H^{(2)} \neq \Delta_{\chi\vec{n}}(P_0)$, the Hopf symmetry generated by P_0 and $\vec{J}$ is broken.

We consider only orbital angular momentum $l = 0$ energy levels for ease of calculation and choose a basis of spin states $|\alpha\rangle_{\vec{n}}$ ($\alpha = \pm 1$, often denoted as just $\pm$) polarized in direction $\vec{n}$:

$$\frac{\vec{\sigma} \cdot \vec{n}}{2} |\alpha\rangle_{\vec{n}} = \frac{\alpha}{2} |\alpha\rangle_{\vec{n}}, \qquad \sigma_i = \text{Pauli matrices}. \tag{11}$$

Then, if $|\nu\rangle$ are the radial single electron states for principal quantum number ν, we write

$$|\nu\rangle \otimes |\alpha\rangle_{\vec{n}} = |\nu, \alpha\rangle_{\vec{n}}, \tag{12}$$

$$|\nu, \alpha\rangle_{\vec{n}} \otimes |\nu', \beta\rangle_{\vec{n}} = |\nu, \alpha; \nu', \beta\rangle_{\vec{n}}. \tag{13}$$

The energy of $|\nu, \alpha\rangle_{\vec{n}}$ is called E_ν, while that of $|\nu, \alpha; \nu', \beta\rangle_{\vec{n}}$, on ignoring $V_{\chi\vec{n}}$ is $E_\nu + E_{\nu'}$.

The normalized twist-antisymmetrized two-electron ground state is

$$|1, 1\rangle_{\chi\vec{n}} = \frac{1}{\sqrt{2}}[|1+, 1-\rangle_{\vec{n}} - e^{i\chi E_1}|1-, 1+\rangle_{\vec{n}}] = -e^{i\chi E_1}\sqrt{2}\frac{1 - \tau_{\chi\vec{n}}}{2}(|1-, 1+\rangle_{\vec{n}}). \tag{14}$$

By the action of Eq. (6) on this state, its energy is $2E_1$.

For $\chi = 0$, the ground state, a spin singlet, is unique. By continuity, it remains so for $\chi \neq 0$. For this reason, replacement of either $|\pm\rangle_{\vec{n}}$ in Eq. (14) by other spin states does not give new answers.

We put two of the electrons in the above ground state.

We put the remaining two electrons in the s-wave levels with $\nu = 2$ and 3. Consider for specificity their state

$$|2+, 3+\rangle_{\chi\vec{n}} = \frac{1}{\sqrt{2}}[|2+, 3+\rangle_{\vec{n}} - e^{i(\chi/2)(E_3 - E_2)}|3+, 2+\rangle_{\vec{n}}]. \tag{15}$$

When the world turns, the Hamiltonian becomes $\Delta_{\chi\vec{m}}(P_0) + V_{\chi\vec{m}}$. The projectors to its antisymmetrized eigenstates, in particular, the projector $|1\ 1\rangle_{\chi\vec{m}\chi\vec{m}}\langle 1\ 1|$ is an observable because the Hamiltonian is an observable. So, in particular, the Hilbert space of states contains $\mathbb{C}|1\ 1\rangle_{\chi\vec{m}}$.

But in the sudden approximation, it contains $|1\ 1\rangle_{\chi\vec{n}}$ as well. We now show that it is not orthogonal to the $\tau_{\chi\vec{m}} = +1$ state

$$\frac{1 + \tau_{\chi\vec{m}}}{2}|1+, 1+\rangle_{\vec{m}} = |1+, 1+\rangle_{\vec{m}}. \tag{16}$$

This follows from [5]

$$|_{\vec{m}}\langle\rho|\alpha\rangle_{\vec{n}}|^2 = \tfrac{1}{2}[1 + (-1)^{(\rho-\alpha)/2}\vec{m}\cdot\vec{n}] \quad (17)$$

so that

$$|_{\vec{m}}\langle 1+, 1+|1\ 1\rangle_{\chi\vec{n}}|^2 = \frac{1}{2}[1 - (\vec{m}\cdot\vec{n})^2]\sin^2\left(\frac{\chi E_1}{2}\right) \neq 0 \quad (18)$$

if $\vec{m} \neq \vec{n}$, $\chi \neq 0$.

Thus $|1\ 1\rangle_{\chi\vec{n}}$, which is in the Hilbert space of states, is linearly independent of the $\tau_{\chi\vec{m}} = -1$ vector $|1\ 1\rangle_{\chi\vec{m}}$. Hence the Hilbert space contains at least one vector with energy $2E_1$ perpendicular to $|1\ 1\rangle_{\chi\vec{m}}$, namely,

$$|1\ 1\rangle_{\chi\vec{n}} - {}_{\chi\vec{m}}\langle 1\ 1|1\ 1\rangle_{\chi\vec{n}}|1\ 1\rangle_{\chi\vec{m}}.$$

It is part of a spin triplet. But $\Delta_{\chi\vec{m}}(J_i)$ are observables and form an angular momentum algebra, and its triplet representation is irreducible. So now the ground state is enhanced to contain the entire triplet of angular momentum one states. The projector

$$P = |1, 1\rangle\langle 1, 1| - |1, 1\rangle_{\chi\vec{m}\chi\vec{m}}\langle 1, 1| \quad (19)$$

to the subspace of these states is also an observable.

We now calculate the rate for the transition

$$|2+, 3+\rangle_{\chi\vec{n}} \rightarrow \frac{1+\tau_{\chi\vec{m}}}{\sqrt{2}}|1\alpha, 1\beta\rangle_{\vec{m}} \quad (20)$$

due to the potential $V_{\chi\vec{m}}$. We can neglect its χ dependence, which only introduces $O(\chi^2)$ corrections in the transition amplitude. The perturbation V_0 is symmetric in electron coordinates as it must commute with τ_0. By assumption, it is spin-independent.

Then, if at time t_i the two electrons were in the $|2+, 3+\rangle_{\chi\vec{n}}$ state, the transition probability $\mathcal{P}(t_f, t_i)$ to any of the three bosonic ground state at time t_f is

$$\mathcal{P}(t_f, t_i) = |\langle 1, 1|\int_{t_i}^{t_f} d\tau e^{i\tau 2E_1}V_0(\tau)e^{-i\tau(E_2+E_3)}|2, 3\rangle|^2 P^{\chi}_{\rm SPIN}, \quad (21)$$

where $\langle 1, 1|V_0|2, 3\rangle$ is the radial matrix element of V_0 and

$$P^{\chi}_{\rm SPIN} = \tfrac{1}{2}|(1 - e^{i(\chi/2)(E_3-E_2)})|^2 \times \{1 - \tfrac{1}{2}|1 - e^{-i\chi E_1}|^2\tfrac{1}{4}[1 - (\vec{m}\cdot\vec{n})^2]\}. \quad (22)$$

Since $\vec{m}$ and $\vec{n}$ keep changing, now average $P^{\chi}_{\rm SPIN}$ over the directions of $\vec{m}$ and $\vec{n}$ by using the standard rotationally invariant measure $d\Omega = \frac{1}{4\pi}d\cos\theta d\cos\phi$. Then $\langle P^{\chi}_{\rm SPIN}\rangle = \frac{1}{3}[5 + \cos(\chi E_1)]\sin^2(\frac{\chi}{4}\Delta E)$, where $\Delta E = E_3 - E_2$.

It is best to work with the branching ratio B_χ of the Pauli-forbidden process to an allowed process to cancel out the details specific to our model and give a formula of general applicability. The factor multiplying the χ-dependent part in the averaged rate is expected to approximate a typical Pauli-allowed process. Thus the branching ratio of a Pauli-forbidden to an allowed process is

$$B_\chi = \frac{1}{3}[5 + \cos(\chi E_1)]\sin^2\left(\frac{\chi}{4}\Delta E\right). \quad (23)$$

THE BOUNDS.—We can now use B_χ for both atomic and nuclear experiments [6–11] to deduce bounds on χ with

$$|\Delta E| \approx \begin{cases} 1\ \text{MeV} & \text{for } ^{12}\text{C and } ^{12}\text{O nucleus} \\ 272\ \text{eV} & \text{for } ^{12}\text{C atom} \\ 1.5\ \text{KeV} & \text{for Cu atom.} \end{cases}$$

Some of the above experiments give only lifetimes for the forbidden processes. To obtain the branching ratio in such cases, we multiply the given rate with the typical lifetimes for such processes. In the case of an atomic process, we use the number 10^{-16} seconds and for a nuclear process we use 10^{-23} seconds.

The bounds are summarized in Table I. They are obtained from the following experimental branching ratios.

Borexino [6] gives a lifetime for the process

$$\tau(^{12}\text{C} \rightarrow {}^{12}\tilde{\text{C}} + \gamma) > 2.1 \times 10^{27}\ \text{years},$$

where $^{12}\tilde{\text{C}}$ is a hypothetical Pauli-forbidden nucleus with an extra nucleon in the filled K shell of ^{12}C. This corresponds to a branching ratio of the order of 10^{-58}.

In the Kamiokande [7] experiment, searches were made for forbidden transitions in ^{16}O nuclei, and they obtain a bound on the ratio of forbidden transitions to normal transitions. This branching ratio is $<2.3 \times 10^{-57}$.

The NEMO Collaboration [8] searches for anomalous $^{12}\hat{\text{C}}$ atoms which are those with 3 K-shell electrons. The bound on the existence of such atoms is $\frac{^{12}\hat{\text{C}}}{^{12}\text{C}} < 2.5 \times 10^{-12}$.

NEMO-2 calculation [9] gives a lifetime $>4.2 \times 10^{24}$ years for a ^{12}C nuclear process, which corresponds to a branching ratio of the order $<10^{-55}$.

Atomic experiments at Maryland introduce new electrons into a copper strip and look for K x rays that would be emitted if one of these electrons were to be captured by a Cu atom and cascade down to the fully filled $1S$ state. The probability for this to occur was found to be less than 1.76×10^{-26} [10].

TABLE I. Bounds on the noncommutativity parameter χ.

Experiment	Type	Bound on χ (length scales)	Bound on χ (energy scales)
Borexino	Nuclear	$\lesssim 10^{-43}$ m	$\gtrsim 10^{24}$ TeV
Kamiokande	Nuclear	10^{-42} m	10^{23} TeV
NEMO	Atomic	10^{-15} m	10^{8} eV
NEMO-2	Nuclear	10^{-41} m	10^{22} TeV
Maryland	Atomic	10^{-22} m	10^{3} TeV
VIP	Atomic	10^{-23} m	10^{4} TeV

An improved version of the experiment at Maryland has been performed by the VIP Collaboration [11]. They find this probability to be less than 4.5×10^{-28}.

REMARKS.—The existence of twisted "symmetric" states for electrons is due to the use of the theory of sudden approximation in our model. As explained earlier, such effects happen as the twisted symmetrization operator itself becomes time-dependent in the model considered here.

Non-Pauli effects are expected for the Moyal twist τ_θ as well [12]. For the Moyal plane, the relative coordinates and x_0 mutually commute [13], forcing us to consider relativistic kinematics where center-of-mass and relative motions influence each other. That is why we considered $\mathcal{B}_{\chi\hat{n}}$ [Eq. (1)], where it is much easier to do the calculation.

Phenomenological models of large extra dimensions and the Randall-Sundrum scenario [14] bring the scale of new fundamental physics from 10^{16} or 10^{19} GeV down to 10 or 100 TeV scales. If the effective four-dimensional (reduced) Planck energy scale is in the TeV range, these time scales are very short and may be of the order of 10^{-18} sec. So for these processes, the Earth's movements are adiabatic (not sudden). By the adiabatic theorem, we expect that $\tau_{\chi\vec{n}}$ eigenstates will smoothly evolve into $\tau_{\chi\vec{m}}$ eigenstates of the same eigenvalue. No non-Pauli effects can thus be hoped for.

Lifetimes for non-Pauli transitions, which create Pauli-forbidden levels, are much longer than the age of the Universe in our model. So if there were only Pauli-allowed levels at the initiation of the Universe, there will not be a significant amount of non-Pauli levels now. Hence, no conflict with experiment from the lack of abundance of these levels is expected.

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