Adiabatic Markovian Dynamics

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We propose a theory of adiabaticity in quantum Markovian dynamics based on a decomposition of the Hilbert space induced by the asymptotic behavior of the Lindblad semigroup. A central idea of our approach is that the natural generalization of the concept of eigenspace of the Hamiltonian in the case of Markovian dynamics is a noiseless subsystem with a minimal noisy cofactor. Unlike previous attempts to define adiabaticity for open systems, our approach deals exclusively with physical entities and provides a simple, intuitive picture at the Hilbert-space level, linking the notion of adiabaticity to the theory of noiseless subsystems. As two applications of our theory, we propose a general framework for decoherence-assisted computation in noiseless codes and a dissipation-driven approach to holonomic computation based on adiabatic dragging of subsystems that is generally not achievable by nondissipative means.

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Introduction.—The adiabatic theorem is a simple and powerful result that has been known since the early days of quantum mechanics [1,2]. It states roughly that a closed system in an eigenstate of a continuously perturbed Hamiltonian remains in an instantaneous eigenstate in the limit of slow perturbations if the corresponding eigenvalue is separated from the rest of the spectrum by a gap. Quantum adiabaticity has applications in many areas, including quantum chemistry [3], geometric phases [4,5], quantum Hall effect [6], STIRAP [7], and quantum phase transitions [8]. More recently, the adiabatic theorem has been the subject of increased interest in relation to quantum information processing, where it has served as a basis for a variety of schemes, including holonomic quantum computation [9] and adiabatic quantum algorithms [10].

Given the importance of the concept of adiabaticity in closed quantum systems, it is natural to ask how this concept extends to the dynamics of systems interacting with an environment. This question is of particular interest from the point of view of quantum information processing where decoherence is a major obstacle to the construction of reliable quantum devices, and at the same time nonunitary processes are an important tool for quantum control. In Ref. [11], Sarandy and Lidar proposed an approach to the adiabatic dynamics of open quantum systems, defining adiabaticity as the regime in which the operator subspaces corresponding to the instantaneous Jordan blocks of the generator of the dynamics evolve independently (for adiabaticity in weakly open systems, see Ref. [12]). This definition is motivated by the formal analogy between the Schrödinger equation and the time-dependent Markovian master equation written in a coherence basis, both being first-order linear vector differential equations with the difference that the generator of the master equation is generally not diagonalizable (hence the Jordan decomposition). But while in closed systems the phenomenon of adiabaticity concerns the decoupled evolution of eigenspaces of the Hamiltonian which themselves are Hilbert spaces containing physical states, the Jordan blocks correspond to generally nonorthogonal subspaces of the space of linear operators that need not contain density matrices or even observables and may decay to zero even when mutually decoupled. In the present Letter, we propose a different approach, based primarily on physical considerations, which yields an inequivalent picture of open-system adiabaticity that links adiabatic dynamics to the theory of noiseless subsystems [13].

Taking as a ground the basic physical characteristic of adiabatic closed-system evolutions—namely, that these are quasistatic evolutions where under sufficiently slow changes of the Hamiltonian a system in a stationary state evolves so as to remain in a stationary state with respect to the changed Hamiltonian—we look for a generalization of this phenomenon to the case of Markovian dynamics. The key insight of our approach is that the natural generalization of the eigenspaces of the Hamiltonian corresponding to distinct eigenvalues are noiseless subsystems whose noiseful cofactors support unique fixed states. A decomposition of the Hilbert space into such subsystems arises naturally from the asymptotic behavior of the Lindblad semigroup [14]. We define adiabaticity as the regime in which the stationary states over such a noiseless subsystem and its cofactor remain stationary with respect to the Lindbladian as it changes. We derive an adiabatic theorem based on this definition.

To illustrate the utility of our formalism, we propose two applications. One is a framework for decoherence-assisted computation in noiseless codes which generalizes the approach of Beige *et al.* [15] to subsystems and general noise models. The other is a dissipation-driven approach to holonomic quantum computation based on adiabatic "dragging" of subsystems [16] along paths that are generally not achievable by nondissipative means.

Generalization of eigenspaces.—Our starting point is the observation that the eigenstates of a Hamiltonian H are the stationary state vectors of its dynamics. In particular, all stationary density matrices under the evolution $d\rho/dt = -i[H,\rho]$ (we set $\hbar = 1$) have the direct-sum form $\rho = \oplus_i p_i \rho_i$, $\sum_i p_i = 1$, $p_i \geq 0$, where ρ_i are density matrices over the eigenspaces $\mathcal{H}_i$ of H corresponding to distinct eigenvalues. In more general quantum processes, the stationary states are organized as operators over noiseless subsystems tensored with a fixed density matrix over the corresponding noiseful cosubsystem [17]. Consider a time-homogenous finite-dimensional Markovian dynamics described by the Lindblad equation [18]

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i \left(L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right) \equiv \mathcal{L}\rho, \tag{1}$$

where L_i are Lindblad operators. As shown in Ref. [14], Eq. (1) induces a decomposition of the Hilbert space

$$H = \bigoplus_{ij} \mathcal{H}^A_{ij} \otimes \mathcal{H}^B_j \oplus \mathcal{K}, \tag{2}$$

where $\mathcal{H}^A_{ij}$ are noiseless subsystems [13], $\mathcal{H}^B_j$ are noiseful subsystems that support unique fixed states, and $\mathcal{K}$ is a decaying subspace. More particularly, it was shown that for any initial state $\rho(0)$, the solution of Eq. (1) satisfies

$$\exists \{p_k, \rho^A_k\}: \lim_{t \to \infty} \| \rho(t) - \bigoplus_j p_j e^{-iH^A_j t} \rho^A_j e^{iH^A_j t} \otimes \varrho^B_j \| = 0, \tag{3}$$

where ρ^A_j are density matrices on the unitarily noiseless subsystems $\mathcal{H}^A_j = \oplus_i \mathcal{H}^A_{ij}$ evolving under the Hamiltonians H^A_j, ϱ^B_j are fixed full-support states on $\mathcal{H}^B_j$, and $\sum_k p_k = 1$, $p_k \geq 0$. The noiseless subsystems $\mathcal{H}^A_{ij}$ are the eigenspaces of H^A_j. The stationary states have the form $\rho = \oplus_{ij} p_{ij} \rho^A_{ij} \otimes \varrho^B_j$, $\sum_{ij} p_{ij} = 1$, $p_{ij} \geq 0$, where ρ^A_{ij} are density matrices on $\mathcal{H}^A_{ij}$. This suggests that the subsystems $\mathcal{H}^A_{ij}$ whose cofactors $\mathcal{H}^B_j$ support unique fixed states ϱ^B_j can be thought of as the generalization of eigenspaces corresponding to distinct eigenvalues.

How do we find the decomposition (2) for a given Lindbladian $\mathcal{L}$? An algorithm for finding the noiseless subsystems of a completely positive trace-preserving (CPTP) map that runs in time $O[(\dim \mathcal{H})^6]$ was described in Ref. [17] (see also Ref. [19]). It is based on finding the left and right operator eigenspaces corresponding to the eigenvalue 1 of the CPTP map. Since Eq. (1) is equivalent to the continuous application of an infinitesimal CPTP map, the same algorithm can be used here (the eigenvalue 1 of the map translates to eigenvalue 0 of $\mathcal{L}$).

Before we introduce adiabaticity for Markovian dynamics, let us briefly review the closed-system case.

Adiabaticity in closed systems.—Consider a time-dependent Hamiltonian $H(t/T)$ changing along a differentiable curve $H(s)$, $s \in [0,1]$. Let $\epsilon_i(s)$ be an eigenvalue of $H(s)$ with multiplicity m, and $P_i(s)$ be the (twice-differentiable) projector on the corresponding eigenspace $\mathcal{H}_i(s) = P_i(s)\mathcal{H}$. [Note that $m = \text{const}(s)$ implies that $\epsilon_i(s)$ is separated from the rest of the spectrum by a gap. The adiabatic theorem has been extended to cases without a gap [20], but in this Letter we restrict it to the standard formulation.] The eigenspace $\mathcal{H}_i(t/T)$ is said to evolve adiabatically under $H(t/T)$ if any state initially in $\mathcal{H}_i(0)$ remains in $\mathcal{H}_i(t/T)$, $t \in [0,T]$. Equivalently, if we change the basis via a unitary $U(s)$ so that P_i becomes fixed, in the new basis the dynamics is driven by the effective Hamiltonian $H'(t/T) = \tilde{H}(t/T) + \frac{1}{T} V(t/T)$, where $\tilde{H}(s) = U(s)H(s)U(s)^\dagger = \epsilon_i(s)P_i + \tilde{H}^\perp_i(s)$ with $\tilde{H}^\perp_i(s)$ having support on the orthogonal complement of $\mathcal{H}_i$, and $V(s) = i\frac{dU(s)}{ds} U^\dagger(s)$. Adiabaticity then refers to the regime in which any state initially in $\mathcal{H}_i$ remains in $\mathcal{H}_i$ despite the action of $\frac{1}{T} V(t/T)$. The adiabatic theorem states [2] that in the limit of large T, one approaches perfect adiabaticity where the states in $\mathcal{H}_i$ evolve via the unitary $U_i(s) = \mathcal{T} \exp(-i \int_0^s P_i V(q) P_i dq)$ where $\mathcal{T}$ denotes time ordering. The error scales with T as $O(\frac{1}{T\Delta})$, where $\Delta > 0$ is a fixed energy scale (e.g., the energy gap).

Note that unlike the "folk" adiabatic condition which is known to be insufficient [21], this theorem (similarly to the one derived below) is concerned with the scaling of the error as a function of T for a fixed curve $H(s)$.

Adiabaticity in Markovian dynamics.—Consider a time-dependent Lindbladian $\mathcal{L}(t/T)$ changing along a differentiable curve $\mathcal{L}(s)$, $s \in [0,1]$. For every s, $\mathcal{L}(s)$ induces a decomposition of the Hilbert space $\mathcal{H} = \oplus_{ij} \mathcal{H}^A_{ij}(s) \otimes \mathcal{H}^B_j(s) \oplus \mathcal{K}(s)$ as explained earlier. Let $\mathcal{H}^A_{kl}(s)$ and $\mathcal{H}^B_l(s)$ [$\dim \mathcal{H}^A_{kl}(s) = m$, $\dim \mathcal{H}^B_l(s) = n$] be two subsystems of the type above, and let $\mathcal{P}_{kl}(s)$ [$\mathcal{P}_{kl}(s)\rho = \text{Tr}_B\{P^{AB}_{kl}(s)\rho P^{AB}_{kl}(s)\} \otimes \varrho^B_l(s)$ where $P^{AB}_{kl}(s)$ is the projector on $\mathcal{H}^A_{kl}(s) \otimes \mathcal{H}^B_l(s)$ and Tr_B denotes partial trace over $\mathcal{H}^B_l$] be the (twice-differentiable) superoperator projector on the fixed points over $\mathcal{H}^A_{kl}(s) \otimes \mathcal{H}^B_l(s)$.

Note.—Similarly to the closed-system case, the assumption that $\dim \mathcal{H}^A_{kl}(s)$ and $\dim \mathcal{H}^B_l(s)$ are constant implies an analogue of the gap condition (see [22]).

Definition.—The noiseless subsystem $\mathcal{H}^A_{kl}(t/T)$ and its noisy cofactor $\mathcal{H}^B_l(t/T)$ evolve adiabatically under $\mathcal{L}(t/T)$, if any state over $\mathcal{H}^A_{kl}(0) \otimes \mathcal{H}^B_l(0)$ of the form $\rho(0) = \rho(0)^A_{kl} \otimes \varrho^B_l(0)$ evolves to a state $\rho(t) = \rho(t)^A_{kl} \otimes \varrho^B_l(t/T)$ over $\mathcal{H}^A_{kl}(t/T) \otimes \mathcal{H}^B_l(t/T)$, $t \in [0,T]$.

As in the case of closed systems, it is convenient to consider a basis rotated by a unitary $U(s)$, in which $\mathcal{H}^A_{kl}$ and $\mathcal{H}^B_l$ are fixed. In this basis, the master equation is

$$\frac{d\rho}{dt} = -\frac{i}{T}[V(t/T), \rho] + \tilde{\mathcal{L}}(t/T)\rho, \tag{4}$$

where $\tilde{\mathcal{L}}(s)$ is the Lindbladian with $H(s)$ replaced by $U(s)H(s)U(s)^\dagger$ and $L_i(s)$ by $U(s)L_i(s)U(s)^\dagger$, and $V(s) = i\frac{dU(s)}{ds} U(s)^\dagger$. (We will not use a different notation for ρ in this basis but will keep in mind the basis we are working in.) Adiabaticity then means that any state $\rho(0) = \rho^A_{kl}(0) \otimes \varrho^B_l(0)$ remains of the form $\rho(t) = \rho^A_{kl}(t) \otimes \varrho^B_l(t/T)$ despite the perturbation $\frac{1}{T} V(t/T)$.

Theorem.—Consider Markovian dynamics satisfying the above assumptions. In the limit of large T, perfect adiabaticity is approached with an error that scales as $O(\sqrt{\frac{1}{T\Delta}})$, where $\Delta > 0$ is some fixed energy scale. In the adiabatic limit, the states inside $\mathcal{H}_{kl}^A$ evolve under the unitary $U_{kl}^A(s) = \mathcal{T}\exp(-i\int_0^s \mathrm{Tr}_B\{P_{kl}^{AB}V(q)P_{kl}^{AB}I_{kl}^A \otimes \varrho_l^B(q)\}dq)$.

Proof.—Let us divide the total time T into N steps, each of length δt, $T = N\delta t$. We will take $\delta t = N/\Delta$ (hence, $T = N^2/\Delta$) such that when $N \to \infty$, δt is short on the time scale of change of the Lindbladian but long on the time scale for reaching the asymptotic regime of the instantaneous Lindbladian. The differentiability assumptions about $\mathcal{L}(s)$ and $\mathcal{P}_{kl}(s)$ imply that we can write $\tilde{\mathcal{L}}(\frac{t+t'}{T}) = \tilde{\mathcal{L}}(\frac{t}{T}) + O(\frac{1}{N})$, $V(\frac{t+t'}{T}) = V(\frac{t}{T}) + O(\frac{1}{N})$, $t' \in [0, \delta t]$. The evolution of the density matrix during a single time step is then

$$\begin{aligned}\rho(t) &\to \rho(t+\delta t)\\ &= \mathcal{T}e^{\int_0^{\delta t}dt'\tilde{\mathcal{L}}(\frac{t+t'}{T})}\rho(t) - \frac{i}{T}\int_0^{\delta t}dt' e^{\tilde{\mathcal{L}}(t/T)(\delta t - t')}\\ &\quad\times\left[V\left(\frac{t}{T}\right), e^{\tilde{\mathcal{L}}(t/T)t'}\rho(t)\right] + O\left(\frac{1}{N^2}\right). \end{aligned} \tag{5}$$

Assume that the state at time t has the form

$$\rho(t) = \rho_{kl}^A(t) \otimes \left[\varrho_l^B\left(\frac{t}{T}\right) + O\left(\frac{1}{N}\right)\right] + O\left(\frac{1}{N^2}\right). \tag{6}$$

Then the first term on the right-hand side of Eq. (5) is $\mathcal{T}e^{\int_0^{\delta t}dt'\tilde{\mathcal{L}}(\frac{t+t'}{T})}\rho(t) = \rho_{kl}^A(t) \otimes [\varrho_l^B(\frac{t}{T}) + O(\frac{1}{N})] + O(\frac{1}{N^2})$, since $\mathcal{H}_{kl}^A$ is noiseless and $\mathcal{T}e^{\int_0^{\delta t}dt'\tilde{\mathcal{L}}(\frac{t+t'}{T})} = e^{\delta t\tilde{\mathcal{L}}(\frac{t}{T})} + O(\frac{1}{N})$, so for large δt the state on $\mathcal{H}_l^B$ decays to $\varrho_l^B(\frac{t}{T}) + O(\frac{1}{N})$ (see the supplementary material [22] for an exact relation to the decay rate). For the second term, ignoring errors of order $O(\frac{1}{N^2})$, we can use $\rho(t) = \rho_{kl}^A(t) \otimes \varrho_l^B(\frac{t}{T})$. But $e^{\tilde{\mathcal{L}}(\frac{t}{T})t'}$ leaves $\rho(t)$ invariant, so this term becomes $\frac{-i}{T}\int_0^{\delta t}dt' e^{\tilde{\mathcal{L}}(\frac{t}{T})(\delta t - t')}[V(\frac{t}{T}), \rho_{kl}^A(t) \otimes \varrho_l^B(\frac{t}{T})]$. Using noiseless-subsystem properties of the Lindbladian [23,24], in the supplementary material [22] we show that this term is equal to $\frac{-i}{T}\int_0^{\delta t}dt' e^{\tilde{\mathcal{L}}(\frac{t}{T})(\delta t - t')}\mathcal{P}_{kl}[V(\frac{t}{T}), \rho_{kl}^A(t) \otimes \varrho_l^B(\frac{t}{T})] + O(\frac{1}{N^2})$. But $\tilde{\mathcal{L}}(s)\mathcal{P}_{kl} = 0$, so the integral yields $\frac{-i\delta t}{T}\mathcal{P}_{kl}[V(\frac{t}{T}), \rho_{kl}^A(t) \otimes \varrho_l^B(\frac{t}{T})] = -i\frac{\delta t}{T} \times [\mathrm{Tr}_B\{P_{kl}^{AB}V(\frac{t}{T})P_{kl}^{AB}I_{kl}^A \otimes \varrho_l^B(\frac{t}{T})\}, \rho_{kl}^A(t)] \otimes \varrho_l^B(\frac{t}{T})$ (the last inequality can be verified by a simple algebra).

We therefore see that if the initial state is of the form (6), it will remain of this form for all times, up to an error $O(\frac{1}{N}) = O(\sqrt{\frac{1}{\Delta T}})$ resulting from the accumulation of the errors $O(\frac{1}{N^2})$ at every step. Moreover, we see that the reduced density matrix on $\mathcal{H}_{kl}^A$ satisfies the difference equation $\rho_{kl}^A(t+\delta t) - \rho_{kl}^A(t) = -\frac{i\delta t}{T}[\mathrm{Tr}_B\{P_{kl}^{AB}V(\frac{t}{T})P_{kl}^{AB}I_{kl}^A \otimes \varrho_l^B(\frac{t}{T})\}, \rho_{kl}^A(t)] + O(\frac{1}{N^2})$, which in the limit $N \to \infty$ yields the differential equation $\frac{\partial}{\partial s}\rho_{kl}^A(Ts) = -i[\mathrm{Tr}_B\{P_{kl}^{AB}V(s)P_{kl}^{AB}I_{kl}^A \otimes \varrho_l^B(s)\}, \rho_{kl}^A(Ts)]$ describing the effective evolution stated in the theorem.

Note.—Our theorem includes an adiabatic theorem for closed systems as a special case. However, the convergence rate stated in our theorem is weaker than the standard one [the error is $O(\sqrt{\frac{1}{\Delta T}})$ as opposed $O(\frac{1}{\Delta T})$] since our proof captures dissipative cases as well. (In the supplementary material [22], we describe a natural energy scale Δ associated with the curve $\mathcal{L}(s)$, which can be regarded as a generalization of the minimum energy gap.)

Decoherence-assisted computation in noiseless codes.—Computation in noiseless subsystems requires operations that keep the information inside the code [25]. However, the Hamiltonians that preserve the code in general may be rather complicated and may not be naturally available in a particular experimental setup. Thus strategies for achieving encoded universality [26] by other means are of particular interest [27]. An immediate implication of the above theorem is that for the common case of time-homogenous Markovian noise with Lindbladian $\mathcal{L}$ [to play the role of $\tilde{\mathcal{L}}(t/T)$ in Eq. (4)], any Hamiltonian perturbation $\frac{1}{T}V(t/T)$ acting during $t \in [0, T]$ would give rise to (possibly nontrivial) unitary evolutions inside the noiseless subsystems $\mathcal{H}_{ij}^A$ of $\mathcal{L}$ within an arbitrary precision for sufficiently large T. Thus given a set of available interactions $\{V_\mu\}$ that can be turned on with variable strength, for a given subsystem $\mathcal{H}_{kl}^A$ one can produce the set of effective interactions

$$V_\mu^{\mathrm{eff}} = \mathrm{Tr}_B(P_{kl}^{AB}V_\mu P_{kl}^{AB}I_{kl}^A \otimes \varrho_l^B). \tag{7}$$

(Note that preparation of the states on $\mathcal{H}_l^B$ is not needed as they quickly decay to the fixed points.) Encoded universality is achieved if the set $\{V_\mu^{\mathrm{eff}}\}$ spans the Lie algebra $su(m)$ over $\mathcal{H}_{kl}^A$. Remarkably this is possible even if the Hamiltonians $\{V_\mu\}$ commute (see example below).

Such an approach was first proposed in Ref. [15] for noiseless subspaces ($\dim\mathcal{H}_l^B = 1$) under certain noise models that can be interpreted as continuous Zeno measurements projecting onto the subspace. Equation (7) provides a generalization of this idea to noiseless subsystems (that may exist even when no noiseless subspaces exist) and arbitrary time-homogenous Markovian models. As an example, we have studied [22] a *two-level* noiseless subsystem of three spin-$\frac{1}{2}$ particles under collective decoherence [13]. The noiseless subsystem involves highly entangled states, and nonlocal interactions are in principle required to perform operations on the encoded qubit. However, we find that the decoherence process itself can be used to induce an effective universal set of gates on the code by acting with local Hamiltonians.

Holonomic quantum computation via dissipation.—In the previous method, we assumed that the perturbation $\frac{1}{T}V(s)$ is applied by the experimenter. However, the conclusions are valid also if we assume that the description is with respect to an instantaneous basis of a time-dependent noiseless subsystem $\mathcal{H}_{kl}^A(s)$ of $\mathcal{L}(s)$, where the perturbation now arises from the time dependence of the basis. As

$\mathcal{L}(s)$ acts trivially on $\mathcal{H}^A_{kl}(s)$, the effective transformation in $\mathcal{H}^A_{kl}(s)$ is not of dynamical origin. Indeed, in the adiabatic limit, an initial state $\rho^{AB}(0)$ over $\mathcal{H}^A_{kl}(0) \otimes \mathcal{H}^B_l(0)$ transforms via the superoperator $\lim_{\delta s \to 0} \mathcal{P}_{kl}(1)\mathcal{P}_{kl}(1 - \delta s) \ldots \mathcal{P}_{kl}(\delta s)\mathcal{P}_{kl}(0)$ which is a geometric quantity defined via the projectors $\mathcal{P}_{kl}(s)$. But the effective unitary on $\mathcal{H}^A_{kl}(s)$ depends on the choice of basis for $\mathcal{H}^A_{kl}(s)$ and is not gauge invariant. However, if $\mathcal{H}^A_{kl}(s)$ is taken around a loop, $\mathcal{H}^A_{kl}(0) = \mathcal{H}^A_{kl}(1)$, so that the final basis is the same as the initial one, the resultant transformation is a gauge-invariant quantity that generalizes the standard holonomy associated with parallel transport of subspaces [5]. We note that the idea of adiabatically "dragging" a subsystem (rather than a subspace) along suitable paths in order to perform geometric gates inside it has been proposed for the case of Hamiltonian dynamics as a powerful tool for robust computation [16]. However, a subsystem cannot be dragged along an arbitrary path $\mathcal{H}^A(s)$ by a Hamiltonian since some paths necessarily give rise to correlations between $\mathcal{H}^A(s)$ and $\mathcal{H}^B(s)$. This problem does not exist here since the Lindbladian acting on $\mathcal{H}^B(s)$ severs any such correlations. (For dissipation-driven holonomies in subspaces, see Ref. [28].)

The mathematical foundations of these geometric transformations will be studied elsewhere. In the supplementary material [22], we show that the method can be used for universal quantum computation. We use a slowly rotating depolarizing type of Lindbladian as an example.

Conclusion.—We introduced a theory of adiabatic Markovian dynamics that relates the notion of adiabaticity to the theory of noiseless subsystems. We proved an adiabatic theorem for such dynamics and proposed two novel methods of quantum information processing based on it—-decoherence-assisted computation in noiseless subsystems and dissipation-driven holonomic computation—that add to the developing picture of dissipation as a powerful quantum computation primitive [29]. A natural problem for future research would be to find exact bounds on the adiabatic error in Markovian dynamics similar to those obtained for closed systems, e.g., in Ref. [30].

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