Enhancement of the Bootstrap Current in a Tokamak Pedestal

Grigory Kagan[1,2] and Peter J. Catto[1]
[1]Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
[2]Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
(Received 15 January 2010; published 22 July 2010)

The strong radial electric field in a subsonic tokamak pedestal modifies the neoclassical ion parallel flow velocity, as well as the radial ion heat flux. Existing experimental evidence of the resulting alteration in the poloidal flow of a trace impurity is discussed. We then demonstrate that the modified parallel ion flow can noticeably enhance the pedestal bootstrap current when the background ions are in the banana regime. Only the coefficient of the ion temperature gradient drive term is affected. The revised expression for the pedestal bootstrap current is presented. The prescription for inserting the modification into any existing banana regime bootstrap current expression is given.

DOI: 10.1103/PhysRevLett.105.045002 PACS numbers: 52.55.Fa, 52.25.Vy, 52.30.-q

The bootstrap current [1,2] is a key feature of advanced tokamak operating regimes since it can dramatically reduce the need to drive current [3]. Moreover, the stability of tokamaks is sensitive to the details of the bootstrap as well as driven current profiles, especially for the density and temperature pedestal just inside the last closed flux surface [4–7].

Recent impurity flow measurements in the pedestal of Alcator C-Mod indicate that the poloidal flow of the background banana regime ions can be in the direction opposite to the one predicted by conventional neoclassical theory [8]. Any sign change in the poloidal ion flow has important implications for the bootstrap current since it is sensitive to the parallel background ion flow due to momentum exchange between electrons and ions.

However, experimental studies of the pedestal bootstrap current [5–7] measure currents large enough to impact edge stability and claim qualitative agreement with the Sauter, Angioni, and Lin-Liu model [9] based on the conventional neoclassical bootstrap expression [10–12] derived by assuming the poloidal ion gyroradius is small compared to the shortest pedestal scale length. Here we reconcile these seemingly contradictory experimental results for the bootstrap current and ion flow by first demonstrating that the change in the sign of the background poloidal ion flow is due to the strong radial electric in the pedestal [13] enhancing the pedestal bootstrap current. We then show that no contradiction arises because the new formula for the bootstrap current continues to be of the conventional form and therefore of the general form of Sauter, Angioni, and Lin-Liu [9] provided the ion temperature gradient coefficient is allowed to depend on the radial electric field in the pedestal.

The bootstrap current is normally regarded as the most important prediction of neoclassical theory and is enhanced by strong density and pressure gradients because of its diamagnetic nature. Conventional tokamaks have $B/B_p \sim qR/a \gg 1$, where B is the total magnetic field, B_p is the poloidal magnetic field, q is the safety factor, and R and a are the major and minor radii, respectively. As a result, the diamagnetism associated with the trapped and barely passing particles is larger than that associated with their gyromotion since the magnetic drift departure from a flux surface is roughly a poloidal gyroradius $\rho_{pj} = \rho_j B/B_p$ rather than a gyroradius $\rho_j = (2T_jM_j)^{1/2}c/|Z_j|eB$, where the subscript j denotes the species of temperature T_j, mass M_j, and charge Z_j, e is the magnitude of the charge of an electron, and c is the speed of light.

During high confinement operation of Alcator C-Mod the pedestal is found to have radial density and electron temperature variations on the scale of the poloidal ion gyroradius $\rho_{\rm pi}$ [14]. However, the flows in C-Mod are subsonic so the only way to satisfy radial ion pressure balance is for the ions to be nearly electrostatically confined with a somewhat weaker background ion temperature variation than the density [15]. This weaker ion temperature variation also enhances the bootstrap current and is required to minimize entropy production in the pedestal; however, it is not the primary effect of interest here.

The more important effect is the strong radial electric field needed to keep the ion flow subsonic in the pedestal since the poloidal $\vec{E} \times \vec{B}$ drift can compete with the small poloidal projection of the parallel ion streaming, thereby modifying conventional neoclassical results in the banana regime [13]. The ion flow is altered because the passing ion constraint on the ion-ion collision operator [10,11] must be imposed along the $\vec{E} \times \vec{B}$ modified ion trajectory by holding the canonical angular momentum fixed, rather than the poloidal flux function. This difference alters the nonlocal part of the ion distribution function and leads to a poloidal ion flow sensitive to the radial electric field when this passing collisional constraint is evaluated retaining the strong poloidal radial electric field variation along the trajectory. Orbit squeezing [16] does not play a role in modifying the nonlocal portion of the ion distribution function but does, of course, change the localized contribution and, thereby, radial transport.

The preceding discussion indicates that ion behavior in the pedestal can be expected to be rather different from that in the core. Of particular interest for the bootstrap current calculation is the change in the parallel ion flow on a flux surface caused by the strong radial electric field inherent in a subsonic pedestal because of the need to maintain radial pressure balance. Experiments find that in many tokamaks the pedestal width can be of the order of the poloidal ion gyroradius. Thus, for a pedestal ion, variation of the electrostatic energy across a neoclassical orbit is comparable to that of the kinetic energy, causing this orbit to be substantially modified compared to its core counterpart. Of course, electron orbits are essentially unchanged since $\rho_{\rm pe} \ll \rho_{\rm pi}$. However, even though electrons do not feel the pedestal electric field directly, it affects them indirectly through their friction with the modified parallel velocity of the bulk ions. Consequently, the coefficient preceding the ion temperature gradient term in the conventional formula and the Sauter, Angioni, and Lin-Liu form [9] is importantly modified in the pedestal.

As the preceding paragraph suggests, the first step to take in revisiting conventional neoclassical theory is to evaluate the ion orbits accounting for a strong radial electric field. Here it is worth noticing that the trapped and barely passing orbit widths are estimated by $\sqrt{\varepsilon}\rho_{\rm pi}$, where $\varepsilon \equiv a/R$ denotes the inverse aspect ratio. In a realistic tokamak pedestal $\sqrt{\varepsilon}$ is rather close to unity, making the ion orbit width comparable to the characteristic scale of the background electric field. This feature complicates the ion motion evaluation and, more importantly, can result in neoclassical transport being nonlocal. To obtain insight into the impact of the electric field on neoclassical phenomena, while staying within a local treatment, we consider the case of large aspect ratio. Then an ion "samples" the electric potential in the narrow vicinity of a flux surface, thereby allowing us to assume this potential parabolic and to evaluate the ion trajectory.

Knowing the ion orbits from Ref. [13], we proceed to the kinetic calculation. Here, the full Maxwellian Rosenbluth potential form of the like particle collision operator must be employed [17] along with a term that ensures momentum conservation. This more complete operator [18,19] captures both energy and pitch angle scattering ion transitions across the electric field modified trapped-passing boundary that is no longer a cone centered at the origin of the $(v_\perp, v_\|)$ plane. This operator is used to evaluate the passing collisional constraint in the pedestal to determine the nonlocal and localized neoclassical corrections to the leading order ion distribution function, which is a stationary Maxwellian f_{i0}. The Maxwellian remains stationary because the $\vec{E}\times\vec{B}$ drift cancels the ion diamagnetic drift in the pedestal to lowest order, and f_{i0} permits the strong density variation required for near electrostatic ion confinement through its dependence on total energy.

There are two changes that occur when evaluating the nonlocal portion of the constraint equation. The first is that the parallel velocity must be shifted by the quantity

$$u \equiv cI\phi'(\psi)/B \tag{1}$$

proportional to the poloidal $\vec{E}\times\vec{B}$ drift since the deeply trapped particles are at $v_\| + u \approx 0$ rather than at $v_\| \approx 0$. Here ϕ is the electrostatic potential with $\vec{E} = -\phi'\vec{\nabla}\psi = -(\partial\phi/\partial\psi)\vec{\nabla}\psi$ and ψ the poloidal flux function. The second is that the factors that must be introduced to ensure momentum conservation in ion-ion collisions change due to the finite electric field modification of the orbits. As a result, the perturbed ion distribution function becomes

$$f_{i1} = -\frac{Iv_\| f_{i0}}{\Omega_i}\left[\frac{1}{p_i}\frac{dp_i}{d\psi} + \frac{Ze}{T_i}\frac{d\phi}{d\psi} + \left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{1}{T_i}\frac{dT_i}{d\psi}\right] + g, \tag{2}$$

where $g = h_\sigma + (g - h_\sigma)$ vanishes for the trapped particles (but not the passing), $g - h_\sigma$ is the small local term giving an order $\sqrt{\varepsilon}$ correction to the ion flow (that we can neglect), and

$$h_\sigma = \frac{I(v_\| + u)}{\Omega_i}\left[\frac{M(v^2+u^2)}{2T_i} - \sigma\right]\frac{f_{i0}}{T_i}\frac{dT_i}{d\psi}, \tag{3}$$

with $I = RB_t$, with B_t the toroidal magnetic field, $\Omega_i = ZeB/Mc$, $\vec{B} = I\vec{\nabla}\zeta + \vec{\nabla}\zeta\times\vec{\nabla}\psi$, and n_i, T_i, $p_i = n_iT_i$, Z, and M the background ion density, temperature, pressure, charge number, and mass, respectively. Notice that, in addition to $v_\| \to v_\| + u$ and $v^2 \to v^2 + u^2$ in h_σ, the factor σ determined by demanding like particle momentum conservation when evaluating $g - h_\sigma$ becomes

$$\sigma = \frac{\int_0^\infty dy \exp(-y)(y + Mu^2/T_i)^{3/2}[\nu_\perp y + \nu_\| Mu^2/T_i]}{\int_0^\infty dy \exp(-y)(y + Mu^2/T_i)^{1/2}[\nu_\perp y + \nu_\| Mu^2/T_i]}, \tag{4}$$

where $\nu_\perp = 3(2\pi)^{1/2}\nu_{ii}[\mathrm{erf}(x) - \Psi(x)]/2x^3$ and $\nu_\| = 3(2\pi)^{1/2}\nu_{ii}\Psi(x)/2x^3$, with $x = v(M/2T_i)^{1/2}$, $\nu_{ii} = 4\pi^{1/2}Z^4e^4n_i\ell n\Lambda/3M^{1/2}T_i^{3/2}$, $\mathrm{erf}(x) = 2\pi^{-1/2}\times\int_0^\infty dy\exp(-y^2)]$, and $\Psi(x) = [\mathrm{erf}(x) - x\,\mathrm{erf}'(x)]/2x^2$. Using the preceding to obtain the lowest order parallel background ion velocity gives

$$V_{i\|} = -\frac{cI}{B}\left(\frac{d\phi}{d\psi} + \frac{1}{Zen_i}\frac{dp_i}{d\psi}\right) + \frac{7cIBJ(U)}{6Ze\langle B^2\rangle}\frac{dT_i}{d\psi}, \tag{5}$$

where

$$U \equiv u(M/2T_i)^{1/2}, \tag{6}$$

$\langle\ldots\rangle$ denotes a flux surface average, and [13]

$$J(U) = (6/7)[(5/2 - \sigma) + U^2]. \tag{7}$$

Adding the perpendicular ion velocity $\vec{V}_{i\perp} = (c/B^2)(\vec{B} \times \vec{\nabla}\psi)[(d\phi/d\psi) + (Zen_i)^{-1}(dp_i/d\psi)]$ to the parallel ion velocity gives the poloidal ion flow to be

$$V_i^{\rm pol} = \frac{7cIB_pJ(U)}{6Ze\langle B^2\rangle}\frac{dT_i}{d\psi}. \qquad (8)$$

The parameter U accounts for the presence of the equilibrium pedestal electric field and becomes comparable to unity once the spatial scale of the potential ϕ or density n_i is of order $\rho_{\rm pi}$. The shaping function $J(U)$ is introduced to denote the difference between the pedestal and conventional $J = 1$, $U = 0$, and $\sigma = 1.33$ result in the core. In the pedestal, Eqs. (4) and (7) yield J to be a monotonically decreasing function of equilibrium electric field, thereby increasing the bootstrap current. The function J goes negative for $U > 1.2$ to give an additive positive poloidal flow from the ion temperature gradient term. The average pedestal electric field in tokamaks such as Alcator C-Mod or DIII-D corresponds to $U \approx 0.75$ [14,20], and therefore we expect $V_i^{\rm pol}$ to change sign in the pedestal near the electric field minimum. Importantly, our calculation is carried out within the large aspect ratio approximation, so the formulas involving U stay the same to leading order in ε regardless of the point at which B is evaluated.

Before presenting the modification to the bootstrap current due to the novel features of the ion flow outlined in the preceding paragraphs, we discuss the experimental evidence available for this effect. To this end, the impurity flow measurements recently performed at C-Mod [8] turn out to be important since their poloidal flow is sensitive to that flow component of the background ions, and therefore measuring the former determines whether the latter is changed. Consequently, when the C-Mod study revealed impurity poloidal flows noticeably larger in banana regime pedestals than predicted by the conventional core formula, we were able to understand this seemingly contradictory result by retaining finite radial electric field modifications. We next demonstrate how this discrepancy is removed by employing our formula (7) instead of the usual $J = 1$ in the expression (8) for the poloidal ion flow.

The analysis is simplified due to the high charge number and mass, and therefore high collisionality, of the boron impurities used in the experiment. These features make the impurity mean free path much less than parallel connection length qR. As a result, the parallel ion and impurity flows are equal, and the usual formula relating the poloidal velocities of the banana regime background ions and the Pfirsch-Schluter impurities holds [21–24]:

$$V_z^{\rm pol} = V_i^{\rm pol} - \frac{cIB_p}{eB^2}\left(\frac{1}{Zn_i}\frac{dp_i}{d\psi} - \frac{1}{Z_zn_z}\frac{dp_z}{d\psi}\right), \qquad (9)$$

where Z_z, n_z, and p_z are the impurity charge number, density, and pressure, respectively, and higher order terms in the aspect ratio expansion are omitted. The conventional formula for the poloidal ion flow ($J = 1$) makes the sum of $V_i^{\rm pol}$ and the diamagnetic terms tend to cancel on the right side of (9). Therefore, the left side of (9) is relatively small and gives rise to the previously mentioned discrepancy between the experiment and conventional neoclassical formulas. On the other hand, accounting for the electric field makes $V_i^{\rm pol}$ smaller or even negative, thereby allowing the terms on the right side of (9) to add and give a larger prediction for the impurity flow. Hence, the change in $V_i^{\rm pol}$ due to the pedestal radial electric field is in the proper direction and large enough to provide a qualitative understanding of the observations.

Because of the small poloidal gyroradius of electrons, their orbits are insensitive to the background electric field. As a result, the electron physics can be modified only by means of the altered background ion flow. Thus, we can readily adapt the usual techniques of evaluating the bootstrap current (e.g., see [10–12]) by using our electric field modified parallel ion flow result.

Before doing so, we remark that the electric field acts to increase the parallel velocity difference between the ion and electron flows, $V_{i\|} - V_{e\|}$, where $V_{e\|}$ is the parallel electron velocity. In the conventional $J(U) = 1$ limit, the $dT_i/d\psi$ term on the right side of (5) is in the direction opposite to the bootstrap current and therefore the flow difference $V_{i\|} - V_{e\|}$. Upon accounting for the presence of the electric field, $J(U)$ becomes smaller or even negative, making the difference $V_{i\|} - V_{e\|}$ larger, due to the $dT_i/d\psi$ on the right side of (5), and thereby increasing the electron friction with the ions. The electron response to the increased friction reduces the size of the effect by the usual $\sqrt{\varepsilon}$ factor, but the bootstrap current is still enhanced.

By knowing (5), the pedestal bootstrap current can be straightforwardly evaluated in the same way as in the core, with the change being solely due to the modified parallel ion velocity. Therefore, only the coefficient of the ion temperature gradient term is altered. Indeed, if the bootstrap current is written as the sum of pressure and temperature (instead of density and temperature) gradients, the ion temperature gradient term needs only to be multiplied by $J(U)$ to retain electric field effects.

For example, in the arbitrary Z case of Ref. [10], the electric field modified bootstrap current in a quasineutral plasma ($Zn_i = n_e$), with order $\sqrt{\varepsilon}$ corrections ignored, becomes

$$J_\|^{\rm bs} = -1.46\varepsilon^{1/2}\frac{cIB}{\langle B^2\rangle}\left[\frac{Z^2 + 2.21Z + 0.75}{Z(Z + 1.414)}\right] \times\left[\frac{dp}{d\psi} - \frac{(2.07Z + 0.88)n_e}{Z^2 + (Z^2 + 2.21Z + 0.75)}\frac{dT_e}{d\psi} - 1.17J(U)\frac{n_e}{Z}\frac{dT_i}{d\psi}\right], \qquad (10)$$

where n_e and T_e are the electron density and temperature, respectively, and $p = p_e + p_i = n_e(T_e + T_i/Z)$. Equation (10) illustrates the enhancement of the bootstrap current due to the finite radial electric field modification factor $J(U)$. This equation predicts that the pedestal bootstrap current is larger than that given by conventional formulas, since $J(U)$ becomes less than unity as U^2 increases and then goes negative for $U > 1.2$. This new feature is entirely due to the finite radial electric field modification of the parallel ion flow.

The Sauter, Angioni, and Lin-Liu formula [9] can be similarly modified to retain the finite electric field effects by multiplying their αL_{34} ion temperature gradient coefficient by $J(U)$. Consequently, our finite orbit generalization is qualitatively consistent with their form. As a result, we have shown that it is not surprising that measurements [5–7] of the bootstrap current density near the edge fit the phenomenological, but theory-motivated, form

$$J_{\|}^{\mathrm{bs}} = -\frac{cIBp}{\langle B^2 \rangle}\left[\frac{\alpha}{n_e}\frac{dn_e}{d\psi} + \frac{\beta}{T_e}\frac{dT_e}{d\psi} + \frac{\gamma}{T_i}\frac{dT_i}{d\psi}\right] \tag{11}$$

and thereby find reasonable qualitative agreement with the neoclassical Sauter, Angioni, and Lin-Liu model [9], where the dimensionless parameters α, β, and γ account for geometrical and collisionality effects. Our first-principles approach demonstrates that in the banana regime pedestal the parameter γ is also dependent upon the equilibrium axisymmetric electric field but still maintains the general form of Eq. (11).

In conclusion, we have deduced that the bootstrap current in a banana regime pedestal is larger than predicted by conventional neoclassical theory. This favorable result is due to the strong pedestal electric field that modifies the parallel ion flow to increase the velocity difference between the electron and ion bulks. We expect our findings to remain qualitatively correct in the case of a more realistic pedestal, even though, to obtain these results analytically, approximations are made—with the large aspect ratio expansion being the strongest. Indeed, some experimental support for our predictions is provided by impurity flow observations when the pedestal of Alcator C-Mod is in the banana regime. These observations [8] are consistent with the very same finite radial electric field modification of the bulk ion poloidal flow that increases the bootstrap current. Therefore our approach is in reasonable agreement with all experimental observations [5–7] and recovers a bootstrap current formula of the general Sauter, Angioni, and Lin-Liu form [9] once the coefficient preceding the ion temperature gradient term is altered in the prescribed way to account for the strong pedestal electric field.

We are grateful to Kenneth Marr and Bruce Lipschultz of the PSFC for helpful discussions on their impurity measurements, to Paul Bonoli of the PSFC for bringing to our attention the relevant bootstrap current experiments, and to Jesus Ramos of the PSFC for conversations on the extreme sensitivity of stability to small changes in the pedestal bootstrap current. Also, our paper would be flawed if it were not for the conscientiousness of Matt Landreman of the PSFC. This work was performed at the MIT Plasma Science and Fusion Center and supported by the U.S. Department of Energy Grant No. DE-FG02-91ER54109.

[1] A. A. Galeev, Sov. Phys. JETP **32**, 752 (1971).
[2] R. J. Bickerton, J. W. Connor, and J. B. Taylor, Nature (London), Phys. Sci. **229**, 110 (1971).
[3] S. Jardin, C. Kessel, C. Bathke, D. Ehst, T. Mau, F. Najmabadi, and T. Petrie, Fusion Eng. Des. **38**, 27 (1997).
[4] P. Bonoli, R. Parker, M. Porkolab, J. Ramos, S. Wukitch, Y. Takase, S. Bernabei, J. Hosea, G. Schilling, and J. Wilson, Nucl. Fusion **40**, 1251 (2000).
[5] M. Wade, M. Murakami, and P. Politzer, Phys. Rev. Lett. **92**, 235005 (2004).
[6] D. Thomas, A. Leonard, L. Lao, T. Osborne, H. Mueller, and D. Finkenthal, Phys. Rev. Lett. **93**, 065003 (2004).
[7] D. Thomas, A. Leonard, R. Groebner, T. Osborne, T. Casper, P. Snyder, and L. Lao, Phys. Plasmas **12**, 056123 (2005).
[8] K. Marr, B. Lipschultz, P. J. Catto, R. McDermott, M. Reinke, and A. Simakov, Plasma Phys. Controlled Fusion **52**, 055010 (2010).
[9] O. Sauter, C. Angioni, and Y. Lin-Liu, Phys. Plasmas **6**, 2834 (1999).
[10] P. Helander and D. J. Sigmar, *Collisional Transport in Magnetized Plasmas* (Cambridge University Press, Cambridge, England, 2002).
[11] F. Hinton and R. Hazeltine, Rev. Mod. Phys. **48**, 239 (1976).
[12] S. P. Hirshman, Phys. Fluids **21**, 1295 (1978).
[13] G. Kagan and P. J. Catto, Plasma Phys. Controlled Fusion **52**, 055004 (2010); G. Kagan and P. J. Catto, Plasma Phys. Controlled Fusion **52**, 079801 (2010).
[14] R. McDermott, B. Lipschultz, J. Hughes, P. Catto, A. Hubbard, I. Hutchinson, R. Granetz, M. Greenwald, B. LaBombard, and K. Marr, Phys. Plasmas **16**, 056103 (2009).
[15] G. Kagan and P. J. Catto, Plasma Phys. Controlled Fusion **50**, 085010 (2008).
[16] R. Hazeltine, Phys. Fluids B **1**, 2031 (1989).
[17] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, Phys. Rev. **107**, 1 (1957).
[18] I. Abel, M. Barnes, S. Cowley, W. Dorland, and A. Schekochihin, Phys. Plasmas **15**, 122509 (2008).
[19] P. J. Catto and D. R. Ernst, Plasma Phys. Controlled Fusion **51**, 062001 (2009).
[20] J. deGrassie (private communication).
[21] S. Hirshman and D. Sigmar, Nucl. Fusion **21**, 1079 (1981).
[22] Y. Kim, P. Diamond, and R. Groebner, Phys. Fluids B **3**, 2050 (1991).
[23] P. Helander, Phys. Plasmas **8**, 4700 (2001).
[24] P. J. Catto and A. N. Simakov, Phys. Plasmas **13**, 052507 (2006).