Single-Flavor Color Superconductivity in a Magnetic Field

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We investigate the single-flavor color superconductivity in a magnetic field. Because of the absence of the electromagnetic Meissner effect, forming a nonspherical CSC phase, polar, *A*, or planar, does not cost energy of excluding magnetic flux. We found that these nonspherical phases may be reached via a sequence of first order phase transitions under the typical quark density and magnetic field inside a neutron star.

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A cold quark matter will become a color superconductor at sufficiently high baryon density [1]. In the core region of a compact star, the baryon density is expected to be several times higher than that of a normal nuclear matter. The quarks may be released from hadrons and form a quark matter of $\mu \sim 400$–500 MeV, providing an opportunity to the color superconductivity (CSC).

While the pairing force is maximized in the s-wave channel, the antisymmetry of the wave function requires the Cooper pairing between different quark flavors. But the mass of strange quarks and the charge neutrality induce a substantial Fermi momentum mismatch among different flavors and thereby reduce the phase space available for pairing. A number of exotic 2 flavor or 3 flavor CSC phases have been proposed without reaching a consensus solution. The single-flavor CSC (pairing within each flavor) becomes a potential candidate even with the disadvantage of a reduced pairing strength. The dominant spin (the total angular momentum) of the single-flavor Cooper pair is one. Like the superfluidity of ^{3}He, there are a number of different pairing states and we shall focus in this Letter on four of them: the spherical color-spin locked (CSL) [2,3] state and nonspherical polar, *A* and planar ones. Without a magnetic field, the CSL pairing is energetically most favored, even when the angular momentum mixing effect is taken into account [4].

The energy balance among different single-flavor CSC phases will be offset in a magnetic field, which is present in a compact star and could exceed 10^{15} G in magnitude. Only the CSL phase shield the magnetic field [3]. The electromagnetic Meissner effect is absent for nonspherical states (polar, *A*, or planar). Cooling a normal quark matter to the CSL will require an extra amount of work to expel out the magnetic flux. Being free from such a penalty, nonspherical phases may show up under a sufficiently high magnetic field. Obtaining the phase diagram of a single-flavor CSC with respect to magnetic field and temperature is the main scope of the present Letter.

The structure of the Meissner effect in a single-flavor pairing is determined by the pattern of its symmetry breaking [3]. The condensate of a diquark operator takes the form

$$\Phi = \langle \bar{\psi}_C \Gamma^c \lambda^c \psi \rangle, \tag{1}$$

where ψ is the quark field, $\psi_C = \gamma_2 \psi^*$ is its charge conjugate, λ^c with $c = 2, 5, 7$ is an antisymmetric Gell-Mann matrix, and Γ^c is a 4×4 spinor matrix. We may choose $\Gamma^5 = \Gamma^7 = 0$ for the polar and A phases, $\Gamma^2 = 0$ for the planar phase, but none of Γ^c's vanishes for CSL phase. The condensate of CSL breaks the gauge symmetry $SU(3)_c \times U(1)_{em}$ completely. A nonspherical condensate, however, breaks the gauge symmetry partially and the Meissner effect is incomplete. Within the residual gauge group, there exists a U(1) transformation, $\psi \rightarrow e^{-(i/2)\lambda_8\theta - iq\phi}\psi$ with q the electric charge of ψ, $\theta = -2\sqrt{3}q\phi$ for the polar and A phases and $\theta = 4\sqrt{3}q\phi$ for the planar phase. The corresponding gauge field $\mathcal{A}_\mu$ is identified with the electromagnetic field in the condensate. It is related to the electromagnetic field A and the 8th component of the color field A^8 in the normal phase through a rotation

$$\mathcal{A}_\mu = A_\mu \cos\gamma - A_\mu^8 \sin\gamma, \qquad \mathcal{V}_\mu = A_\mu \sin\gamma + A_\mu^8 \cos\gamma, \tag{2}$$

where $\tan\gamma = -2\sqrt{3}q(e/g)$ for polar and A, and $\tan\gamma = 4\sqrt{3}q(e/g)$ for planar with g the QCD running coupling constant. The Meissner effect requires $\vec{\nabla} \times \vec{\mathcal{V}} = \vec{0}$ and thereby imposes a constraint inside a nonspherical CSC [5]

$$\mathbf{B}^8 = -\mathbf{B}\tan\gamma \tag{3}$$

between the color and the ordinary magnetic fields. Expressing the gauge coupling $\bar{\psi}\gamma_\mu(eqA_\mu + A_\mu^8\lambda_8/2)\psi$ in terms of $\mathcal{A}_\mu$ and its orthogonal partner $\mathcal{V}_\mu$, we extract the electric charges with respect to $\mathcal{A}$ in color space,

$$Q = \begin{cases} \frac{3qg}{\sqrt{g^2+12q^2e^2}}\mathrm{diag}(0,0,1) & \text{for polar and } A \\ \frac{3qg}{\sqrt{g^2+48q^2e^2}}\mathrm{diag}(1,1,-1) & \text{for planar.} \end{cases} \tag{4}$$

The thermal equilibrium in a magnetic field $H\hat{\mathbf{z}}$ is determined by minimizing the Gibbs free energy density,

$$\mathcal{G} = \Gamma - BH, \tag{5}$$

where Γ is the thermodynamical potential in the grand canonical ensemble. Ignoring the induced magnetization due to the normal current, we have

$$\Gamma = \frac{1}{2}B^2 + \frac{1}{2}\sum_{l=1}^{8}(B^l)^2 - p, \tag{6}$$

where p is the pressure at $B = 0$, maximized with respect to the gap parameter in the case of a CSC phase. The minimization with respect to B and B^l in a nonspherical phase is subject to the constraint (3). For a hypothetical quark matter of one flavor only, we find that

$$\mathcal{G} = \begin{cases} -p_n - \frac{1}{2}H^2, & \text{for normal phase} \\ -p_{\rm CSL}, & \text{for CSL} \\ -p_i - \frac{1}{2}H^2\cos^2\gamma_i, & \text{for } i = \text{polar}, A, \text{ planar} \end{cases} \tag{7}$$

after the minimization. As will be shown below,

$$p_n < p_A < p_{\rm polar} < p_{\rm planar} < p_{\rm CSL}. \tag{8}$$

The phase corresponding to minimum among $\mathcal{G}$'s above wins the competition and transition from one phase to another is first order below T_c.

The situation becomes more subtle when quarks of different flavors coexist even though pairing is within each flavor. Different electric charges of different quark flavors imply different mixing angles which may not be compactible with each other. Consider a quark matter of u and d flavors with each flavor in a nonspherical CSC state with different mixing angles. Equation (3) imposes two constraints, which are consistent with each other only if $B = B^8 = 0$. Then we end up with an effective Meissner shielding [3], making it fail to compete with the phase with both flavors in CSL states. On the other hand, one may relax the constraints by assuming that the basis underlying the condensate of u quarks differ from that underlying the condensate of d quarks by a color rotation. Consequently the constraint (3) for each flavor reads $B^8 = -B\tan\gamma^u$ and $B'^8 = -B\tan\gamma^d$. If both flavors stay in the polar or planar phases, which allows $\mathbf{B}^{1-3}$ to penetrate in, one may expect that an orthogonal transformation

$$B'^8 = B^8\cos\beta - B^3\sin\beta \qquad B'^3 = B^8\sin\beta + B^3\cos\beta \tag{9}$$

could compromise both constraints. Such a transformation, however, lies outside the color SU(3) group and therefore, the mutual rotation of color basis is not an option. The phases of the two-flavor quark matter (u, d) without Meissner effects, which can compete with (CSL, CSL), include (polar, planar), [polar (normal), normal (polar)], [A (normal), normal (A)] and (normal, normal). Notice the coincidence of the mixing angle of the polar state of u quarks and that of the planar state of d quarks. Also the normal phase does not impose any constraint on the gauge field and can coexist with any nonspherical CSC.

The Gibbs free energies of (normal, normal) and (CSL, CSL) phases remain given by the first and the second equations of (7), but with p_n and $p_{\rm CSL}$ referring to the total pressure of u and d quarks. For nonspherical phases, we have $\mathcal{G} = -p - \frac{1}{2}H^2\cos\gamma$, where p is the total pressure of both flavors with at least one of them in a nonspherical CSC state and γ is their common mixing angle. For normal-CSC combination, γ refers to that of the CSC state. The charge neutrality condition is imposed in all phases, which makes the Fermi sea of d quarks larger than that of u quarks. The color neutrality condition is ignored owing to the small energy gap associated to the single-flavor pairing. The number of combinations to be examined is reduced by two criteria: (1) For two combinations of the same mixing angle, the one with higher pressure wins. (2) For two combinations of the same pressure, the one with smaller magnitude of the mixing angle wins. It follows that there are only four phases to be considered in each case of two and three flavors with zero quark masses, which are shown in Table I. The phase diagram of each case will be determined below and their relevance to the realistic s quark mass will be discussed afterwards.

The pressure of the single-flavor CSC in the absence of a magnetic field has been obtained in the literature at zero temperature within the frame work of the one-gluon exchange. We shall extend the analysis up to the transition temperature T_c, which is universal for all single-flavor pairings. To avoid the technical complexity of the one-gluon exchange, we shall work with a NJL-like effective action which picks up only the dominant pairing channel of the former, the transverse pairing, in the ultrarelativistic limit. The Hamiltonian of the effective action reads [6]

TABLE I. This table shows possible phases under a magnetic field for both two-flavor and three-flavor cases with each flavor forming spin-one CSC or remaining normal state. The critical temperature (the same for CSL, planar, polar, and A) has also been included.

	I	II	III	IV	$T_C(10^{-1}$ MeV)
Two-flavor	CSL_u, CSL_d	$(\mathrm{polar})_u$, $(\mathrm{planar})_d$	$(\mathrm{normal})_u$, $(\mathrm{polar})_d$	$(\mathrm{normal})_u$, $(\mathrm{normal})_d$	1.35
Three-flavor	CSL_u, $\mathrm{CSL}_{d,s}$	$(\mathrm{polar})_u$, $(\mathrm{planar})_{d,s}$	$(\mathrm{normal})_u$, $(\mathrm{polar})_{d,s}$	$(\mathrm{normal})_u$, $(\mathrm{normal})_{d,s}$	0.49

$$\mathcal{H} = \int d^3\mathbf{r}[\bar{\psi}(\vec{\gamma}\cdot\vec{\nabla} - \mu\gamma_4)\psi - G\bar{\psi}\gamma_\mu T^l\psi\bar{\psi}\gamma_\mu T^l\psi], \tag{10}$$

with $T^l = \frac{1}{2}\lambda^l$ and G an effective coupling. Introducing the condensate (1), we find the pressure of each flavor under mean field approximation

$$p = \frac{2T}{\Omega}\sum_{\mathbf{k}}\ln(1+e^{-(|k-\mu|/T)}) - \frac{1}{\Omega}\sum_{\mathbf{k}}(k-\mu-|k-\mu|) - \frac{2}{\Omega}\sum_{\mathbf{k}}(k-\mu-E_{\mathbf{k}}) - \frac{9}{4G}\Delta^2 + \frac{4T}{\Omega}\sum_{\mathbf{k}}\ln(1+e^{-(E_{\mathbf{k}}/T)}), \tag{11}$$

where $E_{\mathbf{k}} = \sqrt{(k-\mu)^2 + \Delta^2 f^2(\theta)}$ with θ the angle between $\mathbf{k}$ and a prefixed spatial direction and Δ given by the solution of the gap equation $(\frac{\partial p}{\partial \Delta})_\mu = 0$. The function $f(\theta)$ is given by

$$f(\theta) = \begin{cases} 1, & \text{for CSL phase} \\ \sqrt{\frac{3}{4}(1+\cos^2\theta)}, & \text{for planar phase} \\ \sqrt{\frac{3}{2}}\sin\theta, & \text{for polar phase} \\ \sqrt{3}\cos^2\frac{\theta}{2}. & \text{for } A \text{ phase} \end{cases} \tag{12}$$

Introducing $\Delta p_s \equiv p_s - p_n \equiv \rho_s(T)\frac{\mu^2\Delta_0^2}{2\pi^2}$ with s labeling different pairing states and Δ_0 the CSL gap at $T = 0$. We have $\rho_{\mathrm{CSL}}(0) = 1$, $\rho_{\mathrm{planar}}(0) = 0.98$, $\rho_{\mathrm{polar}}(0) = 0.88$ and $\rho_{\mathrm{A}}(0) = 0.65$, and $\rho_s(T_c) = 0$ with $T_c = \frac{e^{\gamma_E}}{\pi}\Delta_0$. The function $\rho_s(T)$ for $0 < T < T_c$ of various states is displayed in Fig. 1, which satisfies the inequalities (8). In a multiflavor quark matter, the Fermi momentum of each flavor is displayed from each other to meet the charge neutrality requirement. For an ideal gas of (u, d) quarks and electrons at zero temperature, we find that $\mu_u = 0.87\mu$ and $\mu_d = 1.09\mu$. While for an ideal gas of (u, d, s) quarks and electrons with $m_s \ll \mu$, we obtain that $\mu_u = \mu$, $\mu_d = \mu + \frac{m_s^2}{4\mu}$ and $\mu_s = \mu + \frac{m_s^2}{4\mu}$. The corrections brought about by nonzero temperature and/or gap parameters contribute a higher order term than $O(\mu^2\Delta^2)$ to the pressure and can be neglected here.

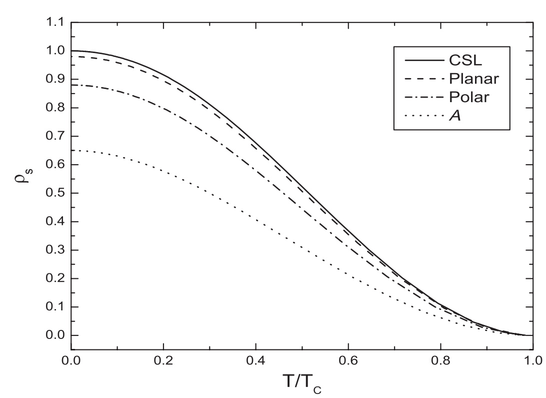

FIG. 1. The function $\rho_s(T)$ for various pairing states.

By balancing the Gibbs free energy of different phases, we obtain the phase diagram with respect to temperature and magnetic field. The two-flavor and three-flavor cases are shown in Fig. 2, where H_0 is defined by $H_0 = \frac{\mu\Delta_0}{\pi}$. If we calibrate the effective coupling G by identifying Δ_0 with that of the one-gluon exchange [2,7]

$$\Delta_0 = 512\pi^4\left(\frac{2}{N_f}\right)^{5/2}\frac{\mu}{g^5}\exp\left(-\frac{3\pi^2}{\sqrt{2}g} - \frac{\pi^2+4}{8} - \frac{9}{2}\right) \tag{13}$$

extrapolated to $\mu = 500$ MeV and $\alpha_s = 1$, we end up with the values of T_c and H_0 in Table I and Fig. 2. For the three-flavor case, we ignored the Fermi momentum mismatch to be consistent with the ultrarelativistic approximation.

Our ultrarelativistic treatment of s quarks in the three-flavor case approximates well if m_s is considerably lower than μ but this may not be the case as was indicated by the numerical work of [8]. For m_s comparable to μ, the mass effect has to be included in the pairing dynamics and the CSC transition temperature for s quarks, T_c' will be lowered. The new phase diagram will consist of the two-flavor CSC for $T_c' < T < T_c$ and the three-flavor CSC for $0 < T < T_c'$. The two-flavor part is obtained by scaling the upper panel of Fig. 2 slightly downward because of the additional screening of the one-gluon exchange by normal s quarks. As long as m_s is sufficiently far from the nonrelativistic limit, the inequality (8) holds for the s quark CSC and the three-flavor region will be occupied by the same phases I–IV in the second row of Table I with the same relative positions. The phase boundaries join contin-

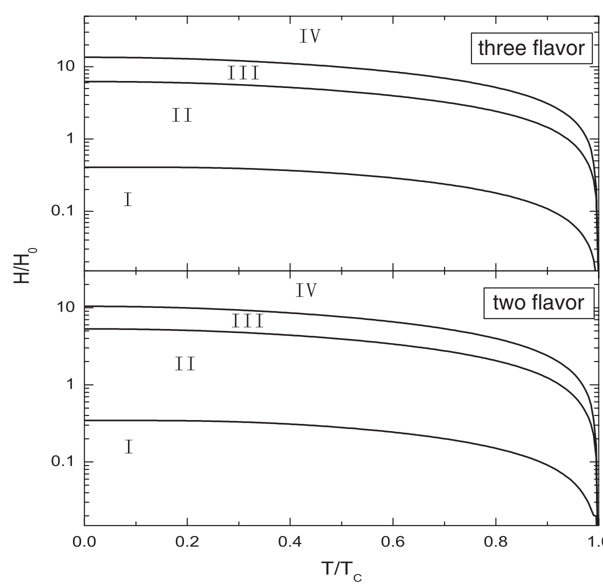

FIG. 2. H-T phase diagram with $H_0 = 5.44 \times 10^{14}$ G, 1.97×10^{14} G for two-flavor and three-flavor cases, respectively.

uously at T_c' with a slight upward jump of the slopes as one crosses from the two-flavor region to the three-flavor region because of the onset of the s quark CSC. The details will be reported elsewhere. The analysis up to now ignores the magnetization $M = \frac{\partial p}{\partial B}$ [9] in the absence of the Meissner effect and we attempt to justify this approximation here. The potential hazard comes from the de Hass–van Alphen (dHvA) effect stemming from the discreteness of the Landau orbit if the mean free path l of quarks is longer than the cyclotron radius, $\mu/(eB)$. Even though the magnetic field in the phase diagrams is weak in the sense $(eB)^2 \ll \mu$, a large magnetization may emerge through the derivative because of the rapid oscillation. The stability condition $\frac{\partial^2 \mathcal{G}}{\partial B^2} > 0$, however, prevents its happening. Along the equilibrium M-B curve constructed by the Maxwell rule, the ratio M/B cannot exceed the order of $\alpha_e^{2/3}$ in the normal phase. This is also expected to be the case in a nonspherical CSC phase. Because of the nonzero charges of the pairing partners Eq. (4), the Landau orbits also impact on the energy gap in the planar phase and a similar issue for color-flavor locking (CFL) has been addressed in the literature [10–12]. Our analytic work reveals that the magnitude of the oscillatory term of the gap is suppressed by $O(\sqrt{eB}/\mu)$ relative to the term at $B = 0$. In the opposite limit where $l \ll \mu/(eB)$, the dHvA oscillation is smeared out by scattering.

To conclude, we have explored the consequences of the absence of the electromagnetic Meissner effect in a nonspherical CSC phase of single-flavor pairing. We found that these nonspherical phases occupy a significant portion of the H-T phase diagram for the plausible magnitude of the magnetic field inside a compact star. The latent heats released as the star cools through the phase boundaries of Fig. 2 is expected to cause observable energy bursts [13]. Since the transverse pairing, which pairs quarks of opposite helicities, breaks the chiral symmetry, the Goldstone modes associated to the nonspherical phases will impact on the transport properties, the neutrino emissivity, and the r-mode instability. Although the Urca process will not be suppressed because of unpaired quarks in the ultrarelativistic limit, including quark masses may change the story.

The nonspherical phases discussed in this Letter are all homogeneous in space. A domain wall structure was suggested in [14] in the context of 2SC and CFL in a magnetic field. The mechanism involves the absence of the Meissner effect, the chiral symmetry breaking, and the axial anomaly. It would be interesting to extend the analysis of [14] to the nonspherical phases. We have not considered the noninert phases discussed in [15]. Nor have we considered the p-wave pairing with mismatch [16], which is unlikely in QCD. In any case, the importance of the nonspherical CSC in a magnetic field, revealed in this Letter, will remain.

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