A Little Inflation in the Early Universe at the QCD Phase Transition

Tillmann Boeckel and Jürgen Schaffner-Bielich
Institut für Theoretische Physik, Universität Heidelberg Philosphenweg 16, D-69120 Heidelberg, Germany
(Received 7 August 2009; revised manuscript received 20 April 2010; published 19 July 2010)

We explore a scenario that allows for a strong first order phase transition of QCD at a non-negligible baryon number in the early Universe and its possible observable consequences. The main assumption is a quasistable QCD-vacuum state that leads to a short period of inflation, consequently diluting the net baryon to photon ratio to today's observed value. A strong mechanism for baryogenesis is needed to start out with a baryon asymmetry of order unity, e.g., as provided by Affleck-Dine baryogenesis. The cosmological implications are direct effects on primordial density fluctuations up to dark matter mass scales of $M_{\rm max} \sim 1$–$10 M_{\odot}$, change in the spectral slope up to $M_{\rm max} \sim 10^6$–$10^8 M_{\odot}$, production of strong primordial magnetic fields and a gravitational wave spectrum with present day peak strain amplitude of up to $h_c(\nu_{\rm peak}) \sim 5 \times 10^{-15}$ around $\nu_{\rm peak} \sim 4 \times 10^{-8}$ Hz.

DOI: 10.1103/PhysRevLett.105.041301 PACS numbers: 98.80.Cq, 12.38.−t, 95.35.+d

The theory of quantum chromodynamics (QCD) predicts a phase transition from a quark-gluon plasma to a hadron gas in the early Universe at a critical temperature $T_{\rm QCD} \approx 150$–200 MeV [1,2]. Only at low net baryon density lattice gauge theory indicates a rapid crossover from the quark-gluon plasma to the hadronic phase. In the standard hot big bang scenario the baryon asymmetry is $\eta_B \sim 10^{-9}$–10^{-10} already before the QCD phase transition and therefore the idea of a first order QCD phase transition in the early Universe has been more or less abandoned. However, most of the QCD phase diagram is actually not well known. There has been recent progress in the attempt to include a finite baryon density on the lattice [3,4] but in general one still has to rely on effective models [5] to tackle the QCD phase diagram. However, a true first order phase transition is expected at finite baryon densities, as indicated by chiral effective models of QCD [6] due to the melting of quark and/or gluon condensates and the phenomenon of color superconductivity [7]. Therefore, we would like to reopen the issue of a first order cosmological phase transition by addressing whether there is a simple scenario in which the QCD phase transition at finite baryon densities can have consequences on cosmological scales.

In this Letter we demonstrate that the scenario of a little inflation at the QCD phase transition at high baryon densities is possible and not in contradiction to present cosmological observations. It has interesting cosmological implications though as it can directly affect primordial density fluctuations on dark matter mass scales below $M_{\rm max} \sim 1$–$10 M_{\odot}$, change the spectral slope up to mass scales of $M_{\rm max} \sim 10^6$–$10^8 M_{\odot}$ due to the change of the global equation of state, produce primordial magnetic fields that may be strong enough to seed the presently observed galactic and extragalactic magnetic fields and produce a spectrum of gravitational waves around a peak frequency of 4×10^{-8} Hz that may be observable via pulsar timing in the future [8,9]. Dark matter properties are also strongly affected as the annihilation cross section for cold dark matter has to be up to 9 orders of magnitude lower to give the right amount of dark matter today, which can be probed at the LHC by detecting the neutralino with an unexpected low annihilation cross section, and thermal warm dark matter masses can be of the order of MeV without exceeding the decoupling degrees of freedom of the standard model. Such a cosmological phase transition would then bear more resemblance to the situation in heavy ion collisions or even the center of neutron stars than to the standard QCD phase transition in the hot big bang scenario. Hence, the upcoming FAIR facility would actually be a probe for the physics of the early Universe in this scenario.

For a first order QCD phase transition in the early Universe to be possible a nonvanishing baryochemical potential μ_B is necessary where $\mu_B/T \sim \mathcal{O}(1)$. The present day baryon asymmetry $\eta_B = (n_B - n_{\bar{B}})/n_\gamma$ has been experimentally found to be $5.9 \times 10^{-10} < \eta_B < 6.4 \times 10^{-10}$ at 98% confidence by combining big bang nucleosynthesis, cosmic microwave background and large scale structure results [10]. The number of baryons in a comoving volume is constant and can be estimated to be $N_B \approx a_i^3 \mu_{Bi} T_i^2 \simeq a_f^3 \mu_{Bf} T_f^2$ where the index i refers to the initial values when the vacuum energy starts to dominate over the radiation energy and f to the final values after reheating. Therefore the initial ratio of the chemical potential to the temperature can be higher by $\frac{\mu_{Bi}}{T_i} \simeq \theta^3 \frac{\mu_{Bf}}{T_f} (\frac{T_f}{T_i})^3$ with $\theta = a_f/a_i$. If the time scale for the decay of the false vacuum is short compared to the Hubble time then $T_i \simeq T_f$ and already for $\theta \sim 10^3$ (corresponding to a little inflationary period with $N \approx 7$ e-foldings) the initial baryon asymmetry η_{Bi} and μ_i/T_i will be of order unity. Hence, the evolution of the early Universe could pass then through the first order chiral phase transition of QCD. A well established mechanism for generating a high baryon number in the early Universe is the Affleck-Dine baryogenesis [11]. The Affleck-Dine mechanism produces in most cases a far too high baryon asymmetry, thus either additional fields or more sophisticated coupling terms have to be introduced to reduce the initial baryon number production or it has to be reduced afterwards. For the latter case an obvious possibility would be a large entropy release that dilutes the baryon to photon ratio, for example, by an inflationary period (see, e.g., Ref. [12]). Affleck-Dine baryogenesis can in fact produce $\eta_B \sim \mathcal{O}(1)$, where this is probably an upper bound [12].

We note that the scenario proposed here has some similarities to thermal inflation as discussed by Lyth and Stewart [13] as both are late time inflation periods in addition to ordinary inflation with a length of only about 10 e-foldings and may thus help to resolve partly the moduli problem. In Ref. [14] the production of quark stars with masses of 10^{-2}–$10 M_\odot$ was proposed within a scenario similar to the one discussed here but at small baryon densities and without addressing the key consequences of such a second late time inflationary period. In [15] it was recently proposed that a large lepton asymmetry could also result in a first order QCD phase transition in the early Universe.

An important issue of this approach is the stability of the barrier between the false and the true vacuum in the effective potential up to very low temperatures. This is indeed the case in chiral models of QCD including gluonic degrees of freedom in the form of a dilaton field in which case the barrier only vanishes in the $T \to 0$ limit [16]. Csernai and Kapusta found in Ref. [17] only small supercooling of about 1% below the critical temperature using values of the surface tension of about $\sigma \sim 50\ \mathrm{MeV/fm^2}$. The nucleation rate Γ depends exponentially on the surface tension as well as on the free energy difference between both phases and its ratio to the Hubble parameter Γ/H exhibits a maximum around $\sim T_c/2$. If Γ/H does not exceed one at this point the phase transition fails and we find that keeping the other parameters used in Ref. [17] the surface tension must exceed $450\ \mathrm{MeV/fm^2}\ \sim 3.7 T_c^3$. However, the precise value of the surface tension at the QCD phase transition at high densities is not known and has been a matter of debate, see, e.g., the discussion in Ref. [18] giving a possible range of $\sigma = 50$–$150\ \mathrm{MeV/fm^2}$ without excluding even smaller or larger values. At very high densities calculations of the surface tension in the first order phase transition between color superconducting phases and nuclear matter arrive at surface tensions of up to $300\ \mathrm{MeV/fm^2}$ [19]. The value of the bag constant used by Csernai and Kapusta is at the upper end of values considered in the literature (i.e., $\mathcal{B} = (235\ \mathrm{MeV})^4$) and a reduction to the value found in the original paper of the MIT group by fits to hadron masses ($\mathcal{B} = (145\ \mathrm{MeV})^4$, see Ref. [20]) also reduces the surface tension needed for nucleation to fail to a value of $124\ \mathrm{MeV/fm^2}$. This of course only covers the initial failure to nucleate, but in general $\mathcal{B}$ and σ will both be temperature dependent. After some supercooling (e.g., 7 e-foldings at most) Γ/H must exceed one for inflation to end and the phase transition to occur. This could either take place due to a strong drop in the surface tension or even due to a complete vanishing of the barrier between the two phases in the effective potential. In the latter case the surface tension goes to zero and a spinodal decomposition takes place. This has been studied, e.g., in [21] for a baglike model. Strong sensitivities of nucleation rates on the surface tension have been also found for high-density matter as encountered in the interior of neutron stars or in core-collapse supernovae [22] so that nucleation time scales can easily be in the range of μs to the age of the Universe.

The equation of state has to fulfill the usual condition $\epsilon + 3p < 0$ to enter an inflationary phase. In the bag model this would be the case below a temperature $T_{\mathrm{inf}} = (30\mathcal{B}/(g\pi^2))^{1/4}$. In the linear-$\sigma$ model or the NJL model this occurs when the thermal contributions to the energy density become smaller than the vacuum contributions like the quark condensate $\langle m_q q\bar{q}\rangle \approx f_\pi^2 m_\pi^2$ and the gluon condensate $\beta_{\mathrm{QCD}}/(2g)\langle G^a_{\mu\nu} G_a^{\mu\nu}\rangle \approx 4\mathcal{B}$. In Ref. [23] the idea of a "quench" in context of heavy ion collisions is discussed, i.e., the chiral field is trapped in a metastable minimum and supercools until the barrier in the effective potential disappears at zero temperature and the field "rolls down" to the true minimum. All in all a delayed chiral phase transition at high baryon densities cannot be excluded for the early Universe with our present poor knowledge of QCD at nonzero baryon densities.

The majority of dark matter candidates is already chemically decoupled from the radiation fluid at the QCD phase transition and thus do not participate in the reheating at the end of the inflationary period. Therefore the dark matter number density is diluted by the same factor θ^3 as the net baryon number. Normally the dark matter mass enclosed inside the Hubble horizon is of the order of $10^{-9} M_\odot$ at $T_{\mathrm{QCD}} \sim 170$ MeV, so any influence on perturbations inside dark matter would not have any consequences on larger scales. An inflationary period at the QCD phase transition can change this drastically, since the amount of dark matter enclosed inside the horizon must be larger by a factor θ^3 initially to give the right amount of dark matter today. For a short inflationary period, as discussed here, there is an additional effect on perturbations that have physical wave numbers $k_{\mathrm{ph}} \lesssim H$ at the beginning of inflation. For general relativistic ideal fluid density perturbations the system of dynamical equations is closed by Einstein's R^0_0 equation that reads $(k^2_{\mathrm{ph}} + \dot{H})\alpha = 4\pi G(\delta\rho + 3\delta p)$ in uniform expansion gauge (see, e.g., [24]) where $\delta\rho$ and δp are the sum of the density and pressure perturbations, respectively, and α is the perturbation of the lapse. The time derivative of the Hubble parameter is given via the second Friedmann equation $\dot{H} = -4\pi G(\epsilon + p) = -4\pi G[\frac{4}{3}\epsilon_{Ri}(\frac{a_i}{a})^4 + \epsilon_{Mi}(\frac{a_i}{a})^3] \propto (\frac{a_i}{a})^q$, where the subscripts refer to matter and radiation with $q = 3$ to 4, respectively, and the index i to the onset of inflation. Comparing this to the first Friedmann equation one finds that $H^2 = \frac{8\pi G}{3}[\epsilon_V + \epsilon_{Ri}(\frac{a_i}{a})^4 + \epsilon_{Mi}(\frac{a_i}{a})^3]$ meaning that the two scales differ by $|\dot{H}/H^2|^{1/2} \simeq (\frac{a_i}{a})^{q/2}$ which would be irrelevant for a long inflationary period (with more than 50 e-foldings) since $\dot{H}^{-1/2}$ then corresponds to an unobservably large length scale. Therefore, one can expect three spectral regimes, $(k_{\rm ph}/H)_i > a_f/a_i$ (always sub-Hubble), $a_f/a_i > (k_{\rm ph}/H)_i > (a_i/a_f)^{q/2}$ (intermediate) and $(k_{\rm ph}/H)_i < (a_i/a_f)^{q/2}$ (unaffected). Translating this to the highest affected mass scale involved we estimate $M_{\rm max} \sim 10^{-8} M_\odot (a_f/a_i)^{3q/2}$. Above this scale the spectrum of density perturbations is given by the primordial spectrum of density perturbations, e.g., a nearly scale invariant spectrum. Note that we do not make a statement about the detailed evolution of perturbations above or below these two scales at this point, we only stress that a cosmologically interesting mass scale appears for a short period of inflation that could lead to observable consequences. For a fully consistent treatment of perturbations one needs to take into account the dynamics of the chiral phase transition in a detailed model and try to estimate the effects of reheating on the amplitude of perturbations. For cold dark matter the dilution of the energy and number densities leads to the possibility of a matter dominated phase before the inflationary phase since the dark matter energy density after reheating is basically fixed by the present day value. Consequently the dark matter density before inflation is larger by the same factor θ^3 as the baryon density. For $\theta \gtrsim 10^3$ a matter dominated phase is present before the QCD phase transition and QCD inflation is naturally limited to a length of $\theta_{\rm max}^{\rm inf} = (\frac{\mathcal{B}}{\epsilon_{DM}(a_f)})^{1/3} \approx 900 (\frac{\mathcal{B}^{1/4}}{235\ {\rm MeV}})^{4/3} (\frac{0.236}{\Omega_{DM0}})^{1/3}$. The highest affected dark matter mass scale would be then $M_{\rm max} \sim 10^6$–$10^8 M_\odot$. One can put a general upper limit on the amount of entropy that is released by demanding that the initial baryon asymmetry is at most of order one, implying that $\theta_{\rm max}^B \lesssim (1/\eta_B(a_E))^{1/3} \approx 1200$ (a complete spectrum of primordial fluctuations would require $\theta \gtrsim 10^{10}$).

We note that for nonrelativistic decoupling of dark matter the weak interaction cross section will no longer give the right amount of dark matter today, the dark matter annihilation cross section has to be much smaller, i.e., $\sigma_{\rm dm}^{\rm annih} \sim \theta^{-3} \sigma^{\rm weak}$ as $\Omega_{DM} \propto 1/\sigma_{\rm dm}^{\rm annih}$ (we ignore logarithmic dependencies on the dark matter mass). This gives the interesting prospect that the little inflation can be probed by the LHC since the discovery of a standard weakly interacting massive particle like the neutralino would exclude the scenario.

For thermally decoupled ultrarelativistic particles the ordinary temperature relation to the radiation background is changed after inflation $T = T_{DM} \theta [g_{\rm eff}^s(T_{\rm Dec})/g_{\rm eff}^s(T)]^{1/3}$. Generalizing the mass limit found in [25] one arrives at $m_{DM}^{\rm max} \approx 51\ {\rm eV} \times \theta^3 (\frac{4}{g_{DM}}) (\frac{g_{\rm eff}^s(T_{\rm Dec})}{106.75}) (\frac{\Omega_{DM}^0 h^2}{0.116})$. This allows for a much higher mass of a thermal relic particle without the need for a large number of additional effective decou pling degrees of freedom beyond those of the standard model.

A vanishing speed of sound during a first order phase transition can also lead to the formation of primordial black holes (PBHs) for a small fraction of Hubble volumes that are sufficiently overdense [26]. The mass spectrum of these PBHs will be strongly peaked around $1 M_\odot$ which corresponds to the total (not just the dark matter) energy density inside the Hubble volume at the phase transition. The abundance of PBHs depends on the spectral index and amplitude of the density fluctuation spectrum that can differ significantly around the Hubble scale in the presented scenario as discussed above. Lumps of quark matter or small quarks stars could be also produced but only with $M \sim 10^{-9} M_\odot$ as we argue that nucleation starts after the little inflationary epoch. For a first order QCD phase transition with bubble nucleation there is a well discussed mechanism for producing magnetic fields via bubble collisions [27]. Since the baryon number is carried by massless quarks and massive nucleons in the respective phases the baryon number will tend to concentrate in the quark phase, at least close to the phase boundary [27]. Because of their finite masses the muon and the strange quark are already slightly suppressed at the critical temperature T_c which leads to a charge dipole layer at the phase boundary. The resulting net positive charge density is $\rho_C^+ = \beta e n_B$ with $\beta \sim 10^{-2}$–10^{-3} for a small η_B and $\beta = 0.2$ for our case. Using the estimates of Ref. [27] we arrive at magnetic fields of strength $B_{\rm QCD} = 10^8$–10^{10} G for low baryon asymmetry although MHD turbulence may readily amplify the initial fields to the equipartition value $B_{\rm eq} = \sqrt{8\pi T^4 v_f^2}$ ([28] and references therein), where v_f is the fluid velocity. In the little inflation scenario the initial value of the baryon contrast between the two phases can be much higher since nucleons will be highly suppressed at $T \sim 170\ {\rm MeV}/\theta \sim 0.2$ MeV, while for a random walk the baryon diffusion length $r_{\rm diff} \propto 1/\sqrt{n_B + n_{\bar{B}}} \sim 4\ \mu{\rm m}\ \theta^{3/2} \sim 10$ cm is larger because n_B and $n_{\bar{B}}$ are reduced by a factor of θ^3. Altogether one can expect that the magnetic field B will easily reach an equipartition value of $B_{\rm eq} \approx 10^{12}$ G, where $v_f \sim 1$ since the released latent heat is much larger than the thermal energy.

The presently observed galactic and extragalactic magnetic fields have strength $B_\lambda^{\rm obs} = 0.1$–$1\ \mu$G, but the required seed fields for an effective galactic dynamo mechanism on scales of 0.1 Mpc are strongly model and parameter dependent and vary over many orders of magnitude $10^{-30}\ {\rm G} \lesssim B_\lambda^{\rm seed} \lesssim 10^{-10}$ G (see [29] and references therein). In Ref. [30] it was argued that for a causal production mechanism the spectrum of the generated magnetic field must be very blue for uncorrelated superhorizon scales, i.e. $B_\lambda^2 \propto \lambda^{-n}$ with $n \geq 2$. Therefore $B_\lambda^{\rm seed}$ can be strongly limited by the allowed additional energy density at big bang nucleosynthesis (BBN) [31]. We find that the produced initial field corresponds to $B_{0.1\ {\rm Mpc}}^{\rm seed} < 10^{-22}$ G which translates to a mean field at the QCD scale of at most $B_{\rm QCD} = 5 \times 10^{13}$ G which is consistent with the above estimates. In [32] it was found that an inverse cascade mechanism due to a nonvanishing helicity of the primordial magnetic field (as one can expect in the presented scenario) is able to successfully seed large scale magnetic fields at the QCD phase transition.

In a first order phase transition nucleation can produce gravitational waves due to bubble collisions and hydrodynamic turbulence as found by [33]. For a nucleation rate of $\Gamma \propto \exp(t/\tau)$ the peak frequency of the spectrum corresponds to a present day frequency of $\nu^B_{\rm peak} \approx 4.0 \times 10^{-8}$ Hz$(\frac{0.1H^{-1}}{\tau})(\frac{T^*}{150\ \mathrm{MeV}})(\frac{g^{\rm eff}}{50})^{1/6}$, where T^* is the reheating temperature. With the above estimates one arrives at a peak strain amplitude $h_c(\nu^B_{\rm peak}) = 4.7\times 10^{-15}(\frac{\tau}{0.1H^{-1}})^2(\frac{150\ \mathrm{MeV}}{T^*})\times(\frac{50}{g^{\rm eff}})^{1/3}$ due to bubble collisions. The kinetic energy of the colliding bubbles is also partially converted to turbulent bulk motion of the plasma stirring gravitational waves at a slightly lower frequency $\nu^T_{\rm peak} \simeq 0.3\nu^B_{\rm peak}$ with a higher peak amplitude $h_c(\nu^T_{\rm peak}) \simeq 2.1 h_c(\nu^B_{\rm peak})$ for a strong first order phase transition [34]. The approximate shape of the strain amplitude spectrum is then $h_c(\nu) \propto \nu^{1/2}$ for $\nu < H$ (uncorrelated white noise) and $h_c(\nu) \propto \nu^{-m}$ for $\nu > \nu^B_{\rm peak}$ where the spectral index m is at most 2 but could easily be close to 1 or even lower due to multibubble collisions [35]. Pulsar timing already limits nucleation with the presently available data to $\tau/H^{-1} < 0.12$ which will improve to $\tau/H^{-1} < 0.06$ for the full data of the Parkes Pulsar Timing Array project [8]. The planned Square Kilometer Array will be about 4 orders of magnitude more sensitive in $\Omega_{\rm gw}(\nu)$ [9] which corresponds to 1 order of magnitude improvement for the bound on τ/H^{-1}. Detection via the Laser Interferometer Space Antenna (LISA) could also be possible if the high frequency tail of the spectrum has a spectral index $m \lesssim 1.4$ and $\tau/H^{-1} \gtrsim 10^{-2}$.

We have here only briefly introduced the idea of a little inflation at the QCD phase transition and sketched the differences from the standard scenario for structure formation, dark matter properties, magnetic fields and gravitational wave production. The main assumptions are a high initial baryon asymmetry before the QCD phase transition and the existence of a quasistable QCD vacuum condensate that dominates the energy budget for a short period. Especially the impact on structure formation in this approach seems to be rather interesting but requires a more thorough field theoretical approach.

We thank Rob Pisarski, Eduardo Fraga, and Ruth Durrer for useful discussions and Arthur Kosowsky for important comments on the gravitation wave signal. This work was supported by the German Research Foundation (DFG) within the framework of the excellence initiative through the Heidelberg Graduate School of Fundamental Physics (HGSFP) and through the Graduate Program for Hadron and Ion Research (GP-HIR) by the Gesellschaft für Schwerionenforschung (GSI), Darmstadt.

[1] Y. Aoki, Z. Fodor, S. D. Katz, and K. K. Szabó, Phys. Lett. B **643**, 46 (2006).
[2] F. Karsch and the RBC-Bielefeld Collaboration, J. Phys. G **34**, S627 (2007).
[3] Z. Fodor and S. D. Katz, J. High Energy Phys. 4 (2004) 050.
[4] C. R. Allton *et al.*, Phys. Rev. D **66**, 074507 (2002).
[5] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).
[6] M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).
[7] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Rev. Mod. Phys. **80**, 1455 (2008).
[8] F. A. Jenet *et al.*, Astrophys. J. **653**, 1571 (2006).
[9] M. Kramer *et al.*, New Astron. Rev. **48**, 993 (2004).
[10] V. Simha and G. Steigman, J. Cosmol. Astropart. Phys. 6 (2008) 016.
[11] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985).
[12] A. D. Linde, Phys. Lett. B **160**, 243 (1985).
[13] D. H. Lyth and E. D. Stewart, Phys. Rev. Lett. **75**, 201 (1995).
[14] N. Borghini, W. N. Cottingham, and R. V. Mau, J. Phys. G **26**, 771 (2000).
[15] D. J. Schwarz and M. Stuke, J. Cosmol. Astropart. Phys. 11 (2009) 025.
[16] B. A. Campbell, J. Ellis, and K. A. Olive, Phys. Lett. B **235**, 325 (1990).
[17] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992).
[18] D. N. Voskresensky, M. Yasuhira, and T. Tatsumi, Nucl. Phys. A **723**, 291 (2003).
[19] M. Alford, K. Rajagopal, S. Reddy, and F. Wilczek, Phys. Rev. D **64**, 074017 (2001).
[20] T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).
[21] L. L. Jenkovszky, V. M. Sysoev, and B. Kämpfer, Z. Phys. C **48**, 147 (1990).
[22] B. W. Mintz, E. Fraga, G. Pagliara, and J. Schaffner-Bielich, Phys. Rev. D **81**, 123012 (2010).
[23] O. Scavenius and A. Dumitru, Phys. Rev. Lett. **83**, 4697 (1999).
[24] C. Schmid, D. J. Schwarz, and P. Widerin, Phys. Rev. D **59**, 043517 (1999).
[25] T. Boeckel and J. Schaffner-Bielich, Phys. Rev. D **76**, 103509 (2007).
[26] K. Jedamzik, Phys. Rev. D **55**, R5871 (1997).
[27] B. Cheng and A. V. Olinto, Phys. Rev. D **50**, 2421 (1994).
[28] G. Sigl, A. V. Olinto, and K. Jedamzik, Phys. Rev. D **55**, 4582 (1997).
[29] L. M. Widrow, Rev. Mod. Phys. **74**, 775 (2002).
[30] C. Caprini and R. Durrer, Phys. Rev. D **65**, 023517 (2001).
[31] R. H. Cyburt, B. D. Fields, K. A. Olive, and E. Skillman, Astropart. Phys. **23**, 313 (2005).
[32] C. Caprini, R. Durrer, and E. Fenu, J. Cosmol. Astropart. Phys. 11 (2009) 001.
[33] A. Kosowsky, M. S. Turner, and R. Watkins, Phys. Rev. Lett. **69**, 2026 (1992).
[34] T. Kahniashvili, A. Kosowsky, G. Gogoberidze, and Y. Maravin, Phys. Rev. D **78**, 043003 (2008).
[35] M. Kamionkowski, A. Kosowsky, and M. S. Turner, Phys. Rev. D **49**, 2837 (1994).