Collective Excitations of a Dipolar Bose-Einstein Condensate

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We have measured the effect of dipole-dipole interactions on the frequency of a collective mode of a Bose-Einstein condensate. At relatively large numbers of atoms, the experimental measurements are in good agreement with zero temperature theoretical predictions based on the Thomas-Fermi approach. Experimental results obtained for the dipolar shift of a collective mode show a larger dependency to both the trap geometry and the atom number than the ones obtained when measuring the modification of the condensate aspect ratio due to dipolar forces. These findings are in good agreement with simulations based on a Gaussian ansatz.

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Interactions strongly affect the properties of quantum degenerate gases [\[1\]](#page-3-0). While in most of the experiments to date short-range interactions dominate, the recent production of Bose-Einstein condensates (BECs) of highly magnetic chromium atoms [\[2,](#page-3-1)[3](#page-3-2)] and the tremendous progress in the manipulation of heteronuclear molecules [[4\]](#page-3-3) have brought considerable interest in particles interacting through long-range, anisotropic, dipole-dipole interactions (DDIs) [[5](#page-3-4)].

The analysis of collective excitations is an excellent tool to analyze the effects of interactions in many-body systems [\[1,](#page-3-0)[6](#page-3-5)]. In trapped BECs where short-range isotropic interactions dominate, the excitations at low energies, being analogous to phonons in homogeneous systems, have a collective character. These excitations are well understood in the framework of the Gross-Pitaevskii equation (nonlinear Schrödinger equation). DDIs add a nonlocal character to this nonlinear framework. In 3D, it has been predicted that relatively small nonlocal interactions from dipoles should lead to modifications of collective mode frequencies, proportional to the ratio between DDIs and the mean-field due to contact interactions in the Thomas-Fermi (TF) regime. This effect of DDIs, which depends on the orientation of the polarization axis of the dipoles with respect to the trap geometry [[7](#page-3-6)[–9\]](#page-3-7), has not been observed to date.

For larger intensities of DDIs in 3D, or in reduced dimensions, the effect of DDIs on the properties of BECs is dramatic. In 3D, the ground state and the collective excitations are strongly modified [[10](#page-3-8)], eventually leading to collapse [\[11](#page-3-9)[,12\]](#page-3-10). In 2D, the excitation spectrum presents a roton structure reminiscent of the physics of liquid helium [\[13\]](#page-3-11). This roton minimum vanishes in 1D, leading to the breakdown of the Landau criterion for superfluidity [\[14,](#page-3-12)[15\]](#page-3-13) (similar to what is found with contact interactions [[16](#page-3-14)]). Understanding the collective modes of trapped dipolar quantum gases has a peculiar interest because collective modes and long-range interactions are key components in the quantum computing toolbox [\[17–](#page-3-15)[19](#page-3-16)].

The recent production of BECs with chromium atoms [\[2,](#page-3-1)[3](#page-3-2)], which have a large magnetic dipole moment (6 Bohr magnetons), makes it possible to experimentally investigate the effects of DDIs on the properties of quantum degenerate gases. The modification of the Thomas-Fermi radii [[20](#page-3-17)] of the BEC [\[21\]](#page-3-18), and its implosion if DDIs become comparable to short-range interactions [[11](#page-3-9)], have already been observed. In this Letter, we report the first measurement of the modification of collective excitation frequencies due to DDIs, in a regime where DDIs are relatively small compared to contact interactions.

In order to experimentally show that a shift of a low energy collective excitation frequency is induced by DDIs, we have measured the frequency of a collective mode of our spin polarized Cr BEC for two orthogonal orientations of the magnetic dipoles. The measured shift is very sensitive to trap geometry, a consequence of an interplay between the anisotropies of the trap and of DDI. Despite the fact that the number of atoms in the BEC is relatively small, the observed shift is in good agreement with zero temperature theoretical predictions based on the Thomas-Fermi approximation [\[8\]](#page-3-19), using the known value of the Cr scattering length $a_6 = 102.5 a_0$ [[22](#page-3-20)]. By operating at an even lower number of atoms, we measure a shift which decreases, and the deviation from the TF prediction is faster than what is observed for the aspect ratio of the expanded BEC, in good agreement with our numerical simulations.

Our all-optical method to produce chromium BECs is described in [\[3\]](#page-3-2). Its main specificity is the direct loading of an optical trap from a magneto-optical trap. Atoms are first loaded in a horizontal $(x \text{ axis})$ infrared (IR) one-beam optical dipole trap produced by a 1075 nm, 50 W fiber laser. Then some of the IR light is transferred to a vertical beam (z axis) by rotation of the angle ϕ of a $\lambda/2$ wave
plate to produce a strongly confining tran in all three plate, to produce a strongly confining trap in all three directions of space, where evaporation is performed. The horizontal and vertical beam waists are, respectively, equal to 40 and 50 μ m, and $\phi = 0$ when all the laser power (30 W) is in the horizontal beam. In contrast to [[3](#page-3-2)], the

horizontal trapping beam is not retroreflected, which increases the stability of the trap: the rms fluctuations of the BEC TF radii are reduced from typically 10% to below 3%. Condensates, with up to 15 000 atoms and no discernible thermal fraction, can be produced in different trap geometries characterized by oscillation frequencies $\omega_{x,y,z}$, by adiabatically changing ϕ after the quantum regime is reached.

Once the BEC is obtained in a trap for a given value of ϕ , we excite shape oscillations of the BEC by modulating the IR laser power with a 20% amplitude during about 15 ms, at a frequency close to the second surface collective mode resonance (here referred to as the intermediate collective mode). This quadrupolelike mode corresponds to an oscillation out of phase of the different TF radii (see Fig. [1](#page-1-0)). We then let the cloud oscillate freely in the crossed optical trap during a variable time. We finally switch off the trap, let the BEC expand for 5 ms, and take destructive absorption images, thus measuring the values of the cloud TF radii. We only report results for the intermediate collective mode: the lower collective mode is predicted to have a larger shift due to DDIs, but we found that for our trap parameters this mode is very hard to excite selectively, due to a quasidegeneracy between its second harmonic and the higher surface mode (monopolelike) frequency [\[23\]](#page-3-21).

A key feature of dipole interactions is its anisotropic nature. In an anisotropic trap, DDI therefore depends on the orientation of the spins (set by the magnetic field) relative to the axis of the trap. We use this property to perform a differential measurement of the shift of the intermediate collective mode due to DDIs: we measure the oscillations of the condensate aspect ratio for two perpendicular orientations θ of the magnetic field. For $\theta =$ 0, the \hat{B} field is horizontal and quasialigned with the imaging beam, while for $\theta = \pi/2$ it is vertical. We then fit the curves by exponentially damped sinusoidal functions, and deduce the corresponding oscillation frequencies $\omega_o(\theta)$. Both the experimental results and the fits are shown in Fig. [1](#page-1-0). We also show the residuals of the fit. The noise on the experimental data corresponds to a 3% rms noise on the TF radii, well below the amplitude of the oscillations. This noise is neither related to the fluctuations in atomic number nor to the way collective excitations are produced. Figure [1](#page-1-0) shows that the residuals to the fit do not increase with oscillation time, a signature of very good short-term (20 ms, the oscillation duration) and long-term (20 s, the cycling time of the experiment) stability.

As shown in Fig. [1](#page-1-0), damping of the collective excitations is rather strong, likely due to the large anharmonicity characteristic of an optical dipole trap. The damping rate depends neither on the magnetic field orientation θ nor on the trap anisotropy set by ϕ .

To deduce the shift due to DDIs on the collective mode frequency from the experimental data, it is important to measure and understand all other systematic shifts associated with varying θ . For this, it is in theory necessary to measure the systematic shift of all three orthogonal vibrational frequencies of the trap. In practice, the knowledge of the shift of the vertical axis is the most important [[24](#page-3-22)], and we have verified that the effect of the shifts of the frequency of the other two modes is negligible. We therefore have excited the dipole mode along the vertical direction and measured the frequency of the center of mass oscillation of the condensate $\omega_z(\theta)$. We deduce the relative shift

FIG. 1 (color online). Influence of DDIs on the intermediate collective mode frequency. Free oscillations of the mode after an excitation in a trap set by $\phi = 27^{\circ}$ ($\lambda = \omega_x/\omega_z = 0.79$) are monitored by measuring the aspect ratio of the BEC, given by the two
experimental TE radii along the y and z axes. The magnetic field is either vertical (bl experimental TF radii along the y and z axes. The magnetic field is either vertical (black diamonds) or parallel to the horizontal imaging beam (red dots). We plot as well the best fits from damped sinusoidal forms. The residuals to the fit are shown below. Inset: The ellipse qualitatively represents the atomic cloud, and the thick arrows show that the TF radius along y oscillates out of phase compared to the other two directions.

FIG. 2 (color online). Variation with the magnetic field orientation of the vertical dipole mode frequency, as a function of the trap geometry. The experimental relative shifts δ_D are plotted versus the angle ϕ (bottom axis) (see text) and versus the corresponding trap anisotropy $\lambda = \omega_x/\omega_z$ (top axis). A fit
assuming a (constant) tensorial shift and a (trap geometry assuming a (constant) tensorial shift and a (trap geometry dependent) magnetic gradient origin for the shifts is also shown.

 $\delta_D = 2[\omega_z(0) - \omega_z(\pi/2)]/[\omega_z(0) + \omega_z(\pi/2)]$, which is plotted in Fig. [2.](#page-2-0)

Figure [2](#page-2-0) shows that systematic effects on the vertical dipole frequency strongly depend on the trap geometry. We have found two independent sources for this systematic shift. The first is the presence of magnetic field gradients depending on the applied magnetic fields. In a parabolic trap, a potential gradient merely shifts the position of the center of the trap; in a Gaussian trap, the oscillation frequency is also modified. The relative frequency shift is $\partial \omega / \omega = \delta\omega/\omega = -3g/m^2w^2\omega^4$, where g is the acceleration due to the potential gradient, m the mass of the atoms, w the waist of the Gaussian potential and ω the tran frequency at waist of the Gaussian potential, and ω the trap frequency at the bottom of the trap. As ω depends on the trap geometry through the angle ϕ , this first systematic shift also depends on ϕ . The second source of systematic shift, Δ , is related to the tensorial light shift in Cr: even though the IR laser is very far detuned compared to the fine structure of the first electronically excited state, the ac Stark shift of Cr atoms slightly depends on both the laser polarization orientation and on the internal Zeeman state m [[25](#page-3-23)]. This second systematic shift is independent of ϕ . As shown in Fig. [2](#page-2-0), we fit our experimental data to a functional form Δ - α/ω^4 , and deduce the tensorial light shift of chromium for our experimental parameters $\Delta = (3 \pm 1)\%$, to be compared to our calculation $\Delta = 1.2\%$ [[25](#page-3-23)]. This represents the first measurement of the tensorial light shift of Cr.

Once we have measured the experimental shift of the collective mode frequency $\delta_{\exp} = 2[\omega_{\mathcal{Q}}(0) - \omega_{\mathcal{Q}}(\pi/2)]/[\omega_{\mathcal{Q}}(0) + \omega_{\mathcal{Q}}(\pi/2)]$ $\left[\omega_0(0) + \omega_0(\pi/2)\right]$, we estimate the shift of the collective mode due to DDIs, δ_{Q} , by subtracting from δ_{exp} the systematic shift of the intermediate collective mode frequency due to δ_D (which we estimate from [\[26\]](#page-3-24)). As shown in Fig. [3,](#page-2-1) we find for various trap geometries a good agreement between the experimentally measured δ_{Q} and a numerical model generalizing the theoretical results of

FIG. 3 (color online). Effect of DDIs on the experimental relative shifts induced by a 90° B field rotation, as a function of the trap geometry: δ_{θ} (red squares) is the shift of the intermediate collective mode frequency, while δ_{σ} (black dots) is the shift of the expanded BEC aspect ratio. Results of numerical simulations based on the TF approximation and with no adjustable parameter are also shown.

[\[8\]](#page-3-19) to nonaxisymmetric parabolic traps in the TF regime (similar to the theoretical work of [[9](#page-3-7)]).

We also plot in Fig. [3](#page-2-1) the measured modification of the aspect ratio due to DDIs for the BEC measured 5 ms after its release from the trap (similar to results reported in [\[21\]](#page-3-18)). For this, we use the BEC shape oscillation experimental data (see Fig. [1](#page-1-0)) and deduce the BEC equilibrium aspect ratio $\sigma(\theta)$ from the mean value of the TF radii. The corresponding relative variation is δ_{σ} = $2[\sigma(0) - \sigma(\pi/2)]/[\sigma(0) + \sigma(\pi/2)]$. We see from Fig. [3](#page-2-1)
that δ is almost constant for various tran geometries that δ_{σ} is almost constant for various trap geometries, which is in reasonable agreement with TF theory. In contrast, the shift of the collective mode δ_{θ} strongly depends on geometry.

The large sensitivity of δ_{Q} to trap geometry comes from the fact that the sign of the shift of the collective mode is approximately set by the sign of the mean field due to DDIs at the center of the BEC [[8](#page-3-19)]. As the sign of the mean field due to DDIs changes (in cylindrical traps) when the trap is modified from oblate (disklike) to elongated (cigarlike), the shift of the collective mode changes sign, too. For our nonaxisymmetric trap, we measure δ_{ρ} in a domain where the sign of dipole-dipole mean field flips (λ close to 1, see
Fig. 3), hence the very large sensitivity illustrated by Fig. 3. Fig. [3\)](#page-2-1), hence the very large sensitivity illustrated by Fig. [3.](#page-2-1) In contrast, as DDIs always stretch the BEC along the direction set by \vec{B} , δ_{σ} keeps the same sign whatever the trap geometry is, hence its much lower relative variation with ϕ .

Although our number of atoms is rather small, our results for δ_{θ} coincide with theoretical predictions based on the TF approximation (see Fig. [3\)](#page-2-1). In our experiment the mean field due to DDIs is only approximately equal to the quantum kinetic energy \hbar^2/mR_{TF}^2 : the fact that our ex-
perimental results follow predictions based on the TF perimental results follow predictions based on the TF approximation is therefore not obvious. To deepen our

FIG. 4 (color online). Variation with the BEC atom number of the relative shifts induced by a 90° B field rotation, for two trap geometries [blue circles, $\phi = 27^\circ$ ($\lambda = 0.79$); red squares, $\phi = 26^\circ$ ($\lambda = 0.73$)] (a) Variation of δ_{α} and comparison with 26° ($\lambda = 0.73$)]. (a) Variation of δ_{Q} and comparison with simulation (for $\phi = 26$ °). (b) Variation of δ , with the corresimulation (for $\phi = 26^{\circ}$). (b) Variation of δ_{σ} , with the corresponding results of our simulations. For both (a) and (b), TF predictions are recovered in the large atom number limit of our numerical simulations. The experimental intermediate collective mode frequency is also plotted (black diamonds). The black line is a guide for the eye.

understanding, we repeated our measurements for BECs with even lower numbers of atoms, obtained by loading less atoms in the optical dipole trap before evaporation. As shown in Fig. [4\(a\),](#page-3-25) we observe a rapid decrease of δ_{θ} as the number of atoms is lowered, which clearly differs from the TF predictions. On the contrary, Fig. [4\(b\)](#page-3-25) shows that the shift of the aspect ratio due to DDIs δ_{σ} is quite insensitive to the number of atoms. We have checked that for these atom numbers the collective frequencies themselves show little shift compared to the TF predictions without DDIs [see Fig. [4\(b\)](#page-3-25)].

Our results show that measuring collective excitations is a more sensitive probe of DDIs than measuring the stretching of the BEC along the axis of the dipoles. Numerical simulations based on a Gaussian ansatz which takes into account the quantum kinetic energy [\[7](#page-3-6)] confirm that δ_{θ} is more sensitive to a reduction of the number of particles than δ_{σ} is. Our numerical results show that it requires about 3 times more atoms to reach the TF predictions when measuring shift of collective excitations compared to when measuring stretching of the BECs. As shown in Fig. [4](#page-3-26) our numerical results (which involve no adjustable parameter) are in relatively good agreement with experimental data. The slight disagreement between theory and experiment for the collective excitation frequency shift may indicate the limits of the Gaussian ansatz.

In conclusion, we have measured the effect of DDIs on a collective mode of a Cr BEC. For a large enough number of particles in the BEC, our results are well explained by TF predictions. In particular, we find a very large sensitivity of the collective mode shift to the anisotropy of the trap, a consequence of the anisotropic character of DDIs. We also find that our results significantly depart from TF predictions for lower numbers of atoms, even when the striction of the BEC due to DDIs is still very well accounted for by a TF theory. This surprising feature is another example of the usefulness of collective modes to characterize quantum degenerate gases. Finally, we have measured for the first time the tensorial light shift of Cr atoms.

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