Aharonov-Bohm Conductance through a Single-Channel Quantum Ring: Persistent-Current Blockade and Zero-Mode Dephasing

A. P. Dmitriev,[1,2] I. V. Gornyi,[1,2] V. Yu. Kachorovskii,[1,2] and D. G. Polyakov[2]

[1]*A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*

[2]*Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*

(Received 4 November 2009; published 12 July 2010)

We study the effect of electron-electron interaction on transport through a tunnel-coupled single-channel ring. We find that the conductance as a function of magnetic flux shows a series of interaction-induced resonances that survive thermal averaging. The period of the series is given by the interaction strength α. The physics behind this behavior is the blocking of the tunneling current by the circular current. The main mechanism of dephasing is due to circular-current fluctuations. The dephasing rate is proportional to the tunneling rate and does not depend on α.

DOI: 10.1103/PhysRevLett.105.036402 PACS numbers: 71.10.Pm, 73.21.Hb

A major focus of interest in nanophysics [1] has been quantum interference effects on one hand and charge-quantization effects on the other, both of which become more prominent with decreasing dimensionality and size of the device. The prime device for specifically probing the interference of electrons is a quantum ring connected to the leads. The conductance of the ring $G(\Phi)$ exhibits the Aharonov-Bohm (AB) effect [2], i.e., changes periodically with the magnetic flux Φ threading the ring—with a period $\Phi_0 = hc/e$—entirely due to the interference of electron trajectories winding around the hole.

A key concept in the study of coherent electron transport is that of dephasing of electron waves, which at low temperature T is due to electromagnetic fluctuations produced by electron-electron (*e-e*) interactions [1]. The AB effect is one of the most convenient tools for studying the dephasing processes, since these directly govern the amplitude of the flux-dependent part of $G(\Phi)$.

The most ideal quantum-ring interferometer would be one made up of single-channel—ultimately one-dimensional (1D)—quantum wires. The basic physics of this deceptively simple setup may, however, become conceptually intricate. Indeed, it is well known that *e-e* interactions in 1D transform the electron gas into a Luttinger liquid (LL) [3]. The issue raised is the nature of the interference and dephasing in this strongly correlated state. Direct confrontation with experiment appears now to be possible since many-electron nanorings with a few or single conducting channels have been manufactured [4,5]. Transport of interacting electrons through the single-channel ring is the subject of this Letter.

We study the AB conductance of a LL ring *weakly* coupled by tunneling contacts to the leads. Throughout the Letter we focus on the high-temperature regime,

$$T \gg \Delta \gg \Gamma, \tag{1}$$

where Δ is the level spacing inside the ring, Γ is the tunneling rate. Our findings are summarized in Fig. 1. The evolution of $G(\Phi)$ with increasing interaction constant α is governed by two effects specific to the single-channel setup: (i) the destructive interference at $\Phi = \Phi_0/2$, inherited from the noninteracting problem [6], and (ii) a peculiar type of interaction of electrons with the circular current inside the ring, which dramatically changes the *shape* of the interference pattern (Fig. 1).

The physics behind this behavior can be outlined as follows. The interplay of (i) and (ii) manifests itself already in an isolated ring. The interaction with the persistent current J (quantized due to charge quantization) leads to a shift $\delta\Phi_J \propto \alpha J$ of the effective flux acting on electrons. This results in the interference-induced blocking of the tunneling current through the ring for specific values of Φ determined by the quantized values of J. We call this phenomenon persistent-current blockade (PCB).

In a tunnel-coupled ring, the circular current J is no longer strictly conserved. Its dynamics (“zero-mode fluctuations”) is responsible for both the peculiar shape of $G(\Phi)$ and the AB dephasing. The novel type of interaction-induced oscillations of $G(\Phi)$ that we predict [Fig. 1(c)]—with a distance between minima controlled by α—arises as a series of the PCB antiresonances, each of which corrresponds to one of the quantized values of J.

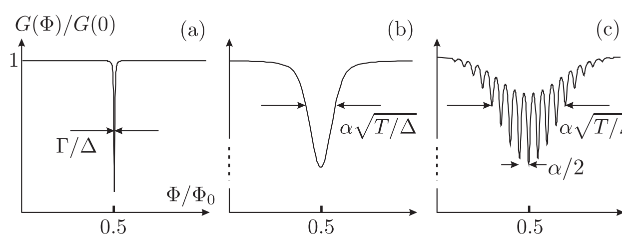

FIG. 1. Schematic evolution of $G(\Phi)$ with increasing interaction strength. (a) $\alpha \ll (\Gamma^2/\Delta T)^{1/2}$: a single deep antiresonance at half-integer flux through the ring; (b) $(\Gamma^2/\Delta T)^{1/2} \ll \alpha \ll \Gamma T/\Delta^2$: suppression of the antiresonance; (c) $\alpha \gg \Gamma T/\Delta^2$: breaking up of the antiresonance into “persistent-current blockade” oscillations. For fixed α, the evolution with increasing T follows (a) → (c) → (b).

The PCB oscillations—in contrast to the Coulomb-blockade oscillations [1]—survive thermal averaging at large T but are suppressed by dephasing. As shown below, the dominant mechanism of dephasing in a single-channel ring is provided by thermal fluctuations of the circular current (which translates into fluctuations of $\delta\Phi_J$). Our main result for the dephasing rate is

$$\gamma_\varphi = 4\Gamma T/\Delta. \tag{2}$$

The dephasing is strongly affected by quantization of charge inside the ring: γ_φ is seen to vanish for $\Gamma \to 0$. Another remarkable feature of γ_φ is that it does not depend on the interaction strength [7]. We stress that this zero-mode dephasing is qualitatively different from dephasing in the much better studied electronic Mach-Zehnder interferometer [8], where for *nonchiral* arms the dephasing rate is given by the single-particle decay rate in a homogeneous LL ($\sim \alpha^2 T$ for spinless electrons [9–11]).

Let us specify the model. Since we are interested in the regime $T \ll \Lambda$, where Λ is the ultraviolet cutoff (e.g., the Fermi energy), we linearize the electron dispersion relation around the Fermi level [3]. The Hamiltonian reads $H = H_{\rm ring} + H_{\rm tun} + H_{\rm leads}$, where ($\hbar = 1$)

$$H_{\rm ring} = \sum_\mu \int_0^L dx\Big(-i\mu v \psi^\dagger_\mu D_x \psi_\mu + \frac{1}{2}V_0 \hat{n}_\mu \hat{n}_{-\mu}\Big) \tag{3}$$

describes the isolated LL ring ($D_x = \partial_x - 2\pi i\phi/L$, $\phi = \Phi/\Phi_0$). In this Letter we focus on the case of spinless electrons. The index $\mu = \pm$ denotes electrons moving clockwise ($+$) and counterclockwise ($-$), L is the circumference of the ring, V_0 the zero-momentum Fourier component of the interaction potential, $\hat{n}_\mu = :\psi^\dagger_\mu \psi_\mu:$ the density in the channel μ. We assume that the Coulomb interaction is screened by a ground plane and take the interaction to be pointlike. The repulsion between electrons with the same μ is then accounted for completely in the renormalization of the velocity v [10]. We characterize the interaction strength by the parameter $\alpha = V_0/2\pi v$.

The tunneling term $H_{\rm tun} = t_0[\psi^\dagger_L \psi(0) + \psi^\dagger_R \psi(L/2)] +$ H.c., connects the electron operators ψ_R (ψ_L) in the right (left) lead at the points of the contacts and $\psi(x) = \psi_+(x) + \psi_-(x)$. The tunneling occurs at $x = 0$ and $L/2$, so that the arms of the interferometer are of the same length. We consider a symmetric setup with both contacts having the same tunneling rate $\Gamma_0 = 8\pi|t_0|^2\rho/L$, where ρ is the (structureless) density of states in the leads at the points of the contacts. Here we assumed that the leads are noninteracting and ballistic; the exact form of $H_{\rm leads}$ describing the leads is then of no importance.

In the absence of interaction, the transmission coefficient $T(\epsilon, \Phi)$ through the tunnel-coupled ring shows a resonance [6] each time the energy ϵ is close to one of the eigenenergies $\epsilon_{n\mu} = (n - \mu\phi)\Delta$ of an isolated ring. At zero T this yields a double resonance in $G(\Phi)$ [6]. In the LL ring at $T \ll \Delta$, the AB resonances are affected by Coulomb blockade and spin-related effects [12–15].

What does not seem to have been generally appreciated in the literature is the behavior of the "noninteracting" Landauer conductance $G_0(\Phi) = (e^2/2\pi) \times \int d\epsilon(-\partial_\epsilon f)T(\epsilon, \Phi)$ in the limit of high temperature $T \gg \Delta$ (f is the thermal distribution function). Of special interest are the points of degeneracy between levels of different chirality μ that occur at integer and half-integer values of ϕ. At $\phi = 1/2$ (which corresponds to the crossing of levels of *different* "parity"),

$$G_0(\Phi) = \frac{e^2\Gamma_0}{2\Delta} \frac{\cos^2(\pi\phi)}{\cos^2(\pi\phi) + (\pi\Gamma_0/2\Delta)^2} \tag{4}$$

exactly vanishes. At $\Gamma_0 \ll \Delta$, the high-T conductance exhibits a sharp (anti)resonance [Fig. 1(a)] [16]. By contrast, the interference contribution vanishes at integer ϕ, where $G_0(\Phi)$ is featureless.

To obtain this behavior in a way that is convenient for introducing interaction, let us write $T(\epsilon, \Phi) = |t_+(\epsilon, \Phi) + t_-(\epsilon, \Phi)|^2$, where $t_\pm$ is the transmission amplitude of electrons injected into the $\psi_\pm$ mode. Returns of electrons to the tunneling contacts described by a 3×3 S matrix are accompanied by changing chirality. Importantly, at $\phi = 1/2$, for each path contributing to t_+ there exists a "mirrored" path (with $\mu \to -\mu$ on each segment) whose contribution to t_- has an opposite sign. It is this destructive interference that leads to the vanishing [6] of $T(\epsilon, \Phi_0/2)$ for arbitrary ϵ. More specifically, at high $T \gg \Delta$, only the products of amplitudes corresponding to paths of equal length (but with an arbitrary sequence of chiralities) are not suppressed by thermal averaging. The conductance can then be written as a sum over the winding numbers $n \geq 0$. A delicate point here is that one cannot neglect backscattering inside the ring at the tunneling contacts even if Γ_0/Δ is small. Doing so would give $G_0(\Phi) \propto \sum_n |A^+_{2n+1} + A^-_{2n+1}|^2$, where $A^\mu_k = e^{i\mu k\pi\phi}(1 + \pi\Gamma_0/2\Delta)^{1-k}$ is the amplitude that preserves the chirality of the injected wave (below $\bar{A}^\mu_k$ is its complex conjugate). This expression contains sharp resonances *both* at $\phi = 0$ and at $\phi = 1/2$. In fact, however, the effect of backscattering is strongly enhanced by multiple returns and leads to $G_0(\Phi) \propto \sum_{n\mu}[|A^\mu_{2n+1}|^2 + (A^\mu_{2n+1}\bar{A}^{-\mu}_{2n+1} - A^\mu_{2n+2}\bar{A}^{-\mu}_{2n+2})/2]$. It is seen that the backscattering removes the resonance at $\phi = 0$ while not affecting the resonance at $\phi = 1/2$.

Our purpose here is to understand how the shape of the AB resonance (4) changes when e-e interactions are turned on. Making use of the scale separation (1), we first integrate out all energy scales larger than T, which takes into account the virtual processes [17] that yield the LL renormalization of the model. The main outcome is the renormalization of the tunneling rate: $\Gamma_0 \to \Gamma(T)$; in particular, $\Gamma(T) \sim \Gamma_0(\Lambda/T)^{(1-K)^2/2K}$ for $\alpha \gg \Gamma_0/\Delta$ [18], where $K = (1-\alpha)^{1/2}(1+\alpha)^{-1/2}$ is the Luttinger constant. Note that at $T \gg \Delta$ two contacts are renormalized independently. Another consequence is that the velocity of single-particle excitations [10] becomes equal to the plasmon velocity $u = v(1-\alpha^2)^{1/2}$ (the level spacing is now $\Delta = 2\pi u/L$). Next, we employ the quasiclassical approximation—justified for $T \gg \Delta$ and $\alpha \ll 1$—in which the effect of e-e interactions on the single-particle transmission amplitudes is described in terms of scattering on the thermal electromagnetic noise created by the bath of other electrons.

It is instructive to first consider the bath with the total number N_μ of electrons in the channel μ being a quantum number. For a linear dispersion relation, the peculiarity of the single-channel ring is that at $\Gamma = 0$ the density profile $n_\mu(x)$ for given μ remains unchanged and rotates as a whole. The forward scattering of electrons of chirality μ is then fully accounted for through the phase they acquire in the time-dependent potential $U_\mu(x,t) = V_0 n_{-\mu}(x+\mu u t)$. In particular, the quasiclassical amplitude of the transition from $x=0$ to $x=L/2$ without winding around the hole is given by $A_1^\mu = \exp\{i\pi\mu\phi + iV_0\int_0^{L/2u} dt n_{-\mu}[x(t)+\mu u t]\}$. A crucial point is that, even though the time integration is taken over the half-period, for $x(t) = \mu u t$ the integral is insensitive to a particular profile of $n_{-\mu}$ and only depends on $N_{-\mu}$. Clearly, this holds true for the amplitude with an arbitrary winding number n. As a result, the interference term in the conductance,

$$A_k^+ \bar{A}_k^- = \exp\{2\pi i k[\phi - \alpha(N_+ - N_-)/2]\}, \tag{5}$$

is not suppressed by thermal averaging over fluctuations of $n_\pm(x,t)$ at fixed $N_\pm$ (it is this averaging that is responsible for the exponential decay of single-particle excitations in an infinite LL). In other words, plasmons in the isolated ring do not lead to any dephasing in our symmetric setup.

It follows from Eq. (5) that, apart from the renormalization of Γ and Δ, the only effect of the interaction of electrons tunneling through the ring with the bath characterized by fixed N_μ is the effective shift of the flux

$$\delta\Phi_J = -\alpha J \Phi_0/2, \tag{6}$$

where $J = N_+ - N_-$ is the (dimensionless) persistent current circulating inside the ring. Physically, the phase shift (6) between two interfering waves stems from the absence of e-e scattering within the same channel μ (“Hartree-Fock cancellation” [10]). In effect, for given J, electrons of opposite chirality see different electrostatic potentials, which translates into the phase difference in Eq. (5). Being inserted in Eq. (4), $\delta\Phi_J$ yields a shift of the AB resonance: the PCB occurs at $\phi = 1/2 - \delta\Phi_J/\Phi_0$; in other words, the persistent current completely blocks the tunneling current through the ring at this value of ϕ.

For a thermodynamic ensemble of the “isolated baths,” the conductance [Eq. (4)] should be averaged over the Gibbs distribution of the zero-mode energies [19,20],

$$\epsilon_{N_+N_-} = (\Delta/4K)[(N-N_0)^2 + K^2(J-2\phi)^2], \tag{7}$$

where N_0 is controlled by the chemical potential and $N = N_+ + N_-$ is the total number of electrons in the ring. Equation (7) describes, quite generally, electrostatics of a 1D ring. The resulting conductance as a function of ϕ shows PCB oscillations with a period α and a Gaussian envelope whose width $w_T = \alpha(T/\Delta)^{1/2}$ is entirely determined by the statistical weights of different values of J.

Taking into account the ergodic tunneling dynamics of the electron bath, i.e., the time dependence of the circular current, leads to PCB oscillations in a *single* ring [21]. In contrast to the isolated ring, each PCB resonance acquires a width induced by a finite lifetime of the state with given J. Importantly, this time is much shorter than the single-electron tunneling lifetime Γ^{-1}. Indeed, the time scale for changing J by unity is given by Γ^{-1} divided by the number of levels T/Δ that participate in the tunneling dynamics. We identify the interaction-induced broadening of the PCB resonances with dephasing [Eq. (2)].

For a quantitative analysis of $G(\Phi)$, we average the product of the amplitudes in Eq. (5) over realizations of $J(t)$. This gives the interaction-induced action $S(t_n)$, where $t_n = 2\pi(n+1/2)/\Delta$ for the winding number n:

$$e^{-S(t)} = \left\langle \exp\left\{-i\alpha\Delta\int_0^t dt'[N_+(t') - N_-(t')]\right\}\right\rangle. \tag{8}$$

We now represent $N_\mu = \sum_j n_j^\mu$ as a sum over individual energy levels inside the ring [22]. The time evolution of the occupation numbers $n_j^\mu = 0, 1$ is telegraph noise with the rates Γf_j and $\Gamma(1-f_j)$ for scattering “in” and “out”, respectively, where f_j is the distribution function in the leads at the energy of the jth level. The phase factor induced by the interaction with the jth level is written as (here we suppress the indexes j and μ for brevity) [23]:

$$\langle e^{i\alpha\Delta\int_0^t dt' n(t')}\rangle = (1-f)(P_{00}+P_{01}) + f(P_{10}+P_{11}),$$

where $P_{kl}(t)$ satisfy the master equation $\dot{P}_{kl} = (-1)^l \times \{[\Gamma(1-f) - il\alpha\Delta]P_{k1} - \Gamma f P_{k0}\}$ and the initial condition $P_{kl}(0) = \delta_{kl}$ (k and l are the initial and final occupation numbers, respectively). Solving this equation we get $S(t) = -2\mathrm{Re}\sum_j \ln[(e^{\lambda_j^+ t}\lambda_j^- - e^{\lambda_j^- t}\lambda_j^+)/(\lambda_j^- - \lambda_j^+)]$, where $\lambda_j^\pm = \lambda - i\alpha\Delta f_j \pm (\lambda^2 + i\alpha\Delta\Gamma f_j)^{1/2}$ and $\lambda = (i\alpha\Delta - \Gamma)/2$. The interference term $\delta G(\Phi) = G(\Phi) - G(0)$ is affected by the action (8) (below $\delta_\phi = \phi - 1/2$):

$$\frac{\delta G(\Phi)}{G(0)} \simeq -\frac{2\pi\Gamma}{\Delta}\sum_{n=0}^{\infty}\cos(2\Delta\delta_\phi t_n)e^{-\Gamma t_n - S(t_n)}. \tag{9}$$

For $\alpha \gg \Gamma/\Delta$, the sum (9) is cut off by $S(t)$ at $t_n \ll \Gamma^{-1}$, so that we can expand $S(t)$ in Γ. The action at $\Gamma = 0$ is given by the thermodynamic average $e^{-S_0(t)} = \langle e^{-i\alpha J\Delta t}\rangle_{\mathrm{Gibbs}}$ over the zero-mode energies (7) and yields PCB resonances with different J. For $T \gg \Delta$, $S_0(t) \simeq \alpha^2 T\Delta\{t^2\}$ where $\{\cdots\}$ denotes a periodic continuation in t from the interval $-\pi/\alpha\Delta < t < \pi/\alpha\Delta$. The linear-in-$\Gamma$ term,

$$S_1(t) \simeq \frac{4\Gamma T}{\Delta}\,\eta(t)\left[t\cos^2\!\left(\frac{\alpha\Delta t}{2}\right) - \frac{\sin(\alpha\Delta t)}{\alpha\Delta}\right] \tag{10}$$

with $\eta(t) = \alpha\Delta\{t\}/\sin(\alpha\Delta t)$, is responsible for the dephasing. For $\alpha \ll (\Delta/T)^{1/2}$, the sum in Eq. (9) can be replaced by an integral. The latter is dominated by the vicinity of the points $t = 2\pi m/\alpha\Delta$ with integer $m \geq 0$, where $e^{-S_0(t)}$ is sharply peaked. At these points for $m \gg 1$, $S_1(t) \simeq \gamma_\varphi t$ with the dephasing rate γ_φ given by Eq. (2). The interference term then reads:

$$\frac{\delta G(\Phi)}{G(0)} \simeq \mathrm{Im}\frac{(\Gamma/2w_T\Delta)\exp(-\delta_\phi^2/w_T^2)}{\sin[\pi(\delta_\phi + 2i\gamma_\varphi/\Delta)/\alpha]}. \tag{11}$$

If $\alpha\Delta \gg \gamma_\varphi$, Eq. (11) yields well-separated Lorentzians [24] [Fig. 1(c)] of width γ_ϕ/Δ, centered at integer δ_ϕ/α. Note that, despite the appearance of the PCB fine structure, the exact period in ϕ remains unity, as it should be. In the opposite limit, $\alpha\Delta \ll \gamma_\varphi$, the broadening of the resonances is larger than the distance between them, so that they merge into a single Gaussian dip of width w_T [Fig. 1(b)]. Equation (11) describes the physically most transparent case of not too large $\alpha \ll (\Delta/T)^{1/2}$, which means that the width w_T of the envelope of the PCB resonances is much smaller than the period of the AB oscillations. At larger α, additional features appear, in particular, related to a possible commensurability between $\delta\Phi_J$ and Φ_0—these will be considered elsewhere [18].

It is worth noting that the tunneling broadens also the plasmon levels inside the ring, which introduces an additional contribution γ_φ^p to the dephasing rate. Averaging the amplitudes A_k^μ over fluctuations of $n_\mu[x(t)]$ that occur on the time scale of Γ^{-1}, we find $\gamma_\varphi^p \sim \alpha^2\Gamma T/\Delta$. It follows that for $\Gamma/\Delta \ll \alpha \ll 1$ the dephasing due to the non-Gaussian zero-mode fluctuations of $J(t)$ is much stronger than that induced by plasmons.

In conclusion, we have demonstrated that e-e interactions lead to profound and unusual effects in transport through a single-channel quantum-ring interferometer tunnel coupled to the leads, originating from the phenomenon of persistent-current blockade. We have shown that the AB conductance $G(\Phi)$ exhibits a series of sharp resonances broadened by dephasing, the distance between which is controlled by the interaction strength. We have calculated the main contribution to the dephasing rate, which is due to tunneling-induced fluctuations of the circular current. The physics described in the Letter remains intact for spinful electrons and ballistic systems with a small number of conducting channels. Our predictions can thus be verified by measuring the conductance of a semiconductor nanoring or a single coil of carbon nanotube.

We thank D. Aristov, D. Bagrets, H. Bouchiat, Y. Imry, D. Khmelnitskii, A. Mirlin, and M. Pletyukhov for valuable discussions. The work was supported by the DFG/CFN, the EUROHORCS/ESF, GIF Grant No. 965, the RFBR, Programs of the RAS, the Dynasty Foundation, and Rosnauka Grant No. 02.740.11.5072.

[1] Yu. V. Nazarov and Ya. M. Blanter, *Quantum Transport: Introduction to Nanoscience* (Cambridge University Press, Cambridge, 2009).
[2] A. G. Aronov and Yu. V. Sharvin, Rev. Mod. Phys. **59**, 755 (1987).
[3] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).
[4] H. R. Shea, R. Martel, and Ph. Avouris, Phys. Rev. Lett. **84**, 4441 (2000); S. Zou *et al.*, Nano Lett. **7**, 276 (2007).
[5] V. Piazza *et al.*, Phys. Rev. B **62**, R10630 (2000); A. Fuhrer *et al.*, Nature (London) **413**, 822 (2001); U. F. Keyser *et al.*, Phys. Rev. Lett. **90**, 196601 (2003); E. B. Olshanetsky *et al.*, JETP Lett. **81**, 625 (2005).
[6] M. Büttiker, Y. Imry, and M. Ya. Azbel, Phys. Rev. A **30**, 1982 (1984); Y. Gefen, Y. Imry, and M. Ya. Azbel, Phys. Rev. Lett. **52**, 129 (1984).
[7] More accurately, α enters in the renormalization of Γ and the condition of the applicability of Eq. (2) $\alpha \gg \Gamma/\Delta$.
[8] Y. Ji *et al.*, Nature (London) **422**, 415 (2003).
[9] K. Le Hur, Phys. Rev. B **65**, 233314 (2002); Phys. Rev. Lett. **95**, 076801 (2005); Phys. Rev. B **74**, 165104 (2006).
[10] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. **95**, 046404 (2005); Phys. Rev. B **75**, 085421 (2007).
[11] Dephasing similar to Refs. [9,10] was found in G. Seelig and M. Büttiker, Phys. Rev. B **64**, 245313 (2001) for the Mach-Zehnder ring capacitively coupled to side gates.
[12] E. A. Jagla and C. A. Balseiro, Phys. Rev. Lett. **70**, 639 (1993).
[13] J. M. Kinaret *et al.*, Phys. Rev. B **57**, 3777 (1998).
[14] M. Pletyukhov, V. Gritsev, and N. Pauget, Phys. Rev. B **74**, 045301 (2006).
[15] M. Eroms, L. Mayrhofer, and M. Grifoni, Phys. Rev. B **78**, 075403 (2008).
[16] The resonance in the energy-averaged conductance at $\phi = 1/2$ was obtained in Ref. [12], where the zero-T case was considered but, to present the results, the authors have averaged them over ϵ in the interval of width Δ.
[17] We use the scheme of separating the virtual and real processes in a LL that was devised in Ref. [10].
[18] A. P. Dmitriev, I. V. Gornyi, V. Yu. Kachorovskii, and D. G. Polyakov (to be published).
[19] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981); D. Loss, Phys. Rev. Lett. **69**, 343 (1992).
[20] As pointed out in Ref. [14], $G(\Phi)$ cannot be decoupled into a product of two single-particle zero-mode averages.
[21] The fluctuating circular current induces fluctuations of $\delta\Phi_J$ and thus of the tunneling current: transport is totally blocked within an interval of time when $J(t)$ is such that $\delta\Phi_J(t)$ compensates the deviation of ϕ from 1/2.
[22] We neglect the inelastic scattering of bath electrons on each other because it does not affect $N_\mu(t)$.
[23] A similar approach was used to describe dephasing of a qubit by a two-level fluctuator, see Y. M. Galperin *et al.*, Phys. Rev. Lett. **96**, 097009 (2006); J. Schriefl *et al.*, New J. Phys. **8**, 1 (2006); C. Neuenhahn *et al.*, Phys. Status Solidi B **246**, 1018 (2009), and references therein.
[24] Equation (11) can be cast in the form of a Gibbs sum over zero-mode states: $\delta G(\Phi)/G(0) \simeq \Gamma\gamma_\varphi\langle[(2\delta_\phi - \alpha J)^2\Delta^2 + \gamma_\varphi^2]^{-1}\rangle_{\mathrm{Gibbs}}$.