Inverse Faraday Effect with Linearly Polarized Laser Pulses

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The inverse Faraday effect is usually associated with circularly polarized radiation; here, we show that it can also occur for linearly polarized radiation. The quasistatic axial magnetic field generated by a laser propagating in plasma can be calculated by considering both the spin and the orbital angular momenta of the laser pulse. A net spin is present when the radiation is circularly polarized and a net orbital angular momentum is present if there is any deviation from perfect rotational symmetry. The orbital angular momentum gives an additional contribution to the axial magnetic field that can enhance or reduce the effect usually attributed to circular polarization and strongly depends on the intensity profile of the Laguerre-Gaussian modes involving the azimuthal and radial mode numbers.

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The inverse Faraday effect involves the generation of a quasistatic axial magnetic field and has always been associated with circularly polarized radiation propagating through an unmagnetized plasma. Several theoretical approaches [1–5] to the inverse Faraday effect have been investigated and confirmed in different experimental conditions [6–9]. It has been found that the induced magnetic field depends on the laser intensity and the electron number density. Lehner [5] discussed the generation of a magnetic field varying as $(n_e I_0)^{1/2}$ by considering the relativistic ponderomotive force in an underdense plasma. Najmudin *et al.* [6] measured a magnetic field of the order 4 ± 0.8 MG due to the interaction of a circularly polarized laser with underdense helium plasma at relativistic intensities $\sim 10^{23}$ W m^{-2}. Deschamps, Fitaire, and Laoutte [7] reported a magnetic field of the order 2×10^{-2} G varying as $n_e I_0$ due to the interaction of circularly polarized microwaves of power ~ 1 MW with plasma having $n_e \sim 3 \times 10^{15}$ m^{-3}. Horovitz and co-workers [8,9] measured an axial magnetic field of the order 10 kG (2 MG) using a neodymium-glass laser (wavelength 1.06 μm) of intensity 10^{17} W m^{-2} (10^{18} W m^{-2}). Sheng and Meyer-ter-Vehn [10] derived an expression for the magnetic field estimating the order of magnitude as 100 MG in the presence of inhomogeneity of laser beam and electron density in overdense plasmas. For a short circularly polarized laser pulse, Gorbunov and Ramazashvili [11] calculated that the magnetic field in a homogeneous plasma should scale as $n_e^{1/2} I_0^2$. In contrast to previous theories, Haines [3] explained the importance of an azimuthal electric field in the generation of an axial magnetic field, considering the photon spin associated circularly polarized radiation, obtaining a dependence of $1/n_e$. According to Haines's approach, magnetic fields in the MG range can be produced by circularly polarized laser pulses with a duration of the order of 1 ps and intensity $I_0 > 10^{22}$ W m^{-2}.

Here we present a more general view of the inverse Faraday effect based on angular momentum and show that it can also occur with linearly polarized radiation. Such a generalization is based on the possibility of laser beams carrying a net orbital angular momentum (OAM). In 1936, Beth [12] and Holbourn [13] were able to measure the mechanical torque due to the exchange of angular momentum of circularly polarized radiation to a half wave plate. The photon angular momentum is transferred to matter when it absorbs radiation [14,15]. Laser absorption in plasma is a very complex area, particularly at high intensities when collisionless processes dominate. The angular momentum of the photon beam essentially consists of spin associated with the polarization state, and the orbital angular momenta created due to the angular beam structure. Recently, Allen *et al.* [16] have highlighted the importance of the OAM of a photon beam, describing it in terms of Laguerre-Gaussian (LG) modes.

Photon beams are usually considered to have planar wave fronts with uniform phase, having the wave vectors and linear momentum along the beam axis. However, helical wave fronts may also exist, when the wave vectors spiral around the beam axis, constituting the existence of OAM. The physics of the helical wave fronts associated with photon states was recently discussed in Refs. [17–19]. It is now well-known that helical wave fronts can be represented in a basis set of orthogonal LG modes and that each LG mode is associated with a well-defined state of photon OAM. Harwit [20] discussed the photon orbital angular momentum in the context of astrophysical phenomena. Mendonça *et al.* [21] studied the excitation of photon OAM states in a plasma, investigating helical disturbances in a static plasma and in a rotating plasma vortex. Stimulated Raman and Brillouin backscattering with OAM have been investigated [22] and plasmon (or longitudinal photon) states involving OAM have been studied [23].

We show in this Letter that an axial magnetic field in a plasma can be enhanced or reduced by using the intensity profiles of LG beams involving a finite OAM and that the excitation of axial magnetic fields becomes possible even with linearly polarized beams.

The average rate of change of the electron angular momentum per unit volume in the presence of photon angular momentum, in a cylindrical coordinate system (r, θ, z), is governed by the following conservation equation,

$$m_e n_e r\left(\frac{d}{dt} + \nu_{ei}\right)U_{e\theta} = -en_e r(E_\theta + U_{ez}B_r - U_{er}B_z) - \frac{dM_z}{dt}, \quad (1)$$

where m_e is the electron mass, n_e is the electron number density, $U_{e\theta}$ is the azimuthal electron velocity, $m_e U_{e\theta} r$ is the angular momentum of the electron in the z direction, E_θ is the azimuthal electric field, ν_{ei} is the electron-ion collision frequency, $B_r(B_z)$ is the radial (axial) component of the magnetic field $\mathbf{B}$, and M_z is the axial component of the photon angular momentum density given by

$$M_z = \frac{l}{\omega c} I + \frac{\sigma_z r}{2\omega c}\left(\frac{\partial I}{\partial r}\right), \quad (2)$$

where $l = 0, \pm 1, \ldots$ is the quantum number of the orbital angular momentum, corresponding to the azimuthal mode number of a LG mode [16,23], σ_z is the quantum number of the spin angular momentum, which is -1 ($+1$) for right (left) circularly polarized light and zero for linearly polarized light, ω is the laser angular frequency, I is the laser intensity, and c is the speed of light in vacuum. Here, we will not distinguish between possible fast (hot or suprathermal) and cold electron populations, considering only net momentum conservation between electrons and photons.

In order to calculate the total value of the axial angular momentum, we integrate over M_z by parts from $r = 0$ to $r = \infty$,

$$\int_0^\infty 2\pi r M_z dr = \frac{2\pi}{\omega c}\left\{\int_0^\infty lrI(r)dr - \sigma_z \int_0^\infty rI(r)dr\right\}, \quad (3)$$

where we have assumed that the intensity goes to zero at infinity. Equation (3) can also be expressed, as

$$\int_0^\infty 2\pi r M_z dr = \frac{(l - \sigma_z)P_L}{\omega c}, \quad (4)$$

where $P_L = 2\pi \int_0^\infty I(r) r dr$ is the total power of the laser pulse. It is clear that Eq. (4) does not vanish for linearly polarized radiation ($\sigma_z = 0$) due to the presence of OAM. Photon beams with helical phase fronts give rise to circularly polarized radiation having both orbital and spin momenta. We also note that the total angular momentum shown in Eq. (4) exactly agrees with the quantum mechanical relation $M_z = (l - \sigma_z)n_\gamma \hbar \equiv (l - \sigma_z)I(r)/\omega c$, where n_γ is the photon number density.

To estimate the axial magnetic field that could be generated we will consider a time scale τ such that $\omega_{pi}^{-1} \gg \tau \gg \omega_{pe}^{-1}$, where the subscripts i and e identify the ion and electron plasma frequencies, respectively. Over such a time scale ion motion may be reasonably neglected and the electrons will have time to reach a steady state ($dU_{e\theta}/dt = 0$). Further assuming that the collision frequency is much less than the electron plasma frequency, which is almost always the case, we can neglect the collisions, which would lead to a subsequent decay of the magnetic field. Balancing the remaining dominant terms involving the azimuthal electric field and the angular momentum density in Eq. (1), we obtain

$$en_e r E_\theta \sim -\frac{dM_z}{dt}. \quad (5)$$

We note that the electron density n_e could well be modified by the ponderomotive force of the laser, but we will not take this into account explicitly here. The z component of Faraday's law can be written as

$$-\frac{1}{r}\frac{\partial}{\partial r} r E_\theta = \frac{\partial B_z}{\partial t}. \quad (6)$$

Inserting Eq. (5) into Eq. (6), we arrive at

$$\frac{\partial B_z}{\partial t} = \frac{1}{en_e r}\frac{\partial}{\partial r}\frac{d}{dt}\left(\frac{lI}{\omega c} + \frac{\sigma_z r}{2\omega c}\frac{\partial I}{\partial r}\right). \quad (7)$$

After integration in time from 0 to t we obtain

$$B_z = -\frac{f_{abs}}{ren_e\omega c}\left\{l\frac{\partial I}{\partial r} + \frac{\sigma_z}{2}\frac{\partial}{\partial r}\left(r\frac{\partial I}{\partial r}\right)\right\}. \quad (8)$$

In deriving Eq. (8), we have used $I(t) - I(0) = -f_{abs}I$. The absorption coefficient f_{abs} of the laser intensity absorbed over a certain axial distance L could be expressed in terms of inverse bremsstrahlung as $f_{abs} = 1 - \exp(-\kappa_{ib}L)$, where κ_{ib} is the damping rate of the laser energy by inverse bremsstrahlung. For weak absorption, $\kappa_{ib}L \ll 1$, the absorption coefficient is approximately $f_{abs} \simeq \kappa_{ib}L$, and for strong absorption, $\kappa_{ib}L \gg 1$, we have $f_{abs} = 1$. As the electron number density $n_e \to 0$ the absorption coefficient f_{abs} will also tend to zero so in practice Eq. (8) does not have a singularity at $n_e = 0$. For linearly polarized radiation $\sigma_z = 0$, but even then an axial magnetic field can exist due to the orbital angular momentum density.

Writing $r = r_0$ and $\partial/\partial r \sim 1/r_0$, Eq. (8) can be expressed to estimate the order of magnitude of the field in terms of practical units, as

$$B_z \sim f_{abs}\left(\frac{\lambda}{r_0}\right)^2\left(\frac{n_e}{n_c}\right)^{-1}\left(\frac{\mu\mathrm{m}}{\lambda}\right)\left(l + \frac{\sigma_z}{2}\right) \times \left\{\frac{I_0\lambda^2}{7.3\times 10^{22}\ \mathrm{W\,m^{-2}}(\mu\mathrm{m})^2}\right\}\mathrm{MG}, \quad (9)$$

where n_c is the nonrelativistic critical density, which is approximately $1.1 \times 10^{15}/\lambda^2$ m^{-3}.

Any laser beam can be described by LG modes [24], which represent a general solution of the paraxial wave equation in cylindrical geometry. They are a natural orthonormal basis set for representing a beam in cylindrical geometry. For instance, if the beam is not a perfect Gaussian then the higher order terms involved in the intensity profiles of LG modes give rise to OAM.

The intensity profile of the LG modes in the focal plane ($z = 0$) is given by

$$I(r) = I_0 \frac{(-1)^{2p} p!}{(l+p)!}\left(\frac{r}{r_0}\right)^{2l} \exp\left(-\frac{r^2}{r_0^2}\right)\left[L_p^l\left(\frac{r^2}{r_0^2}\right)\right]^2, \quad (10)$$

where I_0 is the maximum axial intensity of the beam, r_0 the beam radius, p is the radial mode number, and $L_p^l(X)$ is the associated Laguerre polynomials, where $X = (r/r_0)^2$.

For a circularly polarized Gaussian beam, we have $l = 0 = p$, $\mathrm{LG}_0^0 = 1$, so the magnetic field is

$$B_z = \frac{2 f_{abs} I_0 \sigma_z}{e n_e \omega c r_0^2}\left(1 - \frac{r^2}{r_0^2}\right)\exp\left(-\frac{r^2}{r_0^2}\right), \quad (11)$$

For a finite OAM $l = 1$ and assuming $p = 0$, the intensity profile from Eq. (10) reduces to

$$I(r) = I_0\left(\frac{r}{r_0}\right)^2 \exp\left(-\frac{r^2}{r_0^2}\right). \quad (12)$$

Accordingly, the axial magnetic field becomes

$$B_z = -\frac{2 f_{abs} I_0}{e n_e \omega c r_0^2} f(r), \quad (13)$$

where the radial form function $f(r)$ is defined by

$$f(r) = \left\{1 - \frac{r^2}{r_0^2} + \sigma_z\left(1 - \frac{3r^2}{r_0^2} + \frac{r^4}{r_0^4}\right)\right\}\exp\left(-\frac{r^2}{r_0^2}\right).$$

We now choose some typical experimental values from Refs. [3,6], namely, $I_0 = 7.3 \times 10^{22}$ W m^{-2}, $\omega = 1.79 \times 10^{15}$ s^{-1}, $n_e = 2.1 \times 10^{25}$ m^{-3}, $L = 10^{-3}$ m, $r_0 = 10^{-5}$ m, and $f_{abs} = 1$. Assuming that $l = 0 = p$ and $\sigma_z = -1$ in Eq. (8), the axial magnetic field for a parabolic laser profile can be computed as 2.4 MG, in agreement with Haines's result [3]. Noting that the magnitude of this field is smaller than the experimentally measured value of 4 ± 0.8 MG [6], it is worth mentioning here that Eq. (8) can give a larger magnetic field in the presence of higher order terms of the LG modes, which in this case might have been introduced by the quarter wave plate used to produce circular polarization.

To examine the impact of angular mode number on the total angular momentum density, we plot M_z as a function of r for circular polarization ($\sigma_z = -1$) for the Laguerre-Gauss polynomials LG_0^1, LG_0^2, and LG_1^1 in Fig. 1. It can be seen that the angular momentum density increases for varying l with fixed $p = 0$ and a net variation appears in

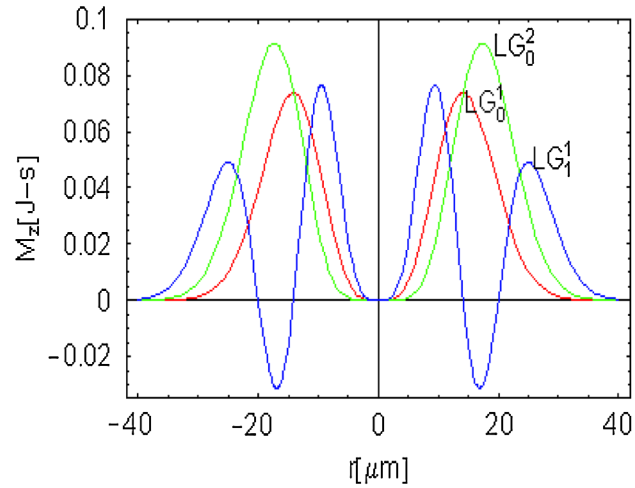

FIG. 1 (color online). The orbital angular momentum density (M_z) as a function of r with circular polarization $\sigma_z = -1$ and laser intensity $I_0 = 7.3 \times 10^{22}$ W m^{-2} for different Laguerre-Gauss polynomials LG_0^1, LG_0^2, and LG_1^1.

the radial direction. On the other hand, a nonzero radial mode number, i.e., $p = 1$, introduces a node in the laser pulse with fixed $l = 1$.

In Fig. 2, we plot the axial magnetic field as a function of r for the polynomials LG_0^0 and LG_0^1 for circular polarizations $\sigma_z = \pm 1$. The azimuthal mode number ($l = 1$) causes an increase in the width of the profile of the magnetic field, which is now in the opposite direction as compared to the Gaussian profile. Figure 3 shows the axial magnetic field generated due to circularly polarized radiation ($\sigma_z = 1$) as well as linearly polarized radiation ($\sigma_z = 0$), with polynomials LG_1^0 and LG_1^{-1}. A node is produced around the Gaussian profile due to radial mode number

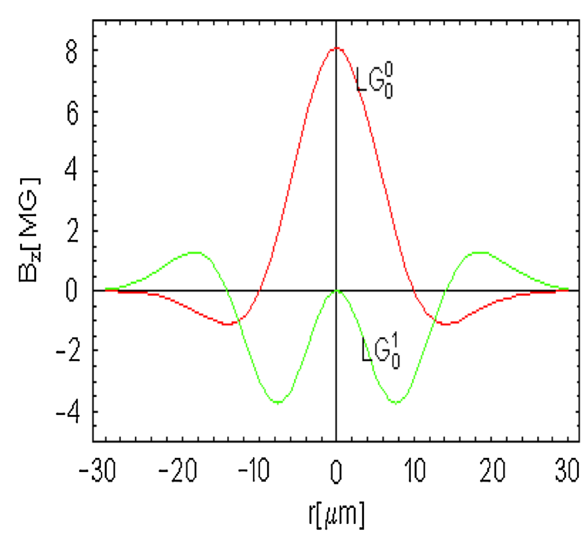

FIG. 2 (color online). The axial magnetic field B_z as a function of r with laser intensity $I_0 = 7.3 \times 10^{22}$ W m^{-2} and the LG polynomials LG_0^0 with $\sigma_z = 1$ and LG_0^1 with $\sigma_z = -1$.

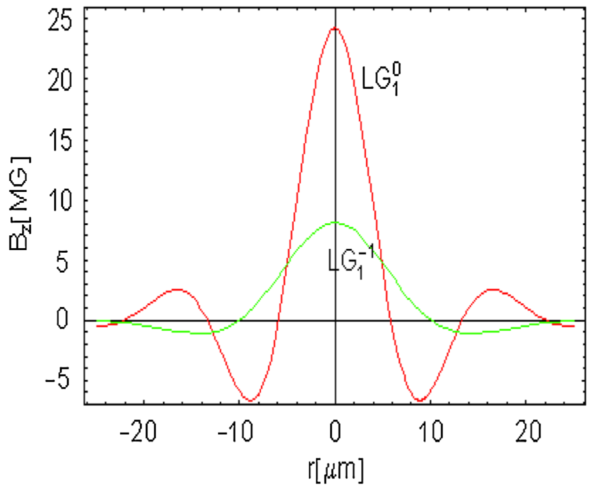

FIG. 3 (color online). The axial magnetic field B_z as a function of r with laser intensity $I_0 = 7.3 \times 10^{22}$ W m^{-2} and the LG polynomials LG_1^0 with $\sigma_z = 1$ and LG_1^{-1} with $\sigma_z = 0$.

$p = 1$. The inverse Faraday effect also takes place for the linearly polarized laser pulses with $l = -1$ and $p = 1$.

To conclude, we have studied a model for the exchange of total angular momentum in laser-plasma interactions. An axial magnetic field can be generated by an orbital angular momentum, associated with deviations from rotational symmetry, as well as spin angular momentum, associated with circular polarization. Orbital angular momentum leads to an inverse Faraday effect even with linear polarization that can exceed that produced by circular polarization, depending upon the azimuthal and radial mode numbers of the LG modes. This has important consequences for our physical understanding of the inverse Faraday effect, and could have interesting implications for the interpretation of electron acceleration and betatron emission in laser wakefield experiments.

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[1] L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. **39**, 1450 (1960) [Sov. Phys. JETP **12**, 1008 (1961)].
[2] A. D. Steiger and C. H. Woods, Phys. Rev. A **5**, 1467 (1972).
[3] M. G. Haines, Phys. Rev. Lett. **87**, 135005 (2001).
[4] G. Shvets, N. J. Fisch, and J.-M. Rax, Phys. Rev. E **65**, 046403 (2002).
[5] T. Lehner, Phys. Scr. **49**, 704 (1994).
[6] Z. Najmudin *et al.*, Phys. Rev. Lett. **87**, 215004 (2001).
[7] J. Deschamps, M. Fitaire, and M. Lagoutte, Phys. Rev. Lett. **25**, 1330 (1970).
[8] Y. Horovitz, S. Eliezer, A. Ludmirsky, Z. Henis, E. Moshe, R. Shpitalnik, and B. Arad, Phys. Rev. Lett. **78**, 1707 (1997).
[9] Y. Horovitz, S. Eliezer, Z. Henis, Y. Paiss, E. Moshe, A. Ludmirsky, M. Werdiger, B. Arad, and A. Zigler, Phys. Lett. A **246**, 329 (1998).
[10] Z. M. Sheng and J. Meyer-ter-Vehn, Phys. Rev. E **54**, 1833 (1996).
[11] L. M. Gorbunov and R. R. Ramazashvili, J. Exp. Theor. Phys. **87**, 461 (1998).
[12] R. A. Beth, Phys. Rev. **50**, 115 (1936).
[13] A. H. S. Holbourn, Nature (London) **137**, 31 (1936).
[14] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms* (Wiley, New York, 1989).
[15] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer-Verlag, Berlin, 1976), 2nd ed.
[16] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A **45**, 8185 (1992).
[17] M. Harris, C. A. Hill, P. R. Tapster, and J. M. Vaughan, Phys. Rev. A **49**, 3119 (1994).
[18] M. J. Padgett, J. Arlt, N. B. Simpson, and L. Allen, Am. J. Phys. **64**, 77 (1996).
[19] J. Leach, M. J. Padgett, S. M. Barnett, S. Franke-Arnold, and J. Courtial, Phys. Rev. Lett. **88**, 257901 (2002).
[20] M. Harwit, Astrophys. J. **597**, 1266 (2003).
[21] J. T. Mendonça, B. Thide, J. E. S. Bergman, S. M. Mohamadi, B. Eliasson, and W. Baan, arXiv:0804.3221v3.
[22] J. T. Mendonça, B. Thidé, and H. Then, Phys. Rev. Lett. **102**, 185005 (2009).
[23] J. T. Mendonca, S. Ali, and B. Thide, Phys. Plasmas **16**, 112 103 (2009).
[24] M. W. Beijersbergen, L. Allen, H. E. L. O. van der Veen, and J. P. Woerdman, Opt. Commun. **96**, 123 (1993).