Damping of Nanomechanical Resonators

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We study the transverse oscillatory modes of nanomechanical silicon nitride strings under high tensile stress as a function of geometry and mode index $m \leq 9$. Reproducing all observed resonance frequencies with classical elastic theory we extract the relevant elastic constants. Based on the oscillatory local strain we successfully predict the observed mode-dependent damping with a single frequency-independent fit parameter. Our model clarifies the role of tensile stress on damping and hints at the underlying microscopic mechanisms.

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The resonant motion of nanoelectromechanical systems has received a lot of recent attention. Their large frequencies, low damping, i.e., high mechanical quality factors, and small masses make them equally important as sensors [1–4] and for fundamental studies [3–9]. In either case, low damping of the resonant motion is very desirable. Despite significant experimental progress [10,11], a satisfactory understanding of the microscopic causes of damping has not yet been achieved. Here we present a systematic study of the damping of doubly-clamped resonators fabricated out of prestressed silicon nitride leading to high mechanical quality factors [10]. Reproducing the observed mode frequencies applying continuum mechanics, we are able to quantitatively model their quality factors by assuming that damping is caused by the local strain induced by the resonator's displacement. We thereby deduce that the high quality factors of strained nanosystems can be attributed to the increase in stored elastic energy rather than a decrease in energy loss. Considering various microscopic mechanisms, we conclude that the observed damping is most likely dominated by dissipation via localized defects uniformly distributed along the resonator.

We study the oscillatory response of nanomechanical beams fabricated from high stress silicon nitride (SiN). A released doubly-clamped beam of such a material is therefore under high tensile stress, which leads to high mechanical stability [12] and high mechanical quality factors [10]. Such resonators therefore have been widely used in recent experiments [6,9]. Our sample material consists of a silicon substrate covered with 400 nm thick silicon dioxide serving as sacrificial layer and a $h = 100$ nm thick SiN device layer. Using standard electron beam lithography and a sequence of reactive ion etch and wet-etch steps, we fabricate a series of resonators having lengths of $35/n$ μm, $n = \{1, \ldots, 7\}$ and a cross section of 100×200 nm^2 as displayed in Figs. 1(a) and 1(b). Since the respective resonance frequency is dominated by the large tensile stress [10,13], this configuration leads to resonances of the fundamental modes that are approximately equally spaced in frequency. Suitably biased gold electrodes processed beneath the released SiN strings actuate the resonators via dielectric gradient forces to perform out-of-plane oscillations, as explained in greater detail elsewhere [12]. The length and location of the gold electrodes is properly chosen to be able to also excite several higher order modes of the beams. The experiment is carried out at room temperature in a vacuum below 10^{-3} mbar to avoid gas friction.

The displacement is measured using an interferometric setup that records the oscillatory component of the reflected light intensity with a photodetector and network analyzer [12,14]. The measured mechanical response around each resonance can be fitted using a Lorentzian line shape as exemplarily seen in the inset of Fig. 2. The thereby obtained values for the resonance frequency f and

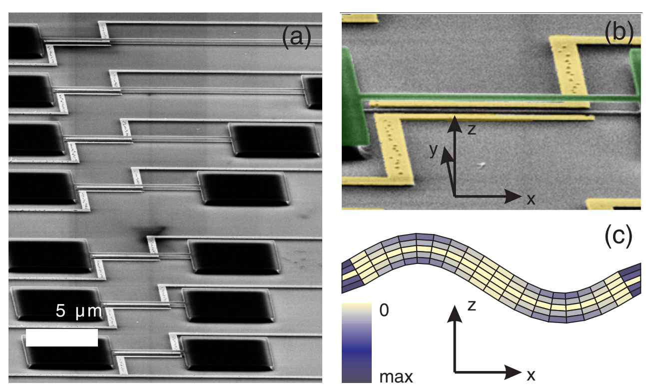

FIG. 1 (color online). *Setup and geometry.* (a) Scanning electron microscope picture of our sample; the lengths of the investigated nanomechanical silicon nitride strings are $35/n$ μm, $n = \{1, \ldots, 7\}$; their widths and heights are 200 nm and 100 nm, respectively. (b) Zoom-in of (a) the resonator (highlighted in green [dark gray]) is dielectrically actuated by the nearby gold electrodes (yellow [light gray]); its displacement is recorded with an interferometric setup. (c) Schematic mode profile and absolute value of the resulting strain distribution (color coded) of the second harmonic.

quality factor Q for all studied resonators and observed modes are displayed in Fig. 2 (filled circles). In order to reproduce the measured frequency spectrum, we apply standard beam theory (see, e.g., [15]). Without damping, the differential equation describing the spatial dependence of the displacement for a specific mode m of beam n $u_{n,m}[x]$ at frequency $f_{n,m}$ writes (with $\rho = 2800\ \mathrm{kg/m^3}$ being the material density [16]; E_1, σ_0 are the (unknown) real Young's modulus and built-in stress, respectively):

$$\frac{1}{12}E_1 h^2 \frac{\partial^4}{\partial x^4} u_{n,m}[x] - \sigma_0 \frac{\partial^2}{\partial x^2} u_{n,m}[x] - \rho(2\pi f_{n,m})^2 u_{n,m}[x] = 0 \quad (1)$$

Solutions of this equation have to satisfy the boundary conditions of a doubly-clamped beam (displacement and its slope vanish at the supports ($u_{n,m}[\pm l/(2n)] = (\partial/\partial x)u_{n,m}[\pm l/(2n)] = 0$, l/n: beam length). These conditions lead to a transcendental equation that is numerically solved to obtain the frequencies $f_{n,m}$ of the different modes.

The results are fitted to excellently reproduce the measured frequencies, as seen in Fig. 2 (hollow squares). One thereby obtains as fit parameters the elastic constants of the microprocessed material $E_1 = 160$ GPa, $\sigma_0 = 830$ MPa, in good agreement with previously published measurements [13].

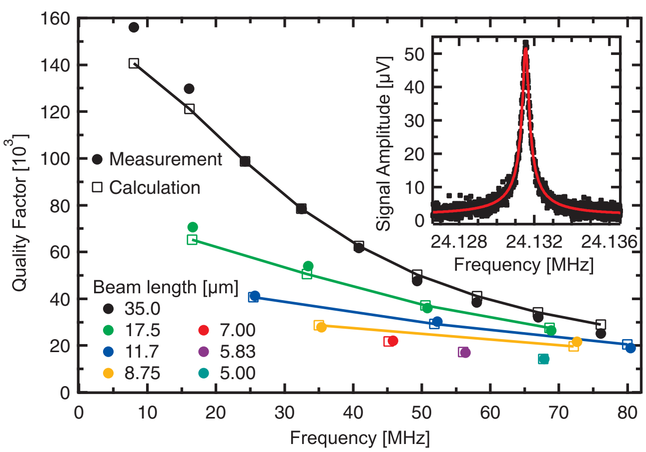

FIG. 2 (color online). *Resonance frequency and mechanical quality factor.* The harmonics of the nanomechanical resonator show a Lorentzian response (exemplary in the inset). Fitting yields the respective frequency and mechanical quality factor. The main figure displays these values for several harmonics (same color) of different beams as indicted by the color. To reproduce the resonance frequencies, we fit a continuum model to the measured frequencies. We thereby retrieve the elastic constants of our (processed) material, namely, the built-in stress $\sigma_0 = 830$ MPa and Young's modulus $E_1 = 160$ GPa. From the displacement-induced, mode-dependent strain distribution, we calculate (except for an overall scaling) the mechanical quality factors. Calculated frequencies and quality factors are shown as hollow squares, the responses of the different harmonics of the same string are connected.

For each harmonic, we now are able to calculate the strain distribution within the resonator induced by the displacement $u[x]$ and exemplarily shown in Fig. 1(c). The measured dissipation is closely connected to this induced strain $\epsilon[x, z, t] = \epsilon[x, z]\exp[i2\pi f t]$. As in the model originally discussed by Zener [17] we now assume also for our case of a statically prestressed beam that the displacement-induced strain and the accompanying oscillating stress $\sigma[x, z, t] = \sigma[x, z]\exp[i2\pi f t]$ are not perfectly in phase; this can be expressed by a Young's modulus $E = E_1 + iE_2$ having an imaginary part. The relation reads again $\sigma[x, z] = (E_1 + iE_2)\epsilon[x, z]$. During one cycle of oscillation $T = 1/f$, a small volume δV of length s and cross section A thereby dissipates the energy $\Delta U_{\delta V} = A s \pi E_2 \epsilon^2$. The total loss is obtained by integrating over the volume of the resonator.

$$\Delta U_{n,m} = \int_V dV \Delta U_{\delta V} = \pi E_2 \int_V dV \epsilon_{n,m}[x, z]^2 \qquad (2)$$

The strain variation and its accompanying energy loss can be separated into contributions arising from overall elongation of the beam and its local bending. It turns out that here the former is negligible, despite the fact that the elastic energy is dominated by the elongation of the string, as discussed below. To very high accuracy we obtain for the dissipated energy $\Delta U_{n,m} \approx \pi/12 E_2 w h^3 \int_l dx(\partial^2/(\partial x)^2 u_{n,m})^2$. A more rigorous derivation can be found in the supplementary information [18]. The total energy depends on the spatial mode [through $\epsilon_{n,m}$, see exemplary Fig. 1(c)] and therefore strongly differs for the various resonances. To obtain the quality factor, one has to calculate the stored energy, e. g., by integrating the kinetic energy $U_{n,m} = \int_l dx A\rho(2\pi f_{n,m})^2 u_{n,m}[x]^2$. The overall mechanical quality factor is $Q = 2\pi U_{n,m}/\Delta U_{n,m}$. A more detailed derivation can be found in [18].

Assuming that the unknown value of the imaginary part E_2 of the elastic modulus is independent of resonator length and harmonic mode, we are left with one fit parameter E_2 to reproduce all measured quality factors and find excellent agreement (Fig. 2, hollow squares). We therefore successfully model the damping of our nanoresonators by postulating a frequency-independent mechanism caused by local strain variation. We wish to point out that the quality factor of, e.g., the second harmonic of a particular beam is significantly higher if compared to the fundamental one of a shorter beam with the same frequency. This can be understood by the fact that the maximum strain and thus local dissipation occurs near the clamping points and a higher harmonic has less clamping points per antinode [see Fig. 1(c)].

Allowing E_2 to depend on frequency, the accordance gets even better, as discussed in detail in [18].

We now discuss the possible implications of our findings, considering at first the cause of the high quality factors in overall prestressed resonators and then the compatibility of our model with different microscopic damping mechanisms. In a relaxed beam, the elastic energy is stored in the flexural deformation and becomes for a small test volume $U_{\delta V} = 1/2AsE_1\epsilon^2$. In the framework of a Zener model, as employed here, this result is proportional to the energy loss [see Eq. (2)] and thus yields a frequency-independent quality factor $Q = E_1/E_2$ for the unstressed beam. In accordance with this finding, Ref. [10] reports a much weaker dependence of quality factor on resonance frequency, in strong contrast with the behavior of their stressed beams.

Similar as in the damping model, the total stored elastic energy in a beam can be very accurately separated into a part connected to the bending and a part coming from the overall elongation. The latter is proportional to the pre-stress σ_0 and vanishes for relaxed beams, refer to [18] for details. Assuming a constant $E = E_1 + iE_2$, Fig. 3 displays the calculation of the elastic energy and the quality factor for the fundamental mode of our longest ($l = 35\ \mu$m) beam as a function of overall built-in stress σ_0. The total elastic energy is increasingly dominated by the displacement-induced elongation $U_{\rm elong} = 1/2\sigma_0 wh \int_l dx(\partial/(\partial x)u[x])^2$. In contrast the bending energy $U_{\rm bend} = 1/24E_1wh^3 \int_l dx(\partial^2/(\partial x)^2 u[x])^2$, which in our model is proportional to the energy loss, is found to increase much slower with σ_0. Thus one expects Q to increase with σ_0, a finding already discussed by Schmid and Hierold for micromechanical beams [19]. However, their model assumes the simplified mode profile of a stretched string and can not explain the larger quality factors of higher harmonics when compared to a fundamental resonance of the same frequency. Including beam stiffness, our model can fully explain the dependence of frequency and damping on length and mode index, as reflected in Fig. 2. It also explains the initially surprising finding [20] that amorphous silicon nitride resonators exhibit high quality factors when stretched whereas having Q factors in the relaxed state that reflect the typical magnitude of internal friction found to be rather universal in glassy materials [21]. More generally we conclude that the increase in mechanical quality factors with increasing tensile stress is not bound to any specific material.

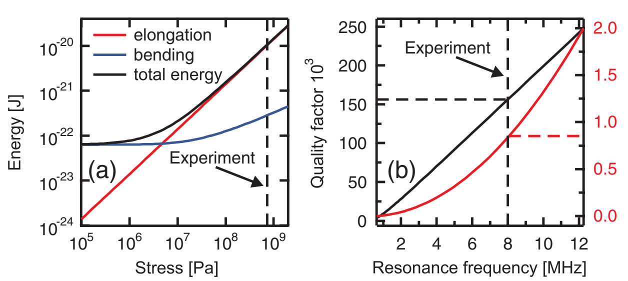

FIG. 3 (color online). *Elastic energy and mechanical quality factor of the beam in dependence of stress.* (a) The elastic energies of the fundamental mode of the beam with $l = 35\ \mu$m are displayed vs applied overall stress separated into the contributions resulting from the overall elongation and the local bending. The dashed line marks the strain of the experimentally studied resonator $\sigma_0 \approx 830$ MPa, there the elongation term dominates noticeably. (b) Quality factor and frequency are calculated for varying stress σ_0. In order to compare the calculation with other published results, quality factor and stress are displayed vs resulting frequency.

Since the resonance frequency is typically easier to access in an experiment, we plot the quality factor vs corresponding resonance frequency in Fig. 3(b), with both numbers being a function of stress. The resulting relation of quality factor on resonance frequency is (except for very low stress) almost linear; experimental results by another group can be seen to agree well with this finding [22]. In addition, we show in [18] that although the energy loss per oscillation increases with applied stress, the linewidth of the mechanical resonance decreases.

We will now consider the physical mechanisms that could possibly contribute to the observed damping. As explained in greater detail in [18], we can safely neglect dampings that are intrinsic to any (bulk) system, namely, clamping losses [23,24], thermoelastic damping [25,26] and Akhiezer damping [26,27], since the corresponding model calculations all predict damping constants significantly smaller than the ones observed.

Therefore, we would like to discuss the influence of localized (defect) states. Mechanisms with discrete relaxation rates will exhibit damping maxima whenever the oscillation frequency matches the relaxation rate [25,26,28]. As our model however is based on a frequency-independent loss mechanism, we therefore conclude that a broad range of states is responsible for the observed damping. This assumption is consistent with a model calculation dealing with the influence of two-level systems on acoustic waves [29] at high temperatures. There, the strain modulates the energy separation of the two states and thereby excites the system out of thermal equilibrium; the subsequent relaxation causes the energy loss. In addition, published quality factors of relaxed silicon nitride nanoresonators [20] cooled down to liquid helium temperature display quality factors that are well within the typical range of amorphous bulk materials [21], therefore the observed damping mechanism can be assumed to reduce to the concept of two-level systems at low temperatures. Moreover, on a different sample chip we measured a set of resonators showing quality factors that are uniformly decreased by a factor of approximately 1.4 compared to the data presented in Fig. 2; the corresponding data are presented in [18]. Their response can still be quantitatively modeled resulting in an increased imaginary part of Young's modulus E_2. We attribute this reduction in quality factor to a nonoptimized RIE-etch step, that leads to an increased density of defect states in the near-surface region of the resonator. In contrast, the above mentioned intrinsic mechanisms are not expected to be influenced by such processing.

We wish to point out some limitations of our simple model description. One is that the above stated simplification to local two-level systems cannot be rigorously applied at elevated temperatures as the concept of two-level systems should be replaced by local excitable systems. The other is that our assumption of a damping mechanism via localized defects distributed uniformly along the resonator cannot differentiate between surface and volume losses (see [18]). In fact, measurements performed on beams with larger width exhibit slightly higher quality factors pointing toward a contribution of surface defects as does the effect of sample processing discussed above, a well-known observation in micro- or nanoresonators, see e.g. [30,31]. At present we cannot conclude on the microscopic nature of the defect states implicitly assumed in our model. These could reflect the amorphous nature of the SiN resonator but also be influenced by near-surface modification.

In conclusion, we systematically studied the transverse mode frequencies and quality factors of prestressed SiN nanoscale beams. Implementing continuum theory, we reproduce the measured frequencies varying with beam length and mode index over an order of magnitude. Assuming that damping is caused by local strain variations induced by the oscillation, independent of frequency, enables us to calculate the observed quality factors with a single interaction strength as free parameter. We thus identify the unusually high quality factors of prestressed beams as being primarily caused by the increased elastic energy rather than a decrease in damping rate. Several possible damping mechanisms are discussed; because of the observed nearly frequency independent damping parameter E_2, we attribute the mechanism to interaction of the strain with local defects of not yet identified origin. One therefore expects that defect-free resonators exhibit even larger quality factors, as recently demonstrated for ultraclean carbon nanotubes [11].

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