# Gaussian Quantum Discord

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We extend the quantum discord to continuous variable systems and evaluate Gaussian quantum discord $C(\varrho)$ for bipartite Gaussian states. In particular, for squeezed-thermal states, we explicitly maximize the extractable information over Gaussian measurements: $C(\varrho)$ is minimized by a generalized measurement rather than a projective one. Almost all squeezed-thermal states have nonzero Gaussian discord: They may be either separable or entangled if the discord is below the threshold $C(\varrho) = 1$, whereas they are all entangled above the threshold. We elucidate the general role of state parameters in determining the discord and discuss its evolution in noisy channels.

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Quantum correlations have been the subject of intensive studies in the past two decades, mainly due to the general belief that they are a fundamental resource for quantum information processing tasks. The first rigorous attempt to address the classification of quantum correlation has been put forward by Werner [1], who put on a firm basis the elusive concept of quantum entanglement. A state of a bipartite system is called entangled if it cannot be written as follows: $\varrho_{AB} = \sum p_k \varrho_{Ak} \otimes \varrho_{Bk}$, where $\varrho_{Ak}$ and $\varrho_{Bk}$ are generic density matrices describing the states of the two subsystems. The definition above has an immediate operational interpretation: Separable states can be prepared by local operations and classical communication between the two parties, whereas entangled states cannot. One might have thought that such classical information exchange could not bring any quantum character to the correlations in the state. In this sense, separability has often been regarded as a synonymous of classicality. However, it has been shown that this is not the case [2,3]. A measure of correlations—quantum discord—has been defined as the mismatch between two quantum analogues of classically equivalent expression of the mutual information. For pure entangled states, quantum discord coincides with the entropy of entanglement. However, quantum discord can be different from zero also for some (mixed) separable state. In other words, classical communication can give rise to quantum correlations. This can be understood by considering that the states $\varrho_{Ak}$ and $\varrho_{Bk}$ above may be physically nondistinguishable, i.e., nonorthogonal, and thus not all the information about them can then be locally retrieved. This phenomenon has no classical counterpart, thus accounting for the quantumness of the correlations in a separable state with positive discord. Quantum discord has been shown to be a property held by almost all quantum states [4] and has recently attracted considerable attention [5–8]. In particular, the vanishing of quantum discord between two systems has been shown to be a requirement for the complete positivity of the reduced subsystem dynamics [9].

While the discord is a fundamental notion allowing for the description of the quantumness of the correlations present in the state of a quantum system, its evaluation requires an optimization procedure over the set of all measurements on a given subsystem, and thus attacking the general case is a formidable task. For this reason, the original definition of the quantum discord [2] involved orthogonal measurements, and its evaluation and the study of its properties has mainly been restricted to final dimensional systems [10]. The purpose of this Letter is to extend the notion of discord to the domain of continuous variable systems. In the following, we focus our analysis on bipartite systems that are described by two-mode Gaussian states, and we explore the concept of discord within the domain of generalized Gaussian measurement, i.e., any measurement that may be achieved by using passive and active linear optics, homodyne detection, and auxiliary modes prepared in Gaussian states [11,12]. We start our discussion by reviewing the main ideas at the basis of the definition of the discord. Let us consider two classical random variables $A$ and $B$ with joint probability $p_{AB}(a, b)$; the total correlations between the two variables are measured by the mutual information. The latter may defined by two equivalent expressions $I(A;B) = H(A) + H(B) - H(A,B)$ and $I(A;B) = H(A) - H(A|B) \equiv H(B) - H(B|A)$, where $H(X) = -\sum_x p_X(x) \log p_X(x)$ is the Shannon entropy of the corresponding probability distribution and the conditional entropy is defined in terms of the conditional probability $p_{A|B}(a|b)$ as $H(A|B) = -\sum_{ab} p_{AB}(a,b) \log p_{A|B}(a|b)$. The idea of quantum discord grows out of the fact that the quantum version of the mutual information of a bipartite state $\varrho_{AB}$ may be defined in two nonequivalent ways. The first is obtained by the straightforward quantization of $I(A;B)$, i.e., $I(\varrho_{AB}) = S(\varrho_A) + S(\varrho_B) - S(\varrho_{AB})$, where $S(\varrho) = -\mathrm{Tr}[\varrho \log \varrho]$ is the von Neumann entropy of the state $\varrho$ and $\varrho_{A(B)} = \mathrm{Tr}_{B(A)}[\varrho_{AB}]$ are the partial traces over the two subsystems. On the other hand, the quantization of the expression based on conditional entropy, i.e., the extractable information, involves the conditional state of a subsystem after a measurement performed on the other one, and this fact has three relevant consequences: (i) The symmetry between the two subsystems is broken; (ii) this quantity depends on the choice of the measurement; (iii) the resulting expression is generally different from $I(\varrho_{AB})$. Let us denote by $\varrho_{Ak} = 1/p_B(k)\mathrm{Tr}_B[\varrho_{AB}\mathbb{I} \otimes \Pi_k]$, with $p_B(k) = \mathrm{Tr}_{AB}[\varrho_{AB}\mathbb{I} \otimes \Pi_k]$, the conditional state of the system $A$ after having observed the outcome $k$ from a measurement performed on the system $B$. In turn, $\{\Pi_k\}$, $\sum_k \Pi_k = \mathbb{I}$ denotes a positive operator-valued measure (POVM) describing a generalized measurement. The quantum analogue of the mutual information defined via the conditional entropy is defined as the upper bound $J_A = \sup_{\{\Pi_k\}} S(\varrho_A) - \sum_k p_B(k) S(\varrho_{Ak})$ taken over all the possible measurements. Finally, the quantum $A$ discord is defined in terms of the mismatch $C(\varrho_{AB}) = I(\varrho_{AB}) - J_A(\varrho_{AB})$. Analogously, one is led to define the $B$ discord through the entropy of conditional states of system $B$. In the following, we show that the extractable information $J(\varrho_{AB})$ for two-mode Gaussian states can be maximized over the class of Gaussian measurements and that the mismatch between $C(\varrho_{AB})$ is actually minimized by a POVM rather than a projective measurement. Since the results for the $B$ discord can be recovered by a reparametrization, from now on we refer to $A$ discord and omit the indication of the subsystem. Recently, a different quantity has been introduced [13], which is essentially a symmetrized version of the discord.

We start our analysis by proving a general result: Quantum discord is invariant under local unitary operations, i.e., $C(U_A \otimes U_B \varrho_{AB} U_A^\dagger \otimes U_B^\dagger) = C(\varrho_{AB})$, $\forall\ \varrho$ and any choice of the local unitaries. The proof simply follows by first noticing that the mutual information $I(\varrho_{AB})$ is written in terms of two single-system entropies and thus it is not changed by the action of local unitaries. Furthermore, extractable information rewrites as $J(\varrho) = S(\varrho_A) - \sum_k p'_B(k) S(\varrho'_{Ak})$, where the primed quantities are evaluated by using the transformed POVM $\Pi'_k = U_B^\dagger \Pi_k U_B$. Since the reparametrization does not change the superior, invariance is proved. This result is relevant since it allows us to focus our analysis on Gaussian states whose covariance matrix

$$\boldsymbol{\sigma} = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$$

can be put in a simplified standard form, i.e., $A = \mathrm{diag}(a, a)$, $B = \mathrm{diag}(b, b)$, and $C = \mathrm{diag}(c_1, c_2)$, by means of local symplectic operations corresponding to local unitaries that preserve the Gaussian character of the state. The quantities $I_1 = \det A$, $I_2 = \det B$, $I_3 = \det C$, and $I_4 = \det \boldsymbol{\sigma}$ are left unchanged by the transformations and are thus referred to as symplectic invariants. The local invariance of the discord has therefore two main consequences. On the one hand, $C(\varrho)$ may be written in terms of symplectic invariants only. On the other hand, it allows us to restrict to states with $\boldsymbol{\sigma}$ already in the standard form. In particular, while the derivation we give for the Gaussian discord is applicable to the general case, for the explicit calculations we will focus on the relevant subclass of states for which $c1 = -c2$, i.e., the squeezed-thermal states (STS) $\varrho = S(r)\nu_1 \otimes \nu_2 S^\dagger(r)$, where $S(r) = e^{r(a^\dagger b^\dagger - ab)}$ is the two-mode squeezing operator and $\nu_j = \sum_k N_j^k (1 + N_k)^{-k-1} |k\rangle\langle k|$, $j = 1, 2$ are chaotic states with $N_j$ the average number of thermal photons. Using this parametrization we have $a = (N_r + \frac{1}{2}) + N_1(1 + N_r) + N_2 N_r$, $b = (N_r + \frac{1}{2}) + N_2(1 + N_r) + N_1 N_r$, and $c_1 = -c_2 = (1 + N_1 + N_2)\sqrt{N_r(1 + N_r)}$, where $N_r = \sinh^2 r$.

The definition of the Gaussian quantum discord is based on the minimization of the mismatch $I(\varrho) - J(\varrho)$ over single-mode generalized Gaussian measurements. A first class of such POVMs may be written as [11,12] $\Pi_{\boldsymbol{X}} = D(\boldsymbol{X})\varrho_M D^\dagger(\boldsymbol{X})$, $\int d\boldsymbol{X} \Pi_{\boldsymbol{X}} = \mathbb{1}$, where $\boldsymbol{X}$ is a two-dimensional real vector and $\varrho_M$ a generic zero mean Gaussian state whose covariance matrix is $(\sigma_M)_{11} = \alpha$ , $(\sigma_M)_{22} = \beta$, $(\sigma_M)_{12} = (\sigma_M)_{21} = \gamma$, with fixed parameters $\alpha, \beta \in \mathbb{R}^+$, $\gamma \in \mathbb{R}$. If one performs the measurement described by $\Pi_{\boldsymbol{X}}$ on, say, mode $B$ of a bipartite Gaussian state, then the distribution of the outcomes $p(\boldsymbol{X})$ is a bimodal Gaussian with covariance matrix $(B + \sigma_M)$, whereas the conditional state $\varrho_{\boldsymbol{X}}$ of mode $A$ is a Gaussian state of mean $X^T(B + \sigma_M)^{-1}C^T$ and covariance matrix given by the Schur complement $\sigma_P = A - C(B + \sigma_M)^{-1}C^T$ [14].

Quantum discord may be written as $C(\varrho) = S(\varrho_B) - S(\varrho) + \inf_{\{\Pi_X\}} \int d\boldsymbol{X} p(\boldsymbol{X}) S(\varrho_{\boldsymbol{X}})$, and the *general form of Gaussian quantum discord* is

$$C(\varrho) = h(\sqrt{I_2}) - h(d_-) - h(d_+) + \inf_{\sigma_M} h(\sqrt{\sigma_P}), \qquad (1)$$

where $h[x] = (x + \frac{1}{2})\log(x + \frac{1}{2}) - (x - \frac{1}{2})\log(x - \frac{1}{2})$ and $d_\pm$ are the symplectic eigenvalues of $\varrho$, expressed by $d_\pm^2 = \frac{1}{2}[\Delta \pm \sqrt{\Delta^2 - 4I_4}]$, $\Delta = I_1 + I_2 + 2I_3$. In deriving the expression for $C(\varrho)$, we have used two facts: (i) The entropy of a Gaussian state depends only on the covariance matrix, and (ii) the covariance matrix $\sigma_P$ of the conditional state does not depend on the outcome of the measurement itself. This facts allows for a simplification of the minimization required to obtain the final general expression of the Gaussian discord. Indeed, for the relevant case of STS, and for any choice of $N_1$, $N_2$, and $N_r$, the minimum of the mismatch $I(\varrho) - J(\varrho)$ is obtained for $\alpha = \beta = 1/2$, $\gamma = 0$, i.e., when the covariance matrix of the measurement is the identity. This corresponds to the coherent state POVM, i.e., to the joint measurement of canonical operators, say, position and momentum, which may realized on the radiation field by means of heterodyne detection [15]. It turns out that the same result is obtained even if we generalize the class of Gaussian measurements to include noncovariant ones $\Pi_{\boldsymbol{Z}} = D(\boldsymbol{X})\varrho_M(\boldsymbol{Y})D^\dagger(\boldsymbol{X})$, where now the vector $\boldsymbol{Z} = (\boldsymbol{X}, \boldsymbol{Y})$ includes the no longer fixed parameeers of the covariance matrix $\sigma_M = \sigma_M(\mathbf{Y})$. Indeed, since the integrand in $\inf_{\{\Pi_Z\}} \int d\mathbf{Z} p(\mathbf{Z}) S(\varrho_{\mathbf{Z}})$ is always positive, we have $\inf_{\{\Pi_Z\}} \int d\mathbf{Z} p(\mathbf{Z}) S(\varrho_{\mathbf{Z}}) \geq \inf_{\{\Pi_Z\}} S(\varrho_{\mathbf{Z}}) = \inf_{\mathbf{Y}} h[\sqrt{\sigma_P(\mathbf{Y})}]$, and the above results apply for any $\mathbf{Z}$. Upon substituting $\sigma_M \to \mathbb{I}/2$, we can now explicitly write the Gaussian discord for the generic bipartite STS in terms of symplectic invariants as

$$C(\varrho) = h(\sqrt{I_2}) - h(d_-) - h(d_+) + h\left(\frac{\sqrt{I_1} + 2\sqrt{I_1 I_2} + 2I_3}{1 + 2\sqrt{I_2}}\right). \tag{2}$$

Upon exchanging $I_1 \leftrightarrow I_2$, one can pass from the $A$ discord to the $B$ discord.

We are now ready to start our discussion about the properties and the operational meaning of Gaussian quantum discord. At first, we notice that $C(\varrho) \neq 0$ as far as $N_r \neq 0$. Given that Gaussian states in standard form are separable for $N_r \leq N_1 N_2/(1 + N_1 + N_2)$, this confirms that for CV Gaussian states there are separable states with nonzero discord. Besides, since $N_r \neq 0 \Leftrightarrow c \neq 0$, we have that bipartite Gaussian states have always nonzero Gaussian discord, except when they are product states. The same condition characterizes the class of tomographically faithful states for reconstruction of quantum operations [16], and this provides an operational meaning for the quantum correlations in separable states with positive discord. The behavior of $C(\varrho)$ for small and large $N_r$ is given by $C(\varrho) \overset{N_r \ll 1}{\simeq} f_1(N_1, N_2) N_r$ and $C(\varrho) \overset{N_r \gg 1}{\simeq} f_2(N_1, N_2) + f_3(N_1, N_2) \log N_r$, respectively, where $f_1$ is a decreasing function of $N_2$ at any fixed value of $N_1$ and $f_2$ and $f_3$ are decreasing functions of both $N_1$ and $N_2$.

We now focus our attention on how $C(\varrho)$ relates with other meaningful properties of the states. In Fig. 1(a), we report $C(\varrho)$ at the separability threshold $N_r = N_1 N_2/(1 + N_1 + N_2)$, as a function of the ratio $N_1/N_T$ for increasing (from bottom to top) values of $N_T = a + b - 1$, which is the total energy of the Gaussian state under investigation ($N_T = N_1 + N_2 + 2N_1 N_2$ at separability threshold). The plot suggests two important facts. First, Gaussian discord is an increasing function of the total energy and is maximized when most of the thermal photons are placed on the unmeasured system, thus maximizing the purity of the measured one. Second, the Gaussian discord for separable states is always smaller than 1. The existence of a bound has been confirmed numerically by the random generation of a large number of bipartite Gaussian states in the standard form: In Fig. 1(b), we report the smaller symplectic eigenvalue $\tilde{d}_-$ of the partially transposed state, obtained by replacing $I_3 \to -I_3$ in the formula for $d_-$, as a function of Gaussian discord. Since a Gaussian state is entangled iff $\tilde{d}_- < \frac{1}{2}$, we have that for $0 \leq C(\varrho) \leq 1$ we have either separable or entangled states, whereas all the states with $C(\varrho) > 1$ are entangled.

The relation between the discord and the entanglement can be further clarified by analyzing the case of symmetric STS, i.e., $N_1 = N_2 = N_s$. Here we focus on the behavior of $C(\varrho)$ with respect to global purity of the state $\mu = (1 + 2N_s)^{-2}$ and $\tilde{d}_- = e^{-2s}(1 + 2N_s)/2$. A first important observation is that for *fixed purity* $C(\varrho)$ turns out to be a growing function of the entanglement, whereas at fixed values of $\tilde{d}_-$ the behavior is more involved. In Fig. 2, we plot $C(\varrho)(\mu, \tilde{d}_-)$ at fixed values of $\tilde{d}_-$. We can distinguish two different cases. For nonentangled states ($\tilde{d}_- \geq 1/2$), $C(\varrho)$ decreases with $\mu$, and it thus is an increasing function of the total energy of the state $N_T = \tilde{d}_- - 1 + (4\tilde{d}_- \mu)^{-1}$. The limiting value is thus reached at infinite energy, and the latter is in general given by $C(\varrho)(\mu \to 0, \tilde{d}_-) = (1 + 2\tilde{d}_-) \ln[(1 + 2\tilde{d}_-)/\tilde{d}_-)] - [1 + (1 + 2\tilde{d}_-) \ln 2]$. Therefore, for nonentangled symmetric states $C(\varrho) \leq 2\log 2 - 1$; the latter bound is also reported in Fig. 1 and defines the limit of the red region corresponding to symmetric separable states with nonzero discord. As for the entangled states ($\tilde{d}_- \leq 1/2$), the behavior of $C(\varrho)$ with $\mu$ is more complex. For states which are highly entangled ($\tilde{d}_- \leq 0.062\,84$) the discord decreases (grows) monotonically with $\mu(N_T)$. Indeed, $J(\varrho) \approx h(\sqrt{I_1})$ when $\tilde{d}_- \to 0$; i.e., the extractable information is maximized, and $C(\varrho)$ is maximum for pure states. For intermediate values of the

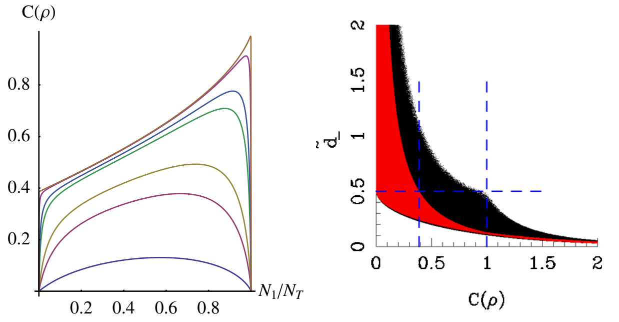


FIG. 1 (color online). Left: Gaussian discord $C(\varrho)$ for STS at separability threshold as a function of the ratio $N_1/N_T$ for increasing values of the energy $N_T$ (bottom to top $N_T = 1$, 5, 10, 50, $10^2$, $10^3$, and $10^5$). Right: Symplectic eigenvalues of the partial transpose $\tilde{d}_-$ versus $C(\varrho)$ for randomly generated STS. The red region corresponds to symmetric states. The separability threshold is $\tilde{d}_- = \frac{1}{2}$ and it corresponds to $C(\varrho) = 1$ [$C(\varrho) = 2\log 2 - 1$ symmetric STS].

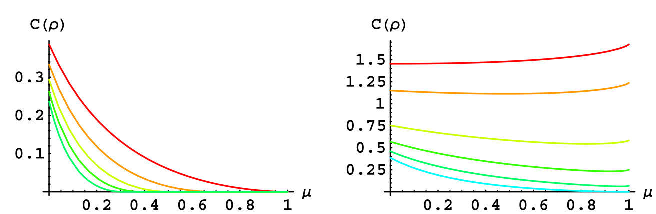


FIG. 2 (color online). Gaussian discord $C(\varrho)$ for symmetric STS as a function of the global purity $\mu$ of the state $\rho$ and $\tilde{d}_-$. Left: Separable states; from bottom to top $\tilde{d}_- = 0.5$,0.6, 0.7, 0.8, and 0.9. Right: Entangled states; from bottom to top $\tilde{d}_- = 0.5$, 0.4, 0.3, 0.3, 0.1, and 0.062 84. See text for description.

entanglement, $\tilde{d}_- \in (0.06284, 0.5)$, $C(\varrho)$ has a nonmonotonic behavior with $\mu(N_T)$. In particular, the maximum discord is reached for $\mu = 1$ (pure states) only for $\tilde{d}_- \geq 0.1282$, while its minimum is reached for intermediate values of $\mu$ that depend the actual value of $\tilde{d}_-$. The overall nonmonotonic behavior of $C(\varrho)$ corresponds to a situation in which, at a fixed value of entanglement, the quantumness of the state as measured by the Gaussian discord varies depending on the total correlations present in the state, and consequently the ordering of the states with respect to their quantumness significantly differs by that given by the entanglement. We also emphasize that by fixing the value of $\tilde{d}_-$ one also fixes the value of the teleportation fidelity $F = (1 + 2\tilde{d}_-)^{-1}$ of coherent states [17]. This means that, by varying the global purity of the state $\rho$ shared by Alice and Bob, the same fidelity can be achieved with different quantum resources as measured by the Gaussian discord.

We finally address the fundamental issue of the evolution of quantum discord in noisy channels. Let us consider bipartite Gaussian states that evolve according to Lindblad master equation $\dot{\varrho} = \frac{1}{2}\sum_j \Gamma_j M_j L[a]\varrho + \Gamma_j(1 + M_j)L[a^\dagger]\varrho$, which describes the Markovian interaction of the two modes with independent thermal reservoirs, $\Gamma_j$ and $M_j$ being the damping factor and the average number of thermal photons of the two reservoirs, respectively. The mapping induced by the master equation is Gaussian, and the covariance matrix of the evolved state is $\boldsymbol{\sigma}_t = \boldsymbol{\Gamma}_t^{1/2}\boldsymbol{\sigma}\boldsymbol{\Gamma}_t^{1/2} + (1 - \boldsymbol{\Gamma}_t)\boldsymbol{\sigma}_\infty$, where $\boldsymbol{\Gamma}_t = \bigoplus_j e^{-\Gamma_j t}\mathbb{1}_2$ and $\boldsymbol{\sigma}_\infty = \text{Diag}(M_1 + \frac{1}{2}, M_1 + \frac{1}{2}, M_2 + \frac{1}{2}, M_2 + \frac{1}{2})$ is the covariance matrix of the reservoir, which also describes the stationary state of the system. If $\boldsymbol{\sigma}_{t=0}$ is in standard form, its parameters evolve as $a' = ae^{-\Gamma_1 t} + (1 - e^{-\Gamma_1 t})(M_1 + \frac{1}{2})$, $b' = be^{-\Gamma_2 t} + (1 - e^{-\Gamma_2 t})(M_2 + \frac{1}{2})$, and $c' = ce^{-(1/2)(\Gamma_1 + \Gamma_2)t}$, i.e., $a' > a$, $b' > b$, and $c' < c$. Since $C(\varrho)$ is a decreasing function of $a$ and $b$ and an increasing function of $c$, we have that Gaussian discord monotonically decreases in noisy channels. On the other hand, it has been shown that the decrease should be smooth since an arbitrary Markovian evolution can never lead to a sudden disappearance of discord [4]. An open question remains the effect of non-Markovian dynamics, which has been proved to produce oscillations in the dynamics of Gaussian entanglement [18,19]. We also expect Gaussian discord to increase if the two parties interact with a common reservoir [20].

In conclusion, in this Letter we have extended the notion of the discord [2] to continuous variable systems and discuss its properties. We have defined the Gaussian discord $C(\varrho)$ for two-mode Gaussian states, and we have shown the general analytical procedure to derive it. In particular, for the relevant subclass of STS, we have shown that the extractable information is maximized by a generalize measurement, i.e., the coherent state POVM corresponding to heterodyne detection. Just as the entanglement, $C(\varrho)$ is invariant under local unitary operations, and it is zero only for (thermal) product states. For separable states $C(\varrho)$ grows with the total energy and it is bounded. Numerical evidences show that in general $C(\varrho) < 1$, while analytical calculations show that for separable symmetric STS the bound reduces to $C(\varrho) < 2\ln 2 - 1$. For symmetric STS we have also shown that the behavior of $C(\varrho)$ strongly depends on the amount of entanglement present in the state: It increases with the total purity only when the entanglement is large, whereas it shows a richer behavior for smaller values of entanglement. Our results pave the way for the general discussion about the quantum discord in continuous variable systems and for its experimental determination with current technology.

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