

Photoinduced Transition between Conventional and Topological Insulators in Two-Dimensional Electronic Systems

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(Received 1 October 2009; revised manuscript received 10 May 2010; published 28 June 2010)

Manipulating the topological properties of insulators, encoded in invariants such as the Chern number and its generalizations, is now a major issue for realizing novel charge or spin responses in electron systems. We propose that a simple optical means, subjecting to a driving laser field with circular polarization, can be fruitfully incorporated to this end. Taking as a prototypical example the two-band insulator first considered by Haldane, we show how the electron system can be tuned through phases associated with different Chern numbers as the laser intensity is adiabatically swept, i.e., a photoinduced analog of the quantum Hall plateau transition. The implications of our findings include the possibility of laser tuning a conventional insulator into a quantum spin Hall system.

DOI: 10.1103/PhysRevLett.105.017401

PACS numbers: 78.67.-n, 73.43.Nq

The past few decades have seen the topology-related quantum aspects of electron systems rise to the forefront of condensed matter physics [1]. The unifying concept behind these developments is the Berry phase, i.e., an extra phase which the wave function picks up as the system undergoes an adiabatic excursion. In a semiclassical description of Bloch electrons, the relevant excursion takes place in momentum space, motivating the introduction of the Berry connection $\mathcal{A}(\mathbf{k}) = i\langle v(\mathbf{k}) | \nabla_{\mathbf{k}} | v(\mathbf{k}) \rangle$, where $v(\mathbf{k})$ is the periodic part of the Bloch wave function; a contour integral of $\mathcal{A}(\mathbf{k})$ gives the Berry phase associated with that path. Also playing a crucial role is the Berry curvature $\Omega(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathcal{A}(\mathbf{k})$, which adds an anomalous-velocity term to the standard equation of motion of an electron [2]. The incorporation of these new entities has deeply enriched modern solid state physics with, e.g., a fully quantum mechanical theory of electron polarization [3,4], the notion of adiabatic quantum pumping [5], an intrinsic mechanism of the anomalous Hall effect [6], and schemes of generating a dissipationless spin current [7].

Largely triggered by the discovery of the quantum spin Hall effect (QSHE), these activities have in more recent years culminated in the conception, classification, and an intensive search for new states of matter collectively referred to as topological insulators (TIs). In its broadest sense, TIs are bulk insulators which nevertheless exhibit novel linear responses to external fields. These responses are dominated by transport through quantized edge or surface channels, whose robustness is “protected” by a set of topological invariants characteristic to that system. A well known example of the latter is the Chern number of a 2D system, $C \equiv \frac{1}{2\pi} \int_{\text{BZ}} \frac{d^2k}{(2\pi)^2} \Omega(\mathbf{k})$, where the integration is to be performed with respect to the first Brillouin zone. Introduced into electron physics by Thouless *et al.* in their pioneering work on the integer quantum Hall effect (IQHE), this integer-valued quantity basically measures the effective “magnetic flux” intrinsic to the Bloch elec-

trons, thus giving rise to a quantized Hall conductivity $\sigma_{xy} = \sum_n C_n \frac{e^2}{h}$, with the summation taken over all occupied bands [8]. Hence a Chern insulator, i.e., a 2D TI with $C_{\text{eff}} \equiv \sum_n C_n \neq 0$, realized in the absence of a magnetic field exhibits a spontaneous quantized Hall effect. States corresponding to different values of C_{eff} are to be regarded as belonging to distinct phases, as is the case with each σ_{xy} -plateau phase of the IQHE. Haldane was the first to devise a minimal two-band model without a magnetic field which accommodates Chern insulator phases [9]. Despite the artificial nature of its construction, this model bears immediate relevance to numerous physical systems (see later discussions). Furthermore, the essence of QSHE is understood in terms of a superposition of two copies of Haldane models (one for each spin component) with a net time-reversal symmetry [10,11]; recent work clarifies how this picture relates to a more generic characterization of the QSHE employing \mathbb{Z}_2 -valued invariants.

From the perspective of material function engineering, a major shortcoming which plagues the search for TIs is that their defining topological invariants, such as C_{eff} , are usually material constants whose values cannot be varied freely within a given sample. This is to be contrasted with the situation in IQHE [8], where an adiabatic sweeping of the magnetic field strength drives the system from a normal ($C_{\text{eff}} = 0$) to a topological ($C_{\text{eff}} \neq 0$) insulator, and vice versa. In this Letter, we pose and answer in the affirmative the question: Is there a generic approach allowing us to externally tune the topological invariant of a TI? Taking up the most basic example of the Haldane model, we show below that an optical means—the application of a monochromatic driving laser field with circular polarization—can be invoked to obtain a photoinduced normal-Chern insulator transition; i.e., by slowly changing the driving laser amplitude (or the laser frequency), one can transport the electron system into different regimes of the C_{eff} vs model parameter phase diagram. This scenario

naturally lends itself to more generic topological phase transitions as we later mention and thereby suggests a route to manipulating the topological properties of various insulators [12].

Let us then illustrate how the claimed result comes about. We begin by noting that our purpose requires us to carefully trace the low-energy sector of the electron system through its evolution, as the laser field intensity is adiabatically swept. This we accomplish by adopting a systematic method previously developed by one of the authors [13]. Aided by the powerful machinery of the Floquet theory, the procedure sketched below is applicable to the adiabatic dynamics of any tight-binding model and was previously used to give a reliable prediction on how the electric polarization of a (normal) insulator renormalizes substantially when subjected to a slowly varying laser field [13]. While the polarization is also a prominent Berry phase effect, it is worth stressing that the adiabatic flow of a *global* topological quantity such as C_{eff} is considerably harder to foresee. Consider then the Hamiltonian for the Haldane model H_H coupled to a circularly polarized driving laser field $H(t) = H_H + H'(t)$, where

$$\begin{aligned}
 H_H = & t_1 \sum_i \sum_{\vec{r} \in A} [a^\dagger(\vec{r})b(\vec{r} + \vec{d}_i) + \text{H.c.}] \\
 & + t_2 \sum_j \left[\sum_{\vec{r} \in A} e^{i\phi} a^\dagger(\vec{r})a(\vec{r} + \vec{\ell}_j) \right. \\
 & \left. + \sum_{\vec{r} \in B} e^{-i\phi} b^\dagger(\vec{r})b(\vec{r} + \vec{\ell}_j) + \text{H.c.} \right] \\
 & + \Delta \sum_{\vec{r} \in A} a^\dagger(\vec{r})a(\vec{r}) - \Delta \sum_{\vec{r} \in B} b^\dagger(\vec{r})b(\vec{r}), \quad (1)
 \end{aligned}$$

$$H'(t) = e\vec{E}(t) \cdot \left[\sum_{\vec{r} \in A} \vec{r} a^\dagger(\vec{r})a(\vec{r}) + \sum_{\vec{r} \in B} \vec{r} b^\dagger(\vec{r})b(\vec{r}) \right]. \quad (2)$$

The electrons reside on a 2D honeycomb lattice comprising two triangular sublattices, which we denote as A and B ; we associate with each the electron creation operators a^\dagger and b^\dagger . The vector \vec{d}_i ($\vec{\ell}_j$) connects nearest (next-nearest) neighbors [see Fig. 1]. The system is at half-filling and is thus a band insulator. The staggered potential Δ and the phase ϕ are each responsible for breaking inversion and

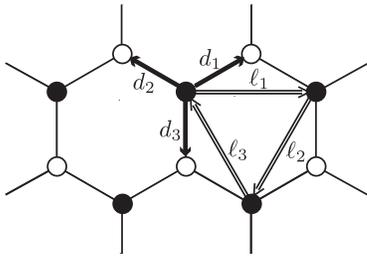


FIG. 1. The lattice structure considered in this study. Solid (open) dots denote sites belonging to the A (B) sublattice. Electrons residing on the A (B) sublattice suffer an on-site electron potential of $+\Delta$ ($-\Delta$).

time-reversal symmetries, which causes the lower-band Chern number to assume the set of possible values $C = 0, \pm 1$ [9]. Turning to the interaction term H' , we choose the wave vector of the laser field to be normal to the lattice plane. The driving laser field with a fixed frequency ω can be expressed as $\vec{E}(t) = E_T(\cos\omega t, \tau \sin\omega t)$, where $\tau = +1$ (-1) corresponds to the left (right) circular polarization. Now let the time dependence of the amplitude E_T be weak enough to justify the use of the adiabatic approximation, which is always possible by preparing a suitable laser source. Then the sinusoidally time-dependent problem maps onto a time-independent eigenvalue problem $\tilde{H}_F|\Phi\rangle = \tilde{\epsilon}|\Phi\rangle$ by virtue of the Floquet theory [14,15]; the eigenvalue $\tilde{\epsilon}$ is the quasienergy in the language of the framework. A straightforward calculation shows that the Floquet Hamiltonian \tilde{H}_F consists of 2×2 block submatrices, i.e., $\tilde{H}_F = (\tilde{H}^{(mn)})$, where the entries for each block (in the Fourier-transformed bases, a_k and b_k) are

$$\begin{aligned}
 \tilde{H}_{11}^{(mn)} = & (\Delta + m\hbar\omega)\delta_{mn} + t_2 J_N(\sqrt{3}\lambda) \\
 & \times \sum_{j=1}^3 e^{i\tau[2N(j-1)\pi]/3} [(-1)^N e^{i(\vec{k}\cdot\vec{\ell}_j + \phi)} \\
 & + e^{-i(\vec{k}\cdot\vec{\ell}_j + \phi)}], \quad (3)
 \end{aligned}$$

$$\tilde{H}_{12}^{(mn)} = t_1 J_N(-\lambda) \sum_{j=1}^3 e^{-i\tau[N(4j-3)\pi]/6} e^{i\vec{k}\cdot\vec{d}_j}, \quad (4)$$

with J_N the Bessel function of the $N \equiv (n - m)$ th order and $\lambda \equiv eaE_T/\hbar\omega$ (a : lattice constant). The remaining matrix elements, $\tilde{H}_{22}^{(mn)}$ and $\tilde{H}_{21}^{(mn)}$, are identical in form to Eqs. (3) and (4), respectively, except for the insertion of -1 in front of Δ and ϕ in the former and λ and \vec{k} in the latter. This Hamiltonian operates on wave functions defined in the space compositely spanned by the electronic degrees of freedom and photons with energy $\hbar\omega$. The diagonal block of the Hamiltonian, $\tilde{H}^{(nn)}$, is the n -photon sector, i.e., the subspace with n photons.

Now suppose that the condition $W \ll \hbar\omega \ll E_G$ is met, where W is the electronic band width and $E_G = 2\Delta$ the energy gap. In this case the admixture of neighboring photon sectors is negligible, which enables us to concentrate on the zero-photon sector [16],

$$\tilde{H}^{(00)} = \sum_k (a_k^\dagger, b_k^\dagger) \tilde{\mathcal{H}} \begin{pmatrix} a_k \\ b_k \end{pmatrix},$$

$$\begin{aligned}
 \tilde{\mathcal{H}} = & 2t_2 J_0(\sqrt{3}\lambda) \cos\phi \sum_j \cos(\vec{k}\cdot\vec{\ell}_j) \mathbf{1} \\
 & + t_1 J_0(\lambda) \sum_i \{\cos(\vec{k}\cdot\vec{d}_i) \sigma_x - \sin(\vec{k}\cdot\vec{d}_i) \sigma_y\} \\
 & + \left[\Delta - 2t_2 J_0(\sqrt{3}\lambda) \sin\phi \sum_j \sin(\vec{k}\cdot\vec{\ell}_j) \right] \sigma_z, \quad (5)
 \end{aligned}$$

where $\mathbf{1}$ is a 2×2 unit matrix and $\sigma_{x,y,z}$ are the Pauli matrices. It is straightforward to see that $\tilde{\mathcal{H}}$ is identical in

form to the Haldane model, save for the fact that the hopping integrals t_1 and t_2 are modulated by factors involving Bessel functions. Hence, expanding $\tilde{H}^{(00)}$ around the two Dirac points $K_{\alpha=\pm} = (\alpha 4\pi/3\sqrt{3}a, 0)$, we finally arrive at our effective Hamiltonian

$$\tilde{\mathcal{H}}_{\alpha} = -3t_2 J_0(\sqrt{3}\lambda) \cos\phi \mathbf{1} + \frac{3a}{2} t_1 J_0(\lambda) [-\alpha k_x \sigma_x + k_y \sigma_y] + [\Delta - 3\sqrt{3}\alpha t_2 J_0(\sqrt{3}\lambda) \sin\phi] \sigma_z, \quad (6)$$

where wave numbers $k_{x,y}$ are now measured relative to the K_{\pm} points. Note that the zero-photon sector is insensitive to the direction of the circular polarization. It is an easy task to read off from the results of Ref. [9] that the Chern number for our effective system described by Eq. (6) is given by

$$\mathcal{C} = \frac{1}{2} \sum_{\alpha=\pm} \alpha \operatorname{sgn}[\Delta + \alpha 3\sqrt{3}t_2 J_0(\sqrt{3}\lambda) \sin\phi]. \quad (7)$$

This expression for the photoinduced modification of the Chern number represents a principal result of this Letter: In addition to the two variables ϕ and Δ/t_2 governing the topological property of the original Haldane model [9], we now have a tunable third variable λ at our disposal which comes from the coupling to the laser field. The extended Chern number phase diagram within our three-dimensional parameter space is depicted in Fig. 2. Points belonging to the interior of the ravioli sheetlike structure fall within one or another of the Chern insulator phases, with either $\mathcal{C} = +1$ or $\mathcal{C} = -1$. Elsewhere, the system is a normal insulator with $\mathcal{C} = 0$. The array of $\mathcal{C} = +1$ and $\mathcal{C} = -1$ phases constitute a checkerboardlike pattern. By simply sweeping the laser amplitude, one can traverse the phase diagram along the λ direction and cross phase boundaries, thus inducing normal-Chern insulator transitions. We have confirmed that the gap closes and reopens in the vicinity of the phase transition point, as it must, since \mathcal{C} is robust to a continuous deformation of the energy bands [12]. We note in passing that the circular polarization was essential in the above; a replacement with a linearly polarized field would fail to yield a massive Dirac-fermion-like energy spectrum as in Eq. (6) [17].

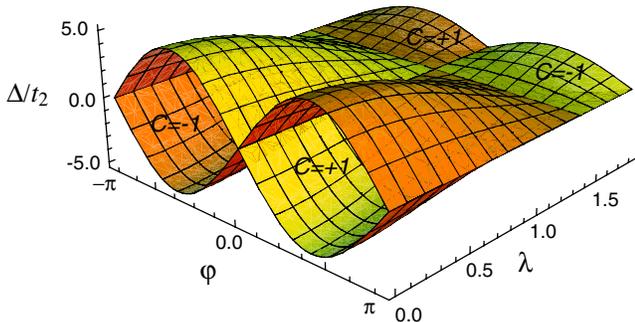


FIG. 2 (color online). The three-dimensional phase diagram of the normal and Chern insulators. (See text for details.)

We now run a consistency check among several conditions which were implicit above; they will place some restrictions on the parameter values for which the transition is expected to be observable. We first note that $|\Delta/t_2| < 3\sqrt{3}$, which derives from the observation that $\lambda = 0$ occupies the largest portion of the phase diagram with $\mathcal{C} \neq 0$. This needs to be consistent with the aforementioned condition for the hopping energy, $W \sim t_1 \ll \hbar\omega \ll E_G$. That, however, is not straightforward, because on top of these $|t_2 J_0(\sqrt{3}\lambda)/t_1 J_0(\lambda)| < 1/3$ should also be satisfied in order to prevent the bands from overlapping [9]. To confront this problem, we will relax the condition $W \ll \hbar\omega \ll E_G$ to $W < \hbar\omega < E_G$, which in turn will generally impose a restriction on the range of values, $[0, \Lambda]$, which the parameter λ can assume without breaking down our scheme. (Note that when $W \ll \hbar\omega \ll E_G$ is satisfied, there are no such restrictions on λ .) We have evaluated this range by numerically checking the agreement between the quasi-energy bands of the Floquet Hamiltonian terminated at a certain order and the eigenenergy bands of $\tilde{H}^{(00)}$. When $t_2/t_1 = 1/3$ and $\Delta/t_1 = 1.3$, we find that our scheme is valid for $\Lambda \approx 0.3$. We therefore confine our discussion to this regime in subsequent discussions. The region where the normal-Chern insulator transition occurs is displayed in Fig. 3(a); in the filled area, one can traverse between the interior and the exterior of the sheet structure by changing λ within the interval $0 < \lambda \leq \Lambda$.

The normal-to-Chern insulator transition that we found can be viewed as a photoinduced analog of the interplateau transition in IQHE, wherein the laser field intensity replaces the role of the external magnetic field. (Recall that each IQHE Hall plateau has a different value of \mathcal{C}_{eff} .) To strengthen this analogy, it is instructive to consider sweeping the system between the two points A and B in Fig. 3(b), which are the sections of Fig. 2 at $\phi = \pm\pi/2$, and explicitly display how the Chern number change actually

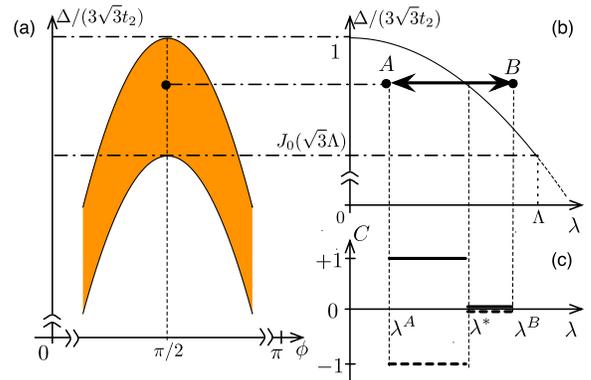


FIG. 3 (color online). (a) Partial section of the phase diagram for $\lambda = 0$. In the filled area, one can traverse between the interior and exterior of the double-sheet structure by changing λ within the interval $0 < \lambda \leq \Lambda$. (b) Section of the phase diagram for $\phi = \pm\pi/2$. (c) Corresponding Chern number as a function of $\lambda \in [\lambda^A, \lambda^B]$ for $\phi = \pi/2$ (thick solid line) and $\phi = -\pi/2$ (thick dashed line).

occurs. The path connecting the two points crosses the phase boundary at a critical point $\lambda = \lambda^*$, where an “interplateau transition” takes place as depicted in Fig. 3(a) in thick solid (dashed) lines. The field strength for the transition around the value of $\lambda^* \sim \mathcal{O}(0.1)$ is estimated to be of the order of $\sim \mathcal{O}(10^6)$ [V/cm]. This is to be compared, e.g., with the corresponding estimate of $\sim \mathcal{O}(10^7)$ [V/cm] [18] made in relation to dynamic localization, a prototypical example of optical manipulation of electron dynamics. The above parallelism loosely conforms with earlier work (*not* straightforwardly applicable to our adiabatic process) suggesting that a circularly polarized light should affect electron motion in a manner analogous to a magnetic field [19].

Let us now size up what the implications of our strategy are for various physically motivated systems. (i) It has been shown [20] that the distribution of the Aharonov-Bohm-like flux inherent in the Haldane model (described by the phase ϕ) can be mimicked by tight-binding electrons on a kagome lattice moving in the background of a spin chirality ordering. This idea has led to a detailed analysis on the anomalous Hall effect in several pyrochlore ferromagnets, such as $\text{Nd}_2\text{Mn}_2\text{O}_7$ [21]. Our method readily carries over to this system; indeed, in the notation of Ref. [20] we find that applying a circular polarized light to this particular model yields $\mathcal{C} = -\text{sgn}[J_0(\lambda) \sin\phi]$ as the counterpart to Eq. (7), from which we expect to encounter the photo-induced transition $\mathcal{C} = -1 \rightarrow \mathcal{C} = +1$ as a function of laser intensity. (ii) It has also been proposed that the Haldane model can be simulated by a fermionic cold atom on an optical lattice [22]. A simple estimation reveals that the energy scale of the laser fields which constitute the lattice (~ 1 [eV]) and that of the additional sweeping field ($\sim 10^{-10}$ [eV] or larger) are easily separated, which makes the implementation of the adiabatic scheme feasible. (iii) The canonical Kane-Mele model [10] for the QSHE, in the absence of a Rashba term, reduces to two copies (up-spin and down-spin) of Haldane models. (The Luttinger model for light or heavy hole bands in a semiconductor also reduces to the same model, provided we replace spins by a conserved pseudospin quantum number [11,23].) The sum of the Chern numbers \mathcal{C}_\uparrow and \mathcal{C}_\downarrow for each spin component cancels in accord with time-reversal symmetry, while the difference $\mathcal{C}_\uparrow - \mathcal{C}_\downarrow$ needs not. The preceding generalizes trivially to this situation, which now leads to a photo-induced normal to quantum spin Hall insulator transition. When the Rashba coupling is present, the \mathbf{Z}_2 invariant which takes over the role of \mathcal{C} can be evaluated following Ref. [24], by using a matrix-valued generalization of the Berry connection. Here again we find that the system can be laser tuned through a normal to quantum spin Hall insulator transition. Details on work along this line will be reported elsewhere.

In summary, we have demonstrated how a photoinduced normal-Chern transition occurs in the Haldane model. The coupling to a driving laser field with circular polarization turns the transfer integrals into tunable variables, which can result in a change in the Chern number. The strategy is reasonably general and suggests an optics-based scheme for manipulating the intrinsic topology of insulators.

We thank M. Arai for helpful discussions. The authors were supported in part by Grant-in-Aid for Scientific Research (C) No. 20540386 and No. 22540340 from MEXT, Japan.

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