Photon-Number Selective Group Delay in Cavity Induced Transparency

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We show that the group velocity of a probe pulse in an ensemble of Λ-type atoms driven by a quantized cavity mode depends on the quantum state-of-the input probe pulse. In the strong-coupling regime of the atom-cavity system the probe group delay is photon-number selective. This can be used to spatially separate the single photon from higher photon-number components of a few-photon probe pulse and thus create a deterministic single-photon source.

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One of the major practical challenges in implementing photon-based quantum cryptography and network quantum computing is the controlled, deterministic generation of single-photon pulses. In the present Letter we propose a scheme where the group delay of a probe pulse in a medium with electromagnetically induced transparency (EIT) is made quantum state dependent. This can be used to build a Fock-state quantum filter and thus to create single-photon pulses on demand.

EIT is an interference effect where the optical properties of a probe field are modified by the presence of a strong, and thus usually classical, coupling field [1,2]. Because of destructive interference induced by the coupling field, an otherwise opaque resonant medium becomes transparent in a narrow spectral region. The transparency is accompanied by a substantial reduction of the group velocity which can be controlled by the intensity of the coupling field [3]. We here consider the case when the classical driving field is replaced by a quantized cavity mode. If the corresponding vacuum Rabi frequency is sufficiently large, i.e., if the cavity-atom system is in the strong-coupling regime, already an empty cavity will induce transparency for the probe field [4]. Furthermore a weak probe pulse will induce cavity enhanced Raman scattering into the resonator mode. Under appropriate conditions almost all excitations will be transferred to the cavity mode and thus its photon-number distribution will be a copy of that of the input probe field. As the probe-field group velocity in the EIT medium depends on the photon number of the drive field, different photon-number components of the probe pulse will experience different group delays. We will show that the differential group delay between the single- and higher-photon-number components can be made large enough to fully separate them spatially during propagation.

It is well known that strong coupling of a cavity mode to atomic dipoles can give rise to nonlinearities that are sufficiently large to induce interactions on the few-photon level and thus can be employed for photonic quantum gates and deterministic single-photon sources [5–8]. However, to achieve strong coupling in the optical domain remains a major technical challenge. The necessity to perform input-optput operations at the same mode for which a high quality factor is needed is a major obstacle. Furthermore, for photon transport optical frequencies are preferable while strong coupling is much easier to achieve for μ waves. The strong requirements of cavity QED can be relaxed when photons interact for a sufficiently long time in a nonlinear medium. It has been suggested that photons propagating in a coherently driven, optically thick medium under conditions of EIT can mutually induce nonlinear phase shifts due to the combination of a strongly reduced group velocity and resonantly enhanced nonlinearities [9–12]. To reach the single-photon level in these schemes it is, however, necessary to let the pulses copropagate for large distances and to confine the light beams transversally to a radius below the wavelength.

The physical mechanism of the present proposal is very different from both approaches. Here strong coupling is required for the coupling field rather than the probe field. As the latter does not need to be coupled in or out, the probe bandwidth is not limited by the high Q value of the cavity mode. Furthermore, the frequency of the coupling field does not need to be in the optical domain. By using molecules or Rydberg atoms an optical probe transition can be combined with a driving transition in the μ-wave regime for which very large cavity couplings have been achieved, e.g., using stripline resonators [13].

Let us consider a medium consisting of an ensemble of three-level atoms with a Λ configuration interacting with two quantum fields (Fig. 1). The probe $\hat{E}$ resonantly couples the transition $|g\rangle - |e\rangle$ with a coupling strength g and propagates along the z axis. The $|s\rangle - |e\rangle$ transition is driven by a cavity field $\hat{a}$. The corresponding vacuum Rabi frequency is denoted as G. The interaction time is assumed to be much shorter than the lifetime of a photon in the cavity mode and thus we may regard the cavity as lossless. All atoms of the medium are initially in the ground state $|g\rangle$. The combination of the probe field with

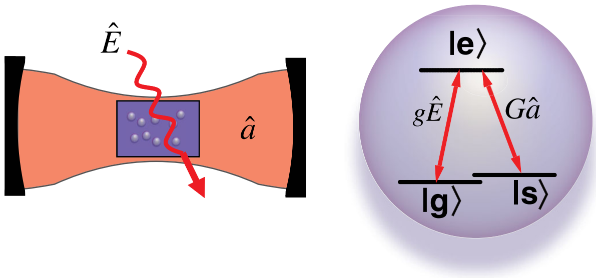

FIG. 1 (color online). Schematic diagram of the system: A quantized probe field $\hat{E}$ interacts with an ensemble of Λ-type atoms driven by a cavity mode $\hat{a}$ (left) in a Raman type coupling scheme (right).

the cavity vacuum field resonantly drives a Raman transition. When the probe field enters the medium it induces Raman scattering resulting in the creation of a collective "spin" excitation of the $|g\rangle - |s\rangle$ transition accompanied by a simultaneous emission of photons into the cavity, provided the cavity coupling is sufficiently large [4–7].

We assume that our medium is (in the absence of EIT) optically thick for the probe field and that all atoms are in the strong interaction regime with the cavity mode [8,14,15]. Although this assumption may seem at first glance rather strong, such systems are state-of-the art technology. Several groups have recently reported experimental results on Bose-Einstein condensates coupled to a cavity in the single-atom strong-coupling regime [16–19]. Furthermore the cavity mode does not have to be an optical field even if the probe field is. One can use polar molecules instead of atoms [20], which can be strongly coupled to stripline microwave resonators [21,22], which in principle enables very high optical depths simultaneously with a strong interaction regime.

We will now analyze the propagation of a probe pulse in such a system. To simplify the discussion we assume the coupling strength of the cavity mode to be the same for all atoms of the ensemble. In this case the Hamiltonian of our system reads in the rotating wave approximation

$$\hat{H} = -\hbar N \int_0^L \frac{dz}{L}[g\hat{E}(z,t)\hat{\sigma}_{\rm eg}(z,t) + \hat{a}(t)G\hat{\sigma}_{\rm es}(z,t) + \text{H.c.}], \qquad (1)$$

where, $\hat{\sigma}_{mn}(z,t) = \frac{1}{N_z}\sum_{j=1}^{N_z}|m\rangle_j\langle n|_j$ are the atomic operators averaged over a small volume around position z containing N_z atoms. $\hat{E}(z,t)$ is the slowly varying operator of the probe field, $\hat{a}(t)$ is the annihilation operator for the cavity mode, N is the total number of atoms in the system, and L is the length of the atomic cloud.

The propagation of the probe field is described by the Maxwell equation for the slowly varying amplitude

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) = igN\hat{\sigma}_{ge}(z,t), \qquad (2)$$

while the cavity mode obeys the Heisenberg equation

$$\frac{d\hat{a}(t)}{dt} = iN\int_0^L \frac{dz}{L} G\hat{\sigma}_{se}(z,t). \qquad (3)$$

In the strong-coupling regime the vacuum Rabi frequency G of the cavity field is large compared to the coupling strength g of the probe field, i.e., $G \gg g$, and already the cavity vacuum prepares a dark state, i.e., causes EIT [4]. If the spectrum of the input probe pulse is within the EIT linewidth, the system will remain in the dark state and thus the population of the excited state can be neglected. Then from Eq. (1) one can find for the dynamics of population of the metastable states

$$\frac{\partial}{\partial t}\hat{\sigma}_{gg}(z,t) = ig\hat{E}^\dagger(z,t)\hat{\sigma}_{ge}(z,t) - ig\hat{\sigma}_{eg}(z,t)\hat{E}(z,t),$$
$$\frac{\partial}{\partial t}\hat{\sigma}_{ss}(z,t) = iG\hat{a}^\dagger(t)\hat{\sigma}_{se}(z,t) - iG\hat{\sigma}_{es}(z,t)\hat{a}(t). \qquad (4)$$

By substituting (2) and (3) into (4) we find that the total number of photons in the system, i.e., in the cavity field $\hat{n}(t) = \hat{a}^\dagger(t)\hat{a}(t)$ and in the probe field $\hat{n}_p(t) = \frac{1}{L}\int_0^L \hat{E}^\dagger(z,t)\hat{E}(z,t)dz$ is fixed by the input and output flux of probe photons.

$$\frac{d}{dt}(\hat{n} + \hat{n}_p) = \frac{c}{L}[\hat{E}^\dagger(0)\hat{E}(0) - \hat{E}^\dagger(L)\hat{E}(L)]. \qquad (5)$$

The right-hand side of (5) is the probe-field photon flux difference at the input and output of the medium. The dynamical equations for the remaining atomic operators are

$$\frac{\partial\hat{\sigma}_{ge}}{\partial t} = -\Gamma\hat{\sigma}_{ge} + iG\hat{a}\hat{\sigma}_{gs} + ig\hat{E}(\hat{\sigma}_{gg} - \hat{\sigma}_{ee}) + \hat{F}_{ge}, \qquad (6)$$

$$\frac{\partial\hat{\sigma}_{se}}{\partial t} = -\Gamma\hat{\sigma}_{se} + ig\hat{E}\hat{\sigma}_{sg} + iG\hat{a}(\hat{\sigma}_{ss} - \hat{\sigma}_{ee}) + \hat{F}_{se}, \qquad (7)$$

$$\frac{\partial(\hat{a}\hat{\sigma}_{gs})}{\partial t} = iG^*\hat{a}\hat{\sigma}_{ge}\hat{a}^\dagger - ig\hat{a}\,\hat{E}\,\hat{\sigma}_{es} + \hat{F}_{gs} + \hat{a}\hat{\sigma}_{gs}\frac{\partial\hat{a}}{\partial t}\hat{a}^\dagger - \frac{\partial\hat{a}}{\partial t}\hat{a}^\dagger\hat{a}\hat{\sigma}_{gs}. \qquad (8)$$

Here Γ is the relaxation rate of the upper level and it is assumed that the decoherence of the lower level transition is negligible on the time scale of interest. We assume that the atomic ensemble is initially prepared in the collective ground state $|g\rangle$. Then by taking into account that the population of the excited and spin states cannot exceed the number of probe photons (n_p) in the system one can give bounds for the diagonal operators $\hat{\sigma}_{ee}$, $\hat{\sigma}_{ss} \le n_p/N$ and $\hat{\sigma}_{gg} \ge 1 - n_p/N$. The number of the photons shall be much smaller than the number of atoms, i.e., $\epsilon \equiv \sqrt{n_p/N} \ll 1$. Keeping only terms proportional to ϵ and neglecting terms $\mathcal{O}(\epsilon^2)$ in Eqs. (6)–(8) yields $\hat{\sigma}_{gg} \approx \hat{1}$, $\hat{\sigma}_{ss} = \hat{\sigma}_{ee} = \hat{\sigma}_{es} = 0$ as well as

$$\frac{\partial}{\partial t}(\hat{a}\hat{\sigma}_{gs}) = iG^*\hat{a}\hat{\sigma}_{ge}\hat{a}^\dagger + \hat{F}_{gs} + \hat{a}\hat{\sigma}_{gs}\frac{\partial \hat{a}}{\partial t}\hat{a}^\dagger - \frac{\partial \hat{a}}{\partial t}\hat{a}^\dagger\hat{a}\hat{\sigma}_{gs}. \quad (9)$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{ge} = -\Gamma\hat{\sigma}_{ge} + iG\hat{a}\hat{\sigma}_{gs} + ig\hat{E} + \hat{F}_{ge}.$$

Note that neglecting terms $\mathcal{O}(\epsilon^2)$ is always justified in the limit of a large atomic ensemble and does not mean neglecting nonlinear interactions. The latter result here from the coupling to the cavity mode, which, as will be shown in the following, has a photon-number distribution that is an exact copy of that of the input probe field in the limit of large atom number. The characteristic length of the probe pulse T is typically large compared to the upper level relaxation time ($\Gamma T \gg 1$) and thus the time derivative in Eq. (9) can be neglected. If the spectrum of the probe pulse lies within the EIT transparency window $\Delta\omega_{\rm EIT}$, i.e., if furthermore

$$T \gg \frac{1}{\Delta\omega_{\rm EIT}} = \frac{\Gamma}{G^2\sqrt{\rm OD}}, \qquad {\rm OD} \equiv \frac{L}{l_{\rm abs}}, \quad (10)$$

where $l_{\rm abs} = c\Gamma/g^2N$ is the resonant absorption length of the medium in the absence of EIT, and OD the optical depth, $\hat{\sigma}_{ge}$ can be adiabatically eliminated, and the Langevin noise operators can be disregarded [23,24]. In this adiabatic limit the atomic dynamics is governed by the equations

$$G\hat{a}\hat{\sigma}_{gs} + g\hat{E} = 0, \quad (11)$$

$$\frac{\partial}{\partial t}(\hat{a}\hat{\sigma}_{gs}) = iG^*\hat{a}\hat{\sigma}_{ge}\hat{a}^\dagger + \hat{a}\hat{\sigma}_{gs}\frac{\partial \hat{a}}{\partial t}\hat{a}^\dagger - \frac{\partial \hat{a}}{\partial t}\hat{a}^\dagger\hat{a}\hat{\sigma}_{gs}. \quad (12)$$

Combining Eqs. (11), (12), and (2), one finally arrives at the following propagation equation of the probe field:

$$\frac{\partial \hat{E}}{\partial t} + \frac{G^2}{g^2N}\hat{a}\left(\frac{\partial \hat{E}}{\partial t} + c\frac{\partial \hat{E}}{\partial z}\right)\hat{a}^\dagger - \hat{E}\frac{\partial \hat{a}}{\partial t}\hat{a}^\dagger + \frac{\partial \hat{a}}{\partial t}\hat{a}^\dagger\hat{E} = 0 \quad (13)$$

The first terms in Eq. (13) describe a probe-field propagation with a cavity dependent group velocity. The last two terms describe dependence of the probe amplitude on the changes of the cavity field, i.e., during the periods of entering and leaving the medium. Since we are not interested in these transients and in order to simplify the discussion we will disregard these terms in the following. Taking into account that cavity and probe operator commute we arrive at an operator-valued group velocity

$$\hat{v}_{\rm gr} = c\frac{G^2(\hat{n}+1)}{G^2(\hat{n}+1) + g^2N}. \quad (14)$$

$\hat{v}_{\rm gr}$ depends on the number of photons $\hat{n} = \hat{a}^\dagger\hat{a}$ in the cavity. On the other hand $\hat{n}$ is determined by the number of probe photons. The relation between these quantities can be derived from (11). This yields

$$G^2\hat{n}(t) = g^2N\int_0^L \frac{dz}{L}\hat{E}^\dagger(t,z)\hat{E}(t,z), \quad (15)$$

which is nothing else than the condition for the system to remain in the dark state. If the number of atoms is large, such that

$$Ng^2 \gg G^2, \quad (16)$$

the number of probe photons in the medium is at all times negligible as compared to the number of photons in the cavity mode. Thus

$$\hat{n}(t) - \hat{n}(0) \approx \frac{c}{L}\int_0^t d\tau[\hat{E}^\dagger(0)\hat{E}(0) - \hat{E}^\dagger(L)\hat{E}(L)]. \quad (17)$$

In the case of an initially empty cavity the propagation of the probe in the medium is entirely determined by the number of probe photons that have entered the medium. This is the main idea of the present Letter.

In the case of a classical driving field and thus a c-number group velocity the spatial length of the probe pulse inside the medium is determined by the product of the group velocity and the pulse duration T: $L_{\rm probe} = v_{\rm gr}T$. In the present case the group velocity is different for the different photon-number components of the pulse. It increases for increasing photon number. For the sake of simplicity we assume that the spatial length of the highest relevant photon-number component of the probe is smaller than the medium length ($L_{\rm probe} \ll L$) and the cavity is initially empty. In this case the whole probe pulse can be loaded into the medium. Once the probe pulse has fully entered, the number of photons in the cavity field becomes equal to the overall number of probe photons at input $\hat{n} = \hat{n}_{p,\rm in} = \frac{c}{L}\int_0^t \hat{E}^\dagger(0,\tau)\hat{E}(0,\tau)d\tau$. Now by making use of inequality (16) one finds the following solution of the propagation equation (13):

$$\hat{E}(z,t) = \hat{E}(0, t - z/\hat{v}_{\rm gr}), \qquad \hat{v}_{\rm gr} = cG^2(\hat{n}_{p,\rm in}+1)/(g^2N). \quad (18)$$

One sees that $\hat{v}_{\rm gr}$ depends on the initial number of photons in the pulse, thus different Fock components of the probe will propagate with different group velocities.

To be specific let us assume that the probe field is initially in a single-mode superposition of Fock states; i.e., the initial state of the probe can be expressed as $|\psi(t)\rangle = \sum_{n=1}^\infty \alpha_n f(t)|n\rangle$, where the common function $f(t)$ describes the shape of the probe field before entering the medium. Note that this function is the same for all Fock components corresponding to a single (pulsed) mode. After propagating through the medium the state is $|\psi(L,t)\rangle = \sum_{n=1}^\infty \alpha_n f(t - \frac{L}{l_{\rm abs}}\frac{\Gamma}{(n+1)G^2})|n\rangle$. Thus different components of the probe will be spatially separated (Fig. 2). This separation is larger for Fock components with a smaller number of photons. Specifically the delay between components

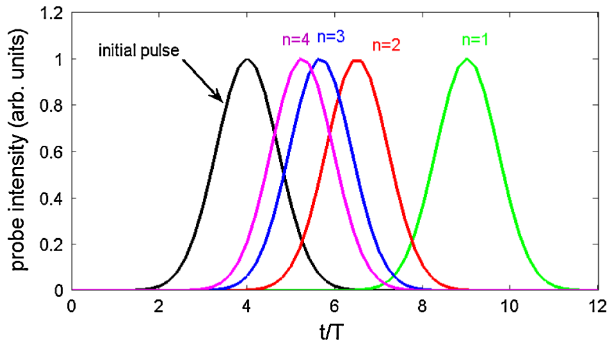

FIG. 2 (color online). Spatial separation of an initial probe pulse into Fock-state components.

with m and $m+1$ photons after propagating over distance L is given by

$$\Delta\tau_m = \frac{L}{l_{\mathrm{abs}}} \frac{1}{(m+1)(m+2)} \frac{\Gamma}{G^2}. \tag{19}$$

Important practical limitations of the present scheme result from dissipation in the form of cavity damping and spontaneous emission. Cavity damping comes into play as soon as the cavity mode is excited, and causes a violation of (5). It can be neglected if

$$n_{p,\mathrm{in}} \kappa T \ll 1, \tag{20}$$

where, κ is the cavity decay rate. Spontaneous decay of the upper state can be disregarded if the interaction is completely adiabatic (10). Hence the technique works optimally if both conditions, (10) and (20), are satisfied which can be realized only in case of strong-coupling $G^2 \gg \kappa\Gamma$. However, we emphasize that since $G \gg g$ no strong coupling is required on the probe transition. In order to separate the single-photon component from components with a larger number of excitations the delay time $\Delta\tau_1$ has to be of the order of T. Thus by combining (10) and (20) we find that in order to effectively separate the single-photon component the following condition has to be satisfied

$$1 \lesssim \frac{\Delta\tau_1}{T} = \frac{L}{l_{\mathrm{abs}}} \frac{\Gamma}{6G^2 T} \ll \sqrt{L/l_{\mathrm{abs}}},$$

and thus a medium with large optical depth L/l_{abs} is required.

Experimental realistic parameters of the cavity are $G \approx 10$ MHz, $\Gamma = 3$ MHz, $\kappa \approx 1$ MHz. State-of-the-art technology enables loading more than 10^5 atoms into the cavity and thus allows us to create an optically thick atomic cloud with optical depth $\mathrm{OD} = L/l_{\mathrm{abs}} \gtrsim 10$. Under these experimental conditions the proof of principle experiments should be realizable with a few μ sec pulses. Better results can in principle be obtained if microwave cavities and polar molecules are used.

In the present Letter we have discussed the propagation of a weak quantum pulse in an atomic Λ-type medium in Raman resonance with a quantized mode of a resonator. We have shown that in this scheme the group velocity of the probe pulse depends on the quantum state-of-the cavity mode, specifically on the number of photons. In the limit of a strong cavity coupling and for a sufficiently large optical depth of the medium for the probe field, the cavity photon statistics is determined by the photon number of the initial probe field. Under these conditions the action of the medium on the probe pulse can be described in terms of a photon-number dependent group velocity. The differential group delay between different Fock-state component can become large enough to spatially separate the single photon from higher photon-number components of the probe. An important application of the latter effect is a quantum state filter which can be employed to build a deterministic single-photon source. The main advantage of the present scheme as compared to other cavity-QED setups lies in the separation of the input-output mode and the cavity mode including the possibility of very different frequencies.

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[1] S. E. Harris, Phys. Today **50**, No. 7, 36 (1997).
[2] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[3] L. Hau *et al.*, Nature (London) **397**, 594 (1999).
[4] J. E. Field, Phys. Rev. A **47**, 5064 (1993).
[5] A. S. Parkins *et al.*, Phys. Rev. Lett. **71**, 3095 (1993).
[6] M. Hennrich *et al.*, Phys. Rev. Lett. **85**, 4872 (2000).
[7] T. Wilk *et al.*, Phys. Rev. Lett. **98**, 063601 (2007).
[8] J. H. Kimble, Phys. Scr. **T76**, 127 (1998).
[9] H. Schmidt and A. Imamoglu, Opt. Lett. **21**, 1936 (1996).
[10] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[11] S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999).
[12] M. D. Lukin and A. Imamoglu, Phys. Rev. Lett. **84**, 1419 (2000).
[13] A. Wallraff *et al.*, Nature (London) **431**, 162 (2004).
[14] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[15] H. Walther *et al.*, Rep. Prog. Phys. **69**, 1325 (2006).
[16] F. Brennecke *et al.*, Nature (London) **450**, 268 (2007).
[17] Y. Colombe *et al.*, Nature (London) **450**, 272 (2007).
[18] S. Gupta *et al.*, Phys. Rev. Lett. **99**, 213601 (2007).
[19] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, Phys. Rev. Lett. **104**, 073602 (2010).
[20] E. Kuznetsova *et al.*, Phys. Rev. A **78**, 012313 (2008).
[21] P. Rabl *et al.*, Phys. Rev. Lett. **97**, 033003 (2006).
[22] D. Petrosyan and M. Fleischhauer, Phys. Rev. Lett. **100**, 170501 (2008).
[23] M. Fleischhauer and M. D. Lukin, Phys. Rev. A **65**, 022314 (2002).
[24] G. Nikoghosyan and G. Grigoryan, Phys. Rev. A **72**, 043814 (2005).