How to Measure the Transmission Phase through a Quantum Dot in a Two-Terminal Interferometer

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(Received 17 March 2010; published 23 June 2010)

Measurement of the transmission phase through a quantum dot (QD) embedded in an arm of a two-terminal Aharonov-Bohm (AB) interferometer is inhibited by phase symmetry, i.e., the property that the linear response conductance of a two-terminal device is an even function of the magnetic field. It is demonstrated that in a setup consisting of an interferometer with a QD in each of its arms, with one of the QDs capacitively coupled to a nearby quantum point contact (QPC), phase symmetry is broken when a finite voltage bias is applied to the QPC. The transmission phase via the uncoupled QD can then be deduced from the amplitude of the odd component of the AB oscillations.

DOI: 10.1103/PhysRevLett.104.256801 PACS numbers: 73.23.−b, 73.63.Kv

Measuring the transport amplitude t through a quantum dot (QD), as a function of energy (or gate voltage), can give detailed information about the energy structure, the wave functions, and the many-body correlations in the QD. While the absolute value of the transmission $|t|^2$ can be easily measured [1], and has been employed extensively to characterize the QD, measurements of the transmission phase are much more subtle. The standard experimental approach is to embed the QD in an Aharonov-Bohm (AB) interferometer [2–12], and study the change in the phase of the AB oscillations, as a function of the QD parameters, e.g., gate voltage. However, the relation between the AB phase and the transmission phase is not straightforward [13]. In particular, when the AB interferometer is connected to two terminals, then the Onsager-Büttiker relations dictate that the linear response conductance must be an even function of magnetic flux [14–16], which means that the phase of AB oscillations can assume only the values 0 or π (*phase-symmetry*), independent of the transmission phase through the QD. This obstacle has been overcome experimentally by employing an open interferometer, i.e., an interferometer with more than two leads. However, this approach requires advanced technology and suffers from low signal due to particle losses to the other leads, and so far only a single group has been reporting measurements of the transmission phase using this approach [4,6,9,11]. Alternatively, it has been suggested that the phase may be extracted from a multiparameter fit to the shape of AB oscillations [17].

In this Letter we propose a new way to measure directly the transmission phase via a QD in a two-terminal AB interferometer, by coupling it to a nearby quantum point contact (QPC). The proposed measurement setup (Fig. 1) resembles the so-called "which path" interferometer [18,19], which in our case consists of a two-terminal AB interferometer containing a QD in each of its arms, and a QPC capacitively coupled to one of the QDs (QD2), as shown in Fig. 1. The QPC is expected to reduce the amplitude of the AB oscillations [19] (due to the dephasing caused by collecting the which-path information) but more importantly in the present context, it causes breaking of the phase symmetry when a finite bias is applied to it. (This, in fact, is a special case of breaking of the phase symmetry due to a nonequilibrium environment [20].) This phase-symmetry breaking will enable a direct measurement of the transmission phase through the QD not coupled to the QPC (QD1).

In this geometry, and under QPC bias, when a level in QD1 is swept across the Fermi level of the interferometer leads, the AB phase smoothly flows between the values 0 and π. Thus, breaking of the phase symmetry by coupling to a nonequilibrium environment can be observed experimentally, which, to our knowledge, has not been done so far. The observed phase of the AB oscillations, however, is not the transmission phase via QD1. Nevertheless, as we demonstrate below, the transmission phase through the QD1 can still be extracted from the amplitude of the odd component of the AB oscillations. In the following we present the model describing our which-path detector, dis-

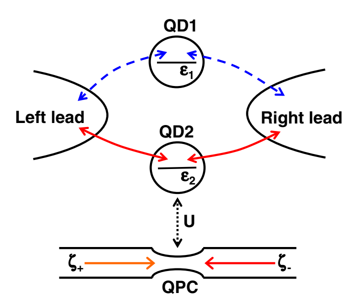

FIG. 1 (color online). Schematics of which-path interferometer studied in this Letter. The Aharonov-Bohm interferometer contains a quantum dot in each arm. In order to measure the transmission phase through one of the dots (QD1), a quantum point contact is coupled electrostatically to the other dot (QD2), in order to break the phase symmetry.

cuss breaking of the phase symmetry, and propose a straightforward method of measuring the transmission phase via the QD.

We describe the system by the following Hamiltonian:

$$\mathcal{H} = \sum_{\alpha} \epsilon_\alpha d_\alpha^\dagger d_\alpha + \sum_{k\in L,R} \epsilon_k c_k^\dagger c_k + \sum_{p,\nu} E_{p\nu} a_{p\nu}^\dagger a_{p\nu} + \sum_{\alpha,k,\mu} (t_{\alpha\mu} d_\alpha^\dagger c_{k\mu} + \text{H.c.}) + U d_2^+ d_2 \sum_{p,\nu;p'\nu'} a_{p\nu}^\dagger a_{p'\nu'}. \tag{1}$$

The first term describes the noninteracting QDs forming the interferometer—d_α is the operator that destroys an electron in QD $\alpha(\alpha = 1, 2)$. For simplicity we treat a single level in each dot, but the calculation and the results can be trivially extended to multilevel dots. The second term describes the leads: c_k destroys an electron in state k. The states in the leads are filled up to their respective chemical potentials $\mu_{L,R}$. The third term describes the QPC, the states in which are taken to be right or left movers ($\nu = \pm$) labeled by wave numbers p, p'. $a_{p\nu}$ is the corresponding electron destruction operator. Right- and left-moving bands are filled up to their respective chemical potentials ζ_ν. The fourth term accounts for tunneling between the leads and the QDs, characterized by coupling strengths $\Gamma^\mu_{\alpha\beta} = 2\pi N_\mu t_{\alpha\mu} t^*_{\beta\mu}$, where N_μ is the density of states in lead μ. The AB flux enters via the phases of these complex tunneling matrix elements $t_{\alpha\mu}$ such that $t^*_{1L} t_{1R} t^*_{2R} t_{2L} = t_{1L} t^*_{1R} t_{2R} t^*_{2L} e^{i2\varphi}$, where $\varphi = 2\pi\Phi/\Phi_0$, Φ is the magnetic flux threading the interferometer, and $\Phi_0 = hc/e$ is the flux quantum. The last term describes the electrostatic interaction between QD2 and the QPC, manifested in additional scattering potential when an electron occupies QD2, which reduces the QPC transmission by $\Delta T = 2\pi g_c$. [$g_c = 2\pi(\rho_0 U)^2$ determines the rate at which QPC electrons are backscattered, $g_c eV_{\text{QPC}}/\hbar$ [19]; ρ_0 is the density of states in the QPC.]

The current via the interferometer was calculated using standard techniques [21,22]. Interaction with the QPC was treated as a second-order self-energy in QDs Green's functions. The linear response conductance for zero bias on the QPC is shown in Fig. 2(a). The AB oscillations are even in magnetic flux, and there is a phase jump that occurs around $\epsilon_1 \approx -0.7$ μeV, where the oscillations change from having a minimum at zero magnetic field to having a maximum. (Although visually the jump occurs around $\epsilon_1 \approx -0.4$ μeV, its correct location can be determined from Fig. 2(c).)

When a finite bias is applied to the QPC, the phase symmetry is broken. Figure 2(b) depicts the difference in the conductance between $V_{\text{QPC}} = 400$ μeV and $V_{\text{QPC}} = 0$. The asymmetric component of the AB oscillations is now evident. The amplitude of the odd component of the oscillations, shown in Fig. 2(d), now takes experimentally measurable values. The phase of the main harmonic of the AB oscillations, extracted by Fourier transform, is shown in Fig. 2(c) for different values of V_{QPC}. This phase changes abruptly at zero bias, but flows smoothly between 0 and π as the bias on the QPC increases.

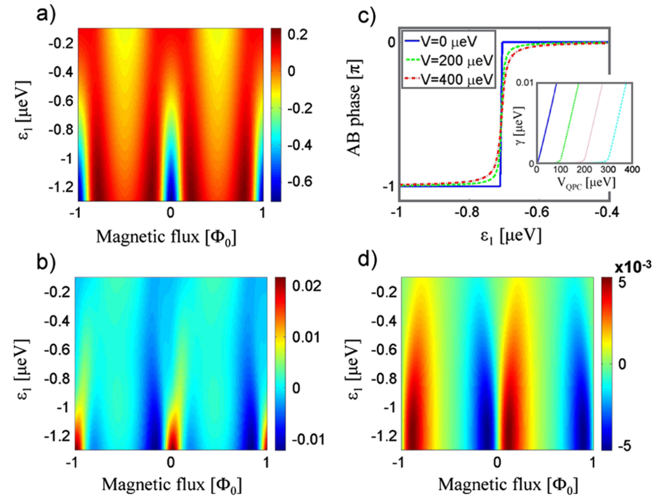

FIG. 2 (color online). (a) Linear response conductance as a function of magnetic flux (horizontal axis) and the energy of the level in the reference arm ϵ_1 (vertical axis) for $V_{\text{QPC}} = 0$ μeV, and (b) its change when the QPC bias is $V_{\text{QPC}} = 400$ μeV. (Other parameters are $\Gamma^L_{11} = \Gamma^L_{22} = 1$ μeV, $\Gamma^R_{11} = \Gamma^R_{22} = 5$ μeV, $g_c = 1.32 \times 10^{-4}$, $\epsilon_2 = 1.5$ μeV, $\epsilon_F = 0$ μeV.) (c) Phase of AB oscillations as a function of ϵ_1 for different values of QPC bias; since $\epsilon_2 > \epsilon_F$, the transmission coefficient is not symmetric in respect to the Fermi level and the phase jump is shifted to $\epsilon_1 = -0.7$ μeV. Inset: dephasing rate, $\gamma(\epsilon_F)$ as a function of the QPC bias, V_{QPC}; different traces correspond to (from left to right) $\epsilon_2 = 1.5$ μeV, and 300 μeV; (d) odd part of the AB conductance at $V_{\text{QPC}} = 400$ μeV.

The magnitude of the odd component of the AB oscillations is proportional to the strength of the coupling between the QPC and QD2, which also determines the experimentally observed reduction of the visibility of the AB oscillations in which-path experiments [18]. Therefore it should also be possible to observe breaking of the phase symmetry experimentally. Then the antisymmetric component of the AB oscillation which can be extracted by antisymmetrizing the data [23], can be used to deduce the transmission phase through QD1, $\varphi_{\text{QD1}}(\epsilon)$, as a function of its energy, as discussed below.

Assuming that the level in QD2 is far from the Fermi level of the interferometer leads, i.e. $\Gamma_{11}, \Gamma_{22}, |\epsilon_1 - \epsilon_F| \ll |\epsilon_2 - \epsilon_F|$, one can express the odd component of the AB conductance as

$$G^{\text{odd}}(\epsilon_F, \varphi) \simeq \pm \sin\varphi \frac{G_0}{2} \frac{\gamma(\epsilon_F)\sqrt{\Gamma^L_{22}\Gamma^R_{22}}|\Gamma^L_{22} - \Gamma^R_{22}|}{2(\Gamma_{22} + \gamma(\epsilon_2))[(\epsilon_2 - \epsilon_F)^2 + \frac{1}{4}(\Gamma_{22} + \gamma(\epsilon_2))^2]} \Re[t_{\text{QD1}}(\epsilon_F) - t_{\text{QD1}}(\epsilon_2)], \tag{2}$$

where $\gamma(\epsilon)$ is the additional broadening (dephasing rate) of the level in QD2 due to the coupling to the QPC [19,24], and $\Re$ stands for the real part. Under the conditions outlined above, the visibility of AB oscillations is unaffected by the dephasing, and $\gamma(\epsilon_2)$ in the denominator can be ignored along with Γ_{22}, with respect to $(\epsilon_2 - \epsilon_F)^2$. In the numerator $\gamma(\epsilon_F)$ determines the strength of the odd component of the AB oscillations. At zero bias on the QPC $\gamma(\epsilon_F)$ is zero, which reflects the fact that no breaking of the phase symmetry occurs in an interferometer coupled to an equilibrium environment [20]. With increasing the QPC bias $\gamma(\epsilon_F)$ grows slowly (due to finite temperature and Γ_{22}) until it reaches the "ionization threshold," $V_{\rm QPC} = |\epsilon_2 - \epsilon_F|$, after which it grows nearly linearly with bias, i.e., as $\gamma(\epsilon_F) \approx g_c(V_{\rm QPC} - |\epsilon_2 - \epsilon_F|)$ [inset in Fig. 2(c)].

Similarly, the term proportional to $\Re[t_{\rm QD1}(\epsilon_2)]$ in Eq. (2) can be omitted along with $\gamma(\epsilon_2)$. Note also that $G^{\rm odd}$ vanishes in a symmetric device, i.e., when $\Gamma_{22}^L = \Gamma_{22}^R$, because in this case changing the sign of the magnetic field is equivalent to interchanging the leads, leaving the conductance unchanged. Other mechanisms of phase-symmetry breaking are also known for their sensitivity to the device asymmetry [10,23,25,26].

The only factor in Eq. (2), dependent on the energy of the level in the reference arm of the interferometer, ϵ_1, is the real part of the transmission amplitude $t_{\rm QD1}(\epsilon_F)$, which is proportional to $\cos\varphi_{\rm QD1}$, the sought-after transmission phase. The other unknown energy-dependent quantities in this equation can be eliminated by measuring of the conductance via QD1 with QD2 disconnected,

$$G_{\rm QD1}(\epsilon_F) = \frac{G_0}{2}|t_{\rm QD1}(\epsilon_F)|^2. \quad (3)$$

Then the cosine of the transmission phase via QD1 can be immediately extracted as

$$\cos[\varphi_{\rm QD1}(\epsilon_F)] \propto \frac{G^{\rm odd}}{\sqrt{G_{\rm QD1}}}. \quad (4)$$

Equation (2) is also correct for a multilevel QD in the reference arm, given that the above-mentioned conditions are still satisfied. This is true even in the presence of interactions, as long as the interaction induced level broadening is small compared to Γ_{11}, Γ_{22}.

Figure 3 depicts the case of two levels of energies ϵ_{1a} and $\epsilon_{1b} = \epsilon_1 - 15\ \mu$eV and identical widths $\Gamma_{11}^L = 1\ \mu$eV, $\Gamma_{11}^R = 5\ \mu$eV, having the same (left panel) or opposite (right panel) parity. [$\epsilon_2 = 0.2$ meV, all other parameters are the same as those used for Fig. 2. A level is defined to have even (odd) parity if the tunneling matrix elements connecting the level to the two leads, $t_{\alpha L}$ and $t_{\alpha R}$, have the same (opposite) signs.] We see that the amplitude of the odd AB conductance (shown normalized to its maximal value) differs from the real part of the transmission coefficient only by a constant factor (and possibly sign), Figs. 3(b) and 3(e) as expected from Eq. (2). There is an excellent agreement between the transmission phase deduces from Eq. (4) and the one calculated directly, for both cases [Figs. 3(c) and 3(f)].

The case of two levels of the same parity, shown in the left panel of Fig. 3, is of particular interest in connection to the studies of phase lapses [27] (apart from the fact that we do not account here for the Coulomb interaction) due to its important generic feature: an abrupt change of the phase by π, as can be seen in Fig. 3(c).

The abrupt change in the phase may occur only when the transmission coefficient, Fig. 3(a), is identically zero, i.e., when both its real and imaginary components vanish simultaneously, and then the phase is undefined. On the other hand, if only the real part is zero, the phase is $\pi/2$ and usually corresponds to a transmission resonance. The fact that the zeros of $G^{\rm odd}$ correspond to one of these two types of special points is important, since in principle, the trans-

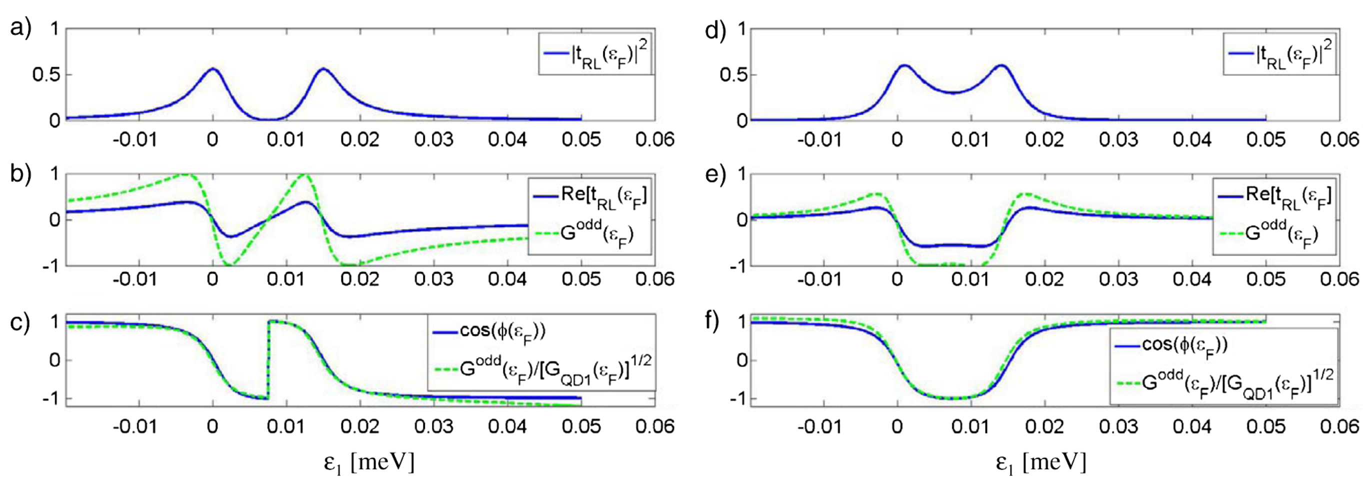

FIG. 3 (color online). (a),(d) Transmission, (b),(e) real part of the transmission coefficient and odd AB conductance (normalized to its maximum), and (c),(f) cosine of the transmission phase and its value extracted from $G^{\rm odd}$ [Eq. (2)] for QD1 with two levels of the same parity (left) or different parity (right) (see parameters in the text).

mission zeros of QD1 may be shifted in respect to those of QD1 with QD2 uncoupled (e.g., due to the level energy shifts induced by electron tunneling back and forth between the two dots), which may lead to unphysical results when naively applying Eq. (4). Therefore, one may have to shift G^{odd} along the energy axis in order to make its zeros coincide with those of G_{QD1}, which we did when plotting Figs. 3(b) and 3(c).

In the case of two levels of different parity, Fig. 3, right panel, the transmission is never equal to zero, and the phase changes smoothly.

Since Eq. (4) is only a proportionality relation, the cosine of the transmission phase has to be normalized to interval $[-1, 1]$. In the case of the levels of the same parity it is conveniently done using the abrupt change of the phase between 0 and π, where the value of cosine should be set to change between 1 and -1. In the absence of such an abrupt jump, e.g., for the two levels of different parities, the cosine may be normalized by its peak value, Fig. 3(f), where phase is expected to take value π.

The normalization factor is the prefactor in Eq. (2), which is proportional to the dephasing rate and therefore dependent on the QPC bias, V_{QPC}. On the other hand, the transmission coefficient through QD1 is independent of this bias; i.e., by measuring G^{odd} at different values of V_{QPC} one should obtain results that differ only by a constant prefactor. Thus, having determined $\Re[t_{\text{QD1}}]$ in one measurement, one can use measurements at different values of V_{QPC} to study the dependence of the dephasing rate, $\gamma(\epsilon_F)$, on V_{QPC}. If, in addition, one is able to measure independently the coupling strengths $\Gamma^{\mu}_{\alpha\alpha}$, one may use Eq. (2) to extract the interaction constant g_c and compare it with that measured by other methods [28].

To conclude, we propose that breaking of the phase symmetry, necessary for measuring the transmission phase via a QD, embedded in an arm of an AB interferometer, can be achieved by coupling the interferometer to a QPC in a which-path geometry, which is equivalent to phase symmetry breaking by coupling to a nonequilibrium environment, predicted in Ref. [20]. Although the phase of the resulting AB oscillations is not the transmission phase via the QD, the latter can be extracted from the amplitude of the odd part of the AB oscillations.

The authors would like to thank Klaus Ensslin and Thomas Ihn for useful discussions. This work was supported in part by the ISF and BSF.

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