Self-Modulation Instability of a Long Proton Bunch in Plasmas

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An analytical model for the self-modulation instability of a long relativistic proton bunch propagating in uniform plasmas is developed. The self-modulated proton bunch resonantly excites a large amplitude plasma wave (wakefield), which can be used for acceleration of plasma electrons. Analytical expressions for the linear growth rates and the number of exponentiations are given. We use full three-dimensional particle-in-cell (PIC) simulations to study the beam self-modulation and transition to the nonlinear stage. It is shown that the self-modulation of the proton bunch competes with the hosing instability which tends to destroy the plasma wave. A method is proposed and studied through PIC simulations to circumvent this problem, which relies on the seeding of the self-modulation instability in the bunch.

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Plasma based particle acceleration, both by lasers and particle beams, has made significant progress over the past years and now routinely accelerates electrons in the GeV energy regime [1]. However, on the path towards the TeV energy range, one still has to solve many problems. The laser technology has to be greatly improved in terms of power, stability, and repetition rate. The electron-beam-driven wakefield acceleration has the fundamental problem of the transformer ratio limit [2]. It arises because an electric field of the same order drags the driver as the one that accelerates the witness beam; the maximum achievable energy gain in one single stage is limited by the energy of particles in the driver beam.

Recently, it has been proposed to harness a TeV proton bunch driver such as the one available from the CERN LHC for the TeV regime of electron acceleration in plasmas [3]. Notwithstanding that the scheme demands a short—of a submillimeter length—proton driver bunch, all the presently available proton bunches are much longer, usually tens of centimeters long. These available long proton bunches at CERN LHC cannot generate a high amplitude wakefield directly. Indeed, for an efficient excitation of wakefields, the proton bunch length must be close to the plasma wavelength $\lambda_p = 2\pi c/\omega_p$, where c is the speed of light and $\omega_p^2 = 4\pi n_e e^2/m_e$ is the square of the plasma frequency, where n_e is the plasma electron density and e and m_e are the electron charge and mass, respectively. In the simplest way, plasma electrons can be understood as an ensemble of oscillators oscillating at plasma frequency. To enforce the resonant swinging of these oscillators, the driver must contain a Fourier component close to the plasma frequency, a criterion easily fulfilled by a short dense driver but not by a long driver. The situation will change, however, if the long driver is modulated at the plasma frequency. Then, even a tenuous proton beam can excite a high amplitude plasma wave.

In this Letter, we develop an analytical model for the self-modulated regime of the proton bunch acceleration of electrons along the lines of the self-modulated laser wakefield accelerator (SMLWFA) concept [4]. The underlying physical mechanism is very much the same as in the SMLWFA concept. A long proton bunch ($L > \lambda_p$) generates a wake within its body, which modulates the bunch itself, leading to the positive feedback and unstable modulation of the whole bunch along the bunch propagation direction. This self-modulation splits the long proton beam into ultrashort bunches of length $\sim\lambda_p$, which resonantly drive the plasma wake. This plasma wake can be used to accelerate electrons, just as in the SMLWFA concept.

The self-modulation of the proton bunch essentially occurs due to the action of the transverse wakefields on the bunch itself. This is different from the modulation caused by the electrostatic two-stream instability, which arises due to relative streaming between the proton bunch and the background plasma, and causes excitation of the longitudinal field. Because of the very anisotropic response of relativistic beam particles to longitudinal and transverse forces, modulation caused by the longitudinal force becomes very inefficient in the case of a relativistic driver. This is precisely the reason that the transverse instabilities, such as the self-modulation instability, have higher growth rates than the longitudinal two-stream instability. In the context of a proton beam modulation, the two-stream instability is analogous to the forward Raman scattering of the laser pulse in laser driven wakefield acceleration. Although, the field of the beam-plasma instabilities is very rich, the analysis of the self-modulation instability of long proton bunches has been missing from the literature till now.

We begin with the analytical theory of the beam self-modulation based on the beam-envelope approach. Asymptotic expressions for the growth rate are obtained, and a simple semianalytical code is developed to simulate the early stage of the instability. To substantiate the analytical results, we also perform fully electromagnetic three-dimensional particle-in-cell (3D PIC) simulations. Finally, we dwell upon the competition of self-modulation instability with the hose instability [5,6] of the proton bunch, and evaluate a proposal, by PIC simulations, to alleviate the effect of hosing instability to produce an efficient excitation of wakefield by proton beams.

Following the approach of Ref. [7], we can write down the two-dimensional expressions for the wakefields of an axisymmtric beam driver of an arbitrary profile by utilizing the Euler variables $\xi = \beta_0 ct - z$, $\tau = t$, where $\beta_0 = v_z/c$ (v_z is the velocity of the bunch), and assuming the quasi-static approximation ($\partial_\tau \simeq 0$) for the beam driver. Inside the body of a long proton bunch ($0 < \xi < L$), these read as

$$E_z(r,\xi) = 4\pi k_p^2 \int_0^\xi \int_0^\infty r' dr' \rho(r',\xi') I_0(k_p r_<) \times K_0(k_p r_>) d\xi' f(\xi') \cos k_p(\xi - \xi'), \quad (1)$$

$$W_\perp(r,\xi) \simeq (E_r - B_\theta)(r,\xi) = 4\pi k_p \int_0^\xi \int_0^\infty r' dr' \partial_{r'} \rho(r',\xi') I_1(k_p r_<) \times K_1(k_p r_>) d\xi' f(\xi') \sin k_p(\xi - \xi'), \quad (2)$$

where $\rho(r,\xi) = \rho_0 \psi(r) f(\xi)$ is the charge density of the bunch, $I_{0(1)}$ and $K_{0(1)}$ are the modified Bessel functions of order 0(1), $r_< = \min(r, r')$ and $r_> = \max(r, r')$, $k_p = \omega_p/c$ is the background plasma wave number, L is the length of the bunch in the $\hat{z}$ direction, and we have assumed $\beta_0 \approx 1$.

The equation for the beam envelope for a long proton bunch with a Heaviside step-function profile [$\psi(r) = \Theta(r_b - r)$, r_b being the radius of the beam envelope] in radial direction and an arbitrary profile $f(\xi)$ in ξ is written as

$$\frac{\partial^2 r_b}{\partial \tau^2} - \frac{\mathcal{M}^2}{r_b^3} = -\frac{\omega_b^2}{\gamma_0} \int_0^\xi r_b(\xi') I_1\{k_p r_b(\xi')\} K_1\{k_p r_b(\xi')\} \times f(\xi') k_p \sin k_p(\xi - \xi') d\xi', \quad (3)$$

where $\gamma_0 = (1 - \beta_0^2)^{-1/2}$ is the relativistic Lorentz factor of the beam, $\omega_b^2 = 4\pi\rho_0 e/m_b$ is the square of the non-relativistic beam-plasma frequency of the proton bunch, $\rho_0 = n_b e$ is the charge density of the proton bunch, m_b being the mass of the beam particle. The beam-envelope radius $r_b = r_b(\xi)$ is a function of ξ on account of pinching caused by the wakefield on the beam. The constant $\mathcal{M}$ arises from the integration of the θ component of the equation of motion for the beam electrons yielding the angular momentum constant, and is associated with the transverse emittance of the beam [8]. For the demonstration of the self-modulation instability of a proton beam, we consider a thin beam ($k_p r_b \ll 1$) with a Heaviside step-function profile [$f(\xi') = \Theta(\xi')$] and take $\mathcal{M} = \omega_{\beta 0} r_{b0}^2$, where $\omega_{\beta 0}^2 = \omega_b^2/2\gamma_0$ and r_{b0} is the initial radius of the beam. The beam-envelope equation, in normalized coordinates ($r_b = r_b/r_{b0}$, $\tau = \omega_{\beta 0}\tau$, $\xi = k_p\xi$), reads as

$$\frac{\partial^2 r_b(\xi)}{\partial \tau^2} - \frac{1}{r_b^3(\xi)} = -\int_0^\xi r_b(\xi') \sin(\xi - \xi') d\xi'. \quad (4)$$

On perturbing Eq. (4) about the equilibrium radius $r_b = 1 + \delta r_b$, and assuming $\delta r_b = \delta \hat{r}_b \exp(i\xi)$, $|\partial \delta \hat{r}_b/\partial \xi| \ll |\delta \hat{r}_b|$, we obtain $(\partial_\xi^2 + 1)(\partial_\tau^2 + \Delta)\delta \hat{r}_b = -\delta \hat{r}_b$, where $\Delta = 3$. We assume the perturbation of the form $\delta \hat{r}_b(\xi,\tau) \sim \exp(i\delta\omega\tau - ik\xi)$, and obtain the dispersion relation $D \equiv (k^2 - 1)(\delta\omega^2 - \Delta) = -1$. The dispersion relation gives two complex k roots (one in the upper half and another in the lower half of the complex-k plane) for real $\delta\omega$ ($\sqrt{\Delta} < \delta\omega < \sqrt{1+\Delta}$). When $\mathrm{Im}(\delta\omega) \to \infty$, (i.e., $|\delta\omega| \to \infty$), the roots of the dispersion relation reach the real k axis, thus confirming the presence of a convective instability [9]. For complex k, the perturbation acquires the asymptotic form $\delta \hat{r}_b(\xi,\tau) \propto \exp(-ik_r\xi)\exp(k_i\xi)$, where k_r and k_i are the real and imaginary parts of the complex-k root. For $k_i > 0$, the perturbation grows spatially in the $\xi > 0$ direction, while for $k_i < 0$ (lower half of the k plane) it grows spatially in the $\xi < 0$ direction, thus representing a spatially amplifying wave.

One can derive asymptotic relations for the instability by following the approach of Bers [10]. For sufficiently late times, $\tau > L_e$, where $L_e \sim 1/\Gamma$ is the e-folding length and Γ is the growth rate of the instability, we solve this dispersion relation by letting $\delta\omega' = \delta\omega - vk$, where $v = \xi/\tau$, and setting $D(\delta\omega', k) = 0$ and $\partial D(\delta\omega', k)/\partial k = 0$ while keeping $\delta\omega'$ constant. This gives $k(\delta\omega^2 - \Delta)^2 = \delta\omega v$. We write $\delta\omega = \delta\omega_1 \pm \sqrt{\Delta}$. For $|\delta\omega_1| \gg \sqrt{\Delta}$ (early-time asymptote), we get $\Gamma_e = (\xi/2\tau)^{1/2}$. The number of exponentiation is given by $N_{ee} = \Gamma_e\tau = (\xi\tau/2)^{1/2}$. For $|\delta\omega_1| \ll \sqrt{\Delta}$ (late-time asymptote), we get the growth rate as $\Gamma_l = 3^{1/3}(\xi/\tau)^{2/3}/4$. The number of exponentiation is given by $N_{el} = \Gamma_l\tau = 3^{1/3}(\xi^2\tau)^{1/3}/4$. The expressions for number of exponentiations in dimensional units can be written as

$$N_{ee} = \left(\frac{\alpha}{2\sqrt{2\gamma_0}} k_p \xi \omega_p \tau\right)^{1/2}, \quad (5)$$

$$N_{el} = \frac{3^{1/3}}{4}\left(\frac{\alpha}{\sqrt{2\gamma_0}} k_p^2 \xi^2 \omega_p \tau\right)^{1/3}, \quad (6)$$

where $\alpha = \sqrt{(n_b/n_e)(m_e/m_b)}$.

We have solved Eq. (4) numerically to demonstrate the self-modulation instability of the beam envelope for a gently rising beam density profile, which flattens at $\xi/\lambda_p = 10$ (here ξ is the unnormalized Euler variable). The initial radius of the beam is $r_{b0}/\lambda_p = 0.1$. The boundary conditions are $r_b(\xi, 0) = 1$, $\partial r_b(\xi, 0)/\partial\tau = 0$. Figure 1(a) shows the self-modulation instability of the beam envelope. The beam's head, $\xi/\lambda_p = 0$, is diffracting, while the self-modulation grows with increasing distance from the front of the beam. The self-modulation instability leads to the generation of strong axial field, depicted in Fig. 1(b). The self-modulation of the beam grows in accordance with both the early-time [Fig. 1(c)] and the late-time [Fig. 1(d)] asymptotes, describing excellent agreement with the analytical scalings.

The propagation of the proton beam in plasmas also suffers from the onset of the hose instability which, in the limit $n_b \ll n_e$, is also known as the transverse two-stream instability [4,5]. For a nonaxisymmetric beam ($\partial_\theta \neq 0$), the hose instability is a concern. For a perfectly axisymmetric beam, such as the one assumed in our calculations, it does not occur. The temporal growth rate of the hose instability of a focused electron beam could be comparable to or less than the growth rate of the self-modulation instability. In a uniform plasma and in the limit of thick plasma skin depth $k_p r_b \ll 1$, the long-time asymptotic of the instability scales as $\Gamma_h \propto (k_\beta z'/\omega_p \tau')^{2/3}\omega_p$, where $k_\beta = k_b/\gamma_0^{1/2}$, $k_b = \omega_b/c$ being the beam betatron wave number, $\tau' = t - z/v_z$, $z' = z$ [4]. The ratio of growth rates for two instabilities scales as $\Gamma_l/\Gamma_h \propto (1/\alpha)^{1/3}(\gamma_0/2)^{1/6} \gg 1$, while $\alpha \ll 1$. Yet, it has time to develop and, if present, can severely affect the wakefield. Thus, there is a big concern that the hosing instability occurring simultaneously with the beam self-modulation may destroy the plasma wakefield. One of the possibilities to circumvent this problem is to preseed the self-modulation instability, so that the beam self-modulation does not have to grow from noise. This seeding would greatly increase both the shot-to-shot reproducibility and the quality of the wakefield. Additionally, the development of the high amplitude wakefield will require much shorter propagation distance, which will further limit the growth of the hose instability. The seed of the self-modulation instability can be accomplished either by a short driver in front of the proton beam [11] or by a modification of the proton beam itself by cutting away the leading part of the proton beam in a "dogleg" device. A dogleg device employs dipole and quadrupole focusing arranged in a way to either compress or stretch the beam. Adjustable trains of short electron bunches have been produced by placing a mask in the dogleg section of the beam line [12]. This method, in principle, can produce a hard-cut proton beam.

Because the hosing instability is not a part of the beam envelope analytic description (3), we have to rely here on 3D PIC simulations, which were performed using the VLPL code [13]. We simulate two cases. In the first case, the beam has a smooth 3D Gaussian density profile $n_{b1}(r, z) = n_b^0 \exp(-z^2/\sigma_z^2)\exp(-r^2/\sigma_r^2)$, where $r^2 = x^2 + y^2$. In the second case, the beam is hard cut in the middle, $n_{b2}(r, z) = n_b^0 \Theta(-z)\exp(-z^2/\sigma_z^2)\exp(-r^2/\sigma_r^2)$, where $\Theta(z)$ is again the Heaviside step function. The plasma is assumed to be singly ionized lithium. The electron density n_e of the plasma is 25 times higher than the peak beam density, i.e., $n_b^0 = 0.04 n_e$. In both cases, the proton beam is assumed to have 24 GeV energy. The beam in the simulations has a rms transverse momentum spread of $\sqrt{\langle p_\perp^2\rangle}/m_b c = 4.5 \times 10^{-3}$. The beam radius is $k_p\sigma_r = 2\pi \times 0.25$, while the beam length is $k_p\sigma_z = 2\pi \times 20$. The beam moves in the $\hat{z}$ direction with a relativistic Lorentz factor $\gamma_0 = 25$.

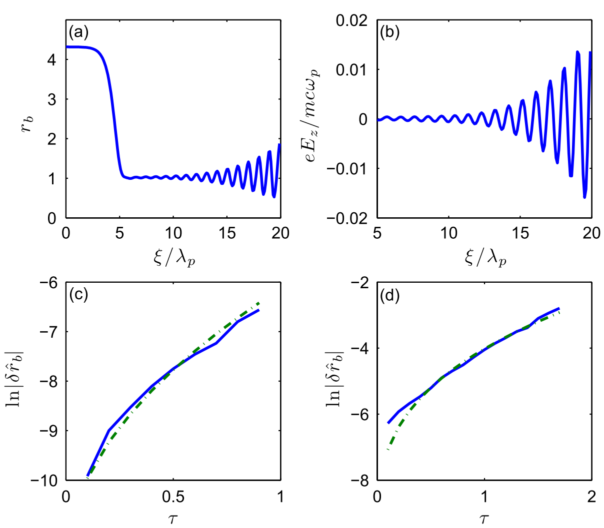

FIG. 1 (color online). Panel (a) shows the beam-envelope radius evolution with ξ/λ_p at a time $\tau = 4.3$. Panel (b) depicts the on-axis axial field generated by the beam. Panels (c) and (d) show the comparison of the amplitude of the beam's radius perturbations from Eq. (4) (solid lines) with the asymptotic relations (dash-dotted lines) for early- and late-time asymptotes, respectively.

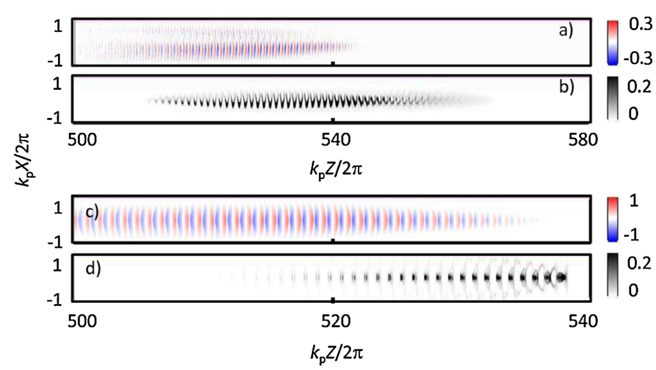

FIG. 2 (color online). 3D PIC simulation results for the smooth Gaussian beam (a),(b) and the hard-cut half-Gaussian (c),(d). Frames (a) and (c) show the accelerating wakefield E_z, while frames (b) and (d) depict the beam density distribution.

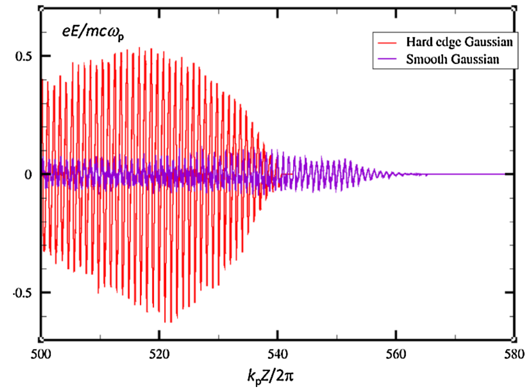

FIG. 3 (color online). Direct comparison of the on-axis wakefields generated after the propagation length of 500 plasma periods for both the smooth Gaussian and the hard-cut half-Gaussian cases.

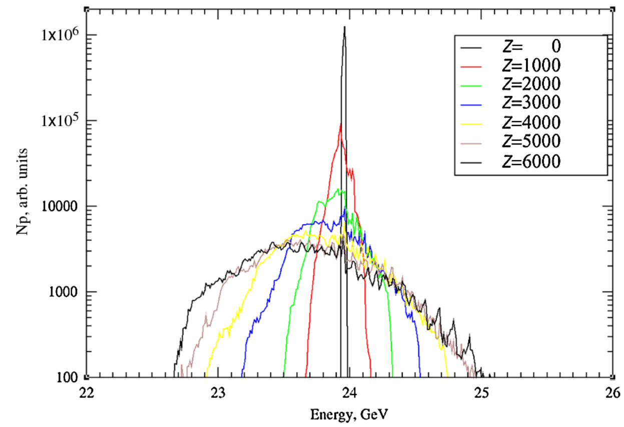

FIG. 4 (color online). Energy spectrum evolution of the hard-cut half-Gaussian proton beam. Spectrum broadening and acceleration of some protons up to 1 GeV is observed.

The simulation results showing the on-axis generated wakefield and the self-modulated beam are given in Fig. 2 in a ZX plane. Figures 2(a) and 2(b) show the smooth Gaussian beam case, while Figs. 2(c) and 2(d) correspond to the second case of the hard-cut Gaussian beam. One sees that in the first case when both instabilities have to grow from noise, the hosing instability competes with the self-modulation instability. Although the beam is split into small beamlets, these density perturbations are located slightly off axis, and even some filamentation can be observed. On the contrary, when the self-modulation instability is seeded by the hard cutting of the beam in the second case, the wakefield is very regular and axisymmetric. To compare the amplitude of the excited wakefields in both cases, we plot the on-axis accelerating field in Fig. 3. The wakefield for the hard-cut Gaussian beam is approximately 5 times larger than that for the smooth Gaussian case. The maximum accelerating field normalized to the wave breaking field is $eE_z/m_e c\omega_p \approx 0.6$. Thus, it is close to the nonlinear regime. Figure 4 shows the energy spectrum evolution of the proton beam. One observes the spectrum broadening as the wakefield is developing. Finally, some protons acquire energy gain of nearly 1 GeV.

In summary, we have demonstrated the self-modulation instability of a long proton bunch, which can resonantly excite the plasma wave needed for the TeV regime of electron acceleration. We have analyzed the seeding of the self-modulation instability in order to alleviate the effects of the hose instability on the plasma wave. The seeding is accomplished by hard cutting a Gaussian beam. The simulation results of the hard-cut Gaussian beam show that in this case the generated wakefield acquires substantially high values. Further, through the combination of the seeding of the self-modulation instability and suitably restricting the beam propagation length in plasmas, one can hope to excite large amplitude plasma waves, and this could pave the way for the successful realization of the TeV regime of electron acceleration scheme.

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[1] A. Pukhov and J. Meyer-ter Vehn, Appl. Phys. B **74**, 355 (2002); J. Faure *et al.*, Nature (London) **431**, 541 (2004); C. G. R. Geddes *et al.*, *ibid.* **431**, 538 (2004); S. P. D. Mangles *et al.*, *ibid.* **431**, 535 (2004); W. P. Leemans *et al.*, Nature Phys. **2**, 696 (2006); S. Kneip *et al.*, Phys. Rev. Lett. **103**, 035002 (2009); I. Blumenfeld *et al.*, Nature (London) **445**, 741 (2007).
[2] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Sci. **24**, 252 (1996).
[3] A. Caldwell, K. Lotov, A. Pukhov, and F. Simon, Nature Phys. **5**, 363 (2009); K. V. Lotov, Phys. Rev. ST Accel. Beams **13**, 041301 (2010).
[4] E. Esarey, J. Krall, and P. Sprangle, Phys. Rev. Lett. **72**, 2887 (1994).
[5] D. H. Whittum, Phys. Fluids B **5**, 4432 (1993).
[6] J. Krall and G. Joyce, Phys. Plasmas **2**, 1326 (1995).
[7] R. Keinigs and M. E. Jones, Phys. Fluids **30**, 252 (1987).
[8] E. P. Lee and R. K. Cooper, Part. Accel. **7**, 83 (1976).
[9] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Course of Theoretical Physics Vol. 10 (Butterworth-Heinemann, Oxford, 1981), 1st ed., p. 275.
[10] A. Bers, *Basic Plasma Physics 1* (North-Holland, Amsterdam, 1983), Chap. 3.2.
[11] K. Lotov, in *Proceedings of the 6th European Particle Accelerator Conference, Stockholm, 1998* (Institute of Physics, London, 1998), pp. 806–808.
[12] P. Muggli, V. Yakimenko, M. Babzien, E. Kallos, and K. P. Kusche, Phys. Rev. Lett. **101**, 054801 (2008).
[13] A. Pukhov, J. Plasma Phys. **61**, 425 (1999).