

Unitarity Triangle without Semileptonic Decays

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(Received 18 January 2010; published 24 June 2010)

The use of semileptonic decays has become standard in constraining the unitarity triangle. Since precise calculations of these are very challenging, we propose an entirely new approach. The ε_K constraint, which depends extremely sensitively on $|V_{cb}|$, is replaced by the interplay between ε_K , $\text{BR}(B \rightarrow \tau\nu)$, and ΔM_{B_s} . Improvements on the $B \rightarrow \tau\nu$ branching ratio and on the lattice determinations of $f_{B_s}\hat{B}_s^{1/2}$ and f_B can increase the effectiveness of this method significantly.

DOI: 10.1103/PhysRevLett.104.251802

PACS numbers: 12.15.Hh, 11.30.Hv

In this decade significant progress has been made in our understanding of flavor physics, thanks in large part to the spectacular performance of the two B factories. We learned that the Cabibbo-Kobayashi-Maskawa (CKM) paradigm [1] is able to simultaneously account for the observed CP violation in the K and B systems up to an accuracy of $O(20\%)$. Impressive as this is, it is still important to understand that this leaves a lot of room for new physics (NP). Indeed, as more data from B factories became available and also as the accuracy in some key theoretical calculations was attained, several $2\text{--}3\sigma$ deviations from the SM have emerged. While this clearly does not represent an unambiguous signal for NP, it does mean that efforts need to continue both on the experiment and on the theory front to seek greater clarity with regard to these anomalies.

In this context use of semileptonic decays in all traditional analysis of the unitarity triangle (UT) to date is a concern. The inclusive $b \rightarrow u$ transitions are not governed by any symmetry and as a result are a special challenge for continuum methods. Exclusive decays are in principle amenable to the lattice and steady, but unfortunately rather slow, progress is being made. The fact that for both $b \rightarrow c$ and for $b \rightarrow u$ inclusive and exclusive methods disagree by $\approx 2\sigma$ casts a shadow of doubt on the results of the UT analysis. This is especially aggravated by the fact that use of the input from ε_K is exceedingly sensitive to V_{cb} , scaling as the fourth power. These observations motivate us to seek alternate approaches, which we will provide herein.

Recent improvements especially in the lattice calculation of B_K [2–5], led to the appearance of a $\sim 2\sigma$ tension that can be interpreted as NP in B_d and/or in K mixing [6–10]. An important difficulty with these analyses is the long standing discrepancy between the extraction of $|V_{cb}|$ and $|V_{ub}|$ from exclusive and inclusive semileptonic decays. From Table I, one sees that inclusive and exclusive determinations differ at the 2σ level. While Ref. [7] demonstrated that $|V_{ub}|$ can be dropped from the fit without affecting the observed tension, it is usually believed that $|V_{cb}|$ from $b \rightarrow c\ell\nu$ decays is essential to include ε_K .

Bearing all this in mind, we propose a new approach to the UT analysis, wherein no use of semileptonic decays is

made. We show that the traditional use of the $\varepsilon_K + |V_{cb}|$ combination can be effectively replaced by the interplay between ε_K , ΔM_{B_s} and $\text{BR}(B \rightarrow \tau\nu)$. We find that even after removing information from semileptonic decays, the $\sim 2\sigma$ tension in the fit survives. More importantly, every experimental and theoretical input to this analysis is now clean and under very good control. The latter point is quite important, because many of the hints for NP that come from precision studies tend to have some problems. A very important exception is the 2.2σ evidence for a CP violating phase in B_s mixing, whose nonzero value would be a clean evidence for NP [13–16].

Present status of the UT fit.—We follow the approach of Refs. [7,10] and utilize the averages calculated in Ref. [11] with some exceptions: we include inclusive $|V_{ub}|$ albeit with an additional 10% model uncertainties [17]; we take a simple (not weighted) average of the determinations of ξ from Fermilab/MILC [18] and HPQCD [19]. Also we take the central value of $f_{B_s}\sqrt{\hat{B}_s}$ from Ref. [11] but adopt the uncertainty quoted in Ref. [19]. We adopt this conservative stance to show that the impact of the approach we champion here remains largely unaffected even if the lattice errors are not as small as currently claimed in the literature.

We summarize the inputs we use in Table I. Below we present explicitly only those formulas that are relevant to the traditional analysis which uses semileptonic decays:

$$\Delta M_{B_s} = \chi_s f_{B_s}^2 \hat{B}_{B_s} A^2 \lambda^4, \quad (1)$$

$$\Delta M_{B_d} = \chi_d f_{B_d}^2 \hat{B}_{B_d} A^2 \lambda^6 [\eta^2 + (-1 + \rho)^2], \quad (2)$$

$$|\varepsilon_K| = 2\chi_\varepsilon \hat{B}_K \kappa_\varepsilon \eta \lambda^6 \{A^4 \lambda^4 (\rho - 1) \eta_2 S_0(x_t) + A^2 [\eta_3 S_0(x_c, x_t) - \eta_1 S_0(x_c)]\}, \quad (3)$$

$$\text{BR}(B \rightarrow \tau\nu) = \chi_\tau f_B^2 |V_{ub}|^2 \simeq \chi_\tau f_B^2 A^2 \lambda^6 (\rho^2 + \eta^2), \quad (4)$$

where we expanded in λ and the χ_i can be extracted from Ref. [10]. The 68% C.L. allowed regions in the (ρ, η) plane are shown in Fig. 1, where we show explicitly that the ε_K , $B \rightarrow \tau\nu$, and $|V_{ub}|$ constraints require $|V_{cb}|$ to be drawn independently. In particular we obtain

TABLE I. Inputs used in the fit. References to the original experimental and theoretical papers and the description of the averaging procedure can be found in Ref. [11]. Statistical and systematic errors are combined in quadrature. We adopt the averages of Ref. [11] for all quantities with the exception of $|V_{ub}|$, $f_{B_s}\sqrt{\hat{B}_s}$ and $\xi = f_{B_s}\sqrt{\hat{B}_s}/f_{B_d}\sqrt{\hat{B}_d}$ (see text).

$ V_{cb} _{\text{excl}} = (38.6 \pm 1.2)10^{-3}$	$\eta_1 = 1.51 \pm 0.24$
$ V_{cb} _{\text{incl}} = (41.31 \pm 0.76)10^{-3}$	$\eta_2 = 0.5765 \pm 0.0065$
$ V_{cb} _{\text{incl+excl}} = (40.3 \pm 1.0)10^{-3}$	$\eta_3 = 0.47 \pm 0.04$
$ V_{ub} _{\text{excl}} = (34.2 \pm 3.7)10^{-4}$	$\eta_B = 0.551 \pm 0.007$
$ V_{ub} _{\text{incl}} = (40.1 \pm 2.7 \pm 4.0)10^{-4}$	$\xi = 1.23 \pm 0.04$
$ V_{ub} _{\text{incl+excl}} = (36.4 \pm 3.0)10^{-4}$	$\lambda = 0.2255 \pm 0.0007$
$\Delta M_{B_d} = (0.507 \pm 0.005) \text{ ps}^{-1}$	$\alpha = (89.5 \pm 4.3)^\circ$
$\Delta M_{B_s} = (17.77 \pm 0.12) \text{ ps}^{-1}$	$S_{\psi K_S} = 0.672 \pm 0.024$
$\varepsilon_K = (2.229 \pm 0.012) \times 10^{-3}$	$\gamma = (78 \pm 12)^\circ$
$m_c(m_c) = (1.268 \pm 0.009) \text{ GeV}$	$\hat{B}_K = 0.725 \pm 0.027$
$m_{t,\text{pole}} = (172.4 \pm 1.2) \text{ GeV}$	$f_B = (192.8 \pm 9.9) \text{ MeV}$
$f_{B_s}\hat{B}_s^{1/2} = (275 \pm 19) \text{ MeV}$	$f_K = (155.8 \pm 1.7) \text{ MeV}$
$\mathcal{B}_{B \rightarrow \tau\nu} = (1.43 \pm 0.37)10^{-4}$ [12]	$\kappa_e = 0.92 \pm 0.01$

$$|V_{ub}|_{\text{fit}} = (3.61 \pm 0.13) \times 10^{-3}, \quad (5)$$

$$\text{BR}(B \rightarrow \tau\nu)_{\text{fit}} = (0.87 \pm 0.11) \times 10^{-4}, \quad (6)$$

$$[\sin 2\beta]_{\text{fit}} = 0.766 \pm 0.036. \quad (7)$$

where each quantity is extracted by removing the corresponding direct determination from the fit (for V_{ub} this means removing information from semileptonic $b \rightarrow u\ell\nu$ decays). Note that $|V_{ub}|_{\text{fit}}$ is quite close to the determination from exclusive decays and that $\text{BR}(B \rightarrow \tau\nu)_{\text{fit}}$ is *smaller* than the corresponding world average $(1.43 \pm 0.37) \times 10^{-4}$ [12] (see also Refs. [20,21]). The relatively low p value ($p_{\text{SM}} = 15\%$) of the fit has been interpreted in terms of NP in K or in B_d mixing [6–11].

Adopting the model independent parametrizations

$$|\varepsilon_K^{\text{NP}}| = C_\varepsilon |\varepsilon_K^{\text{SM}}|, \quad (8)$$

$$M_{12}^{d,\text{NP}} = r_d^2 e^{2i\theta_d} M_{12}^{d,\text{SM}}, \quad (9)$$

$$\text{BR}(B \rightarrow \tau\nu)^{\text{NP}} = r_H \text{BR}(B \rightarrow \tau\nu)^{\text{SM}}, \quad (10)$$

where in the SM we have $(C_\varepsilon, r_H, r_d) = 1$ and $\theta_d = 0$. When considering NP in B_d mixing we allow simultaneous variations of both θ_d and r_d . We find that NP in $|M_{12}^d|$ has a limited effect on the tension between the direct and indirect determination of $\sin(2\beta)$. We obtain

$$C_\varepsilon = 1.28 \pm 0.14 \Rightarrow (2.0\sigma, p = 58\%), \quad (11)$$

$$\theta_d = -(3.9 \pm 1.8)^\circ \Rightarrow (2.1\sigma, p = 61\%), \quad (12)$$

$$r_d = 0.95 \pm 0.04 \Rightarrow (1.1\sigma, p = 61\%), \quad (13)$$

$$r_H = 1.7 \pm 0.5 \Rightarrow (1.4\sigma, p = 29\%). \quad (14)$$

The above results are mutually exclusive in the sense that they are obtained by allowing either NP in K , or in B_d mixing or in $B_u \rightarrow \tau\nu$ and point to a $\sim 2\sigma$ hint for NP. The quoted p values have to be compared with the SM result

($p_{\text{SM}} = 15\%$). The lower p value for NP in $B \rightarrow \tau\nu$ indicates that the tension in the fit is only partially lifted by new contributions to $B \rightarrow \tau\nu$.

Removing semileptonic decays.—Recall that inclusive and exclusive $b \rightarrow (c, u)\ell\nu$ decays are tree-level processes and, therefore most likely, are quite insensitive to the presence of NP. The $|V_{cb}|$ constraint translates into a determination of the parameter A of the CKM matrix. Knowledge of the latter is critical in order to extract information from $|V_{ub}| \propto A$, $\text{BR}(B \rightarrow \tau\nu) \propto A^2$ and $\varepsilon_K \propto A^4$ [see Eqs. (3) and (4)]. The (ρ, η) regions allowed by each of these three observables is obtained with the inclusion of $|V_{cb}|$. Without any information on A these bands would cover the whole (ρ, η) plane. The main role of the determination of $|V_{ub}|$ is to limit the amount of NP contributions to the phase of B_d mixing; in fact, an upper limit on $|V_{ub}|$ implies an upper limit on $S_{\psi K} = \sin 2(\beta + \theta_d)$ [22,23]. The exclusion of the $|V_{ub}|$ constraint is not critical any longer to the presence of the 2σ effects in Eqs. (11) and (12) as emphasized recently in [7]. In particular the prediction that we obtain for the B_d mixing phase in the no- V_{ub} scenario reads $[\sin 2\beta]_{\text{fit}} = 0.840 \pm 0.056$ deviating by 2.8σ from its direct determination. On the other hand, $|V_{cb}|$ appears to be central: employing only its exclusive (inclusive) determination, the 2.0σ significance of the extraction of C_ε shifts to 2.5σ (1.6σ); similarly the 2.2σ effect in B_d mixing shifts to 2.9σ (1.4σ).

We now come to elaborating on the new approach that we are advocating here in which no use of semileptonic decays will be made. Note that the critical issue is the determination of A from $|V_{cb}|$. We find that the interplay of ε_K , $\text{BR}(B \rightarrow \tau\nu)$ and ΔM_{B_s} results in a fairly strong constraint on the (ρ, η) plane even without using semileptonic decays at all. A simple way to understand this result is to use Eqs. (1) and (4) to eliminate A and write

$$|\varepsilon_K| \propto \hat{B}_K (f_{B_s} \hat{B}_s^{1/2})^{-4}, \quad |\varepsilon_K| \propto \hat{B}_K \text{BR}(B \rightarrow \tau\nu)^2 f_B^{-4}, \quad (15)$$

where for simplicity we kept only the dominant contributions to ε_K ($\propto A^4$) and did not explicitly write the dependence of ε_K on ρ, η and all other quantities that are irrelevant to the error budget. Equation (15) shows that the $|V_{cb}|$ constraint can be effectively replaced by either

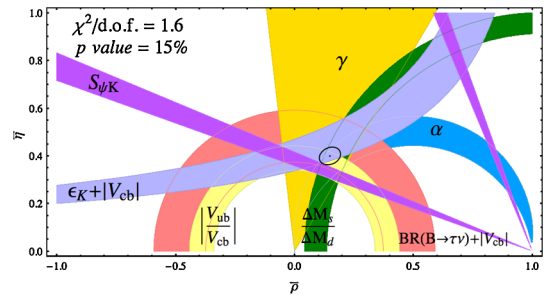


FIG. 1 (color online). Standard UT fit. The contour is obtained using V_{cb} , V_{ub} , ε_K , $B \rightarrow \tau\nu$, γ , ΔM_{B_s} and ΔM_{B_d} .

$f_{B_s} \hat{B}_s^{1/2}$ or $\text{BR}(B \rightarrow \tau\nu) \times f_B^{-2}$. In Fig. 2 we show the complete fit of the UT in absence of semileptonic decays. The fit results for $|V_{ub}|$ and $\text{BR}(B \rightarrow \tau\nu)$ do not deviate significantly from Eqs. (5) and (6). However, it is interesting to note that the extracted value of $[\sin 2\beta]_{\text{fit}} = 0.811 \pm 0.074$ still deviates by 1.9σ from its direct determination. It is also interesting to observe that the result $|V_{cb}|_{\text{fit}} = (43.2 \pm 0.9) \times 10^{-3}$ is slightly larger than the average we quote in Table I: this is yet another manifestation of the tension between K and B_d mixing. Finally, we note that $f_{B_s} \hat{B}_s^{1/2}$, and ξ are largely independent because they are affected by different lattice systematics and we average results from different lattice collaborations thereby reducing the possible correlation between statistical errors. A surprising outcome is the slight preference of the fit for NP in B_d mixing. This can be seen by extracting C_ε , θ_d , r_d , and r_H

$$C_\varepsilon^{\text{noV}_{qb}} = 1.23 \pm 0.30 \Rightarrow (0.8\sigma, p = 39\%), \quad (16)$$

$$\theta_d^{\text{noV}_{qb}} = -(9.0 \pm 4.7)^\circ \Rightarrow (1.9\sigma, p = 87\%), \quad (17)$$

$$r_d^{\text{noV}_{qb}} = 0.95 \pm 0.05 \Rightarrow (0.9\sigma, p = 87\%), \quad (18)$$

$$r_H^{\text{noV}_{qb}} = 1.7 \pm 0.5 \Rightarrow (1.4\sigma, p = 64\%), \quad (19)$$

and noting that NP in B_d mixing yields larger p value than NP in K mixing or in $B \rightarrow \tau\nu$.

As an illustration of the implications of these constraints we consider the impact on two Higgs doublet models. Within these models the $r_H \neq 1$ result translates into a constraint on the mass of H^\pm . In the type-II two Higgs doublet model (2HDM) and in the minimal supersymmetric SM (MSSM) we can write [24] $r_H = (1 - X_H)^2$ where $X_H = (\tan\beta m_{B^\pm}/m_{H^\pm})^2 / (1 + \epsilon_0 \tan\beta)$, $\tan\beta$ is the ratio of the vacuum expectation values of the Higgs bosons that couple to up and down quarks and ϵ_0 summarizes supersymmetric corrections to the $\bar{u}bW^+$ vertex. In the 2HDM we have $\epsilon_0 = 0$; in the MSSM typical values for ϵ_0 range at the 10^{-2} level. A full supersymmetric analysis of Eq. (19) is beyond the scope of this Letter. In Fig. 3 we present the regions of the $(\tan\beta, m_{H^\pm})$ that are allowed for various values of ϵ_0 . In addition to the bounds implied by Eq. (19) we include also constraints from $B \rightarrow D\tau\nu$ [25] and $B \rightarrow X_s\gamma$ [26]. From the observation that X_H is always positive, it follows that the charged Higgs exchange can only reduce the $B \rightarrow \tau\nu$ branching ratio unless $X_H > 2$. Equation (19) implies $X_H = (2.3 \pm 0.2) \vee (-0.3 \pm 0.2)$. At the 1σ level only the $X_H \sim 2$ solution is permitted (note, $X_H > 0$) and the resulting allowed narrow band at low M_{H^\pm} or large $\tan\beta$, is, in turn, excluded by $B \rightarrow D\tau\nu$ data both in the 2HDM and in the MSSM (we follow the numerical analysis of Ref. [25]). At 95% C.L. the solution $X_H = 0$ opens up, corresponding to large M_{H^\pm} . In the 2HDM the $B \rightarrow X_s\gamma$ constraint implies $m_{H^\pm} > 295$ GeV [26]. In the MSSM, chargino loops contributions to the

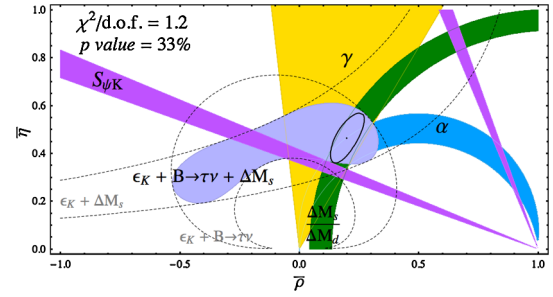


FIG. 2 (color online). Unitarity triangle fit without semileptonic decays. The solid contour is obtained using ε_K , $B \rightarrow \tau\nu$, γ , ΔM_{B_s} and ΔM_{B_d} . The dashed contours show the interplay of the ε_K , ΔM_{B_s} and $\text{BR}(B \rightarrow \tau\nu)$ constraints.

$b \rightarrow s\gamma$ amplitude can compensate charged Higgs effects: the bound on m_{H^\pm} depends strongly on the chosen point in the supersymmetric parameter space [27].

Let us now discuss the dominant sources of uncertainties in this analysis. In the following table we list the most relevant inputs, their errors, and their impact on ε_K :

X :	\hat{B}_K	$ V_{cb} $	$f_{B_s} \hat{B}_s^{1/2}$	$\text{BR}(B \rightarrow \tau\nu)$	f_B
δX :	4%	2.5%	6.9%	26%	5%
$\delta \varepsilon_K$:	4%	10%	27.6%	52%	20%

First of all, note that the impact of \hat{B}_K on the error is subdominant. The use of the semileptonic $b \rightarrow c$ constraint results in a $\sim 10\%$ determination of ε_K , roughly half of the uncertainty obtained by employing only ΔM_{B_s} (i.e., $f_{B_s} \hat{B}_s^{1/2}$). A calculation of $f_{B_s} \hat{B}_s^{1/2}$ at the 2.5% level would reduce the overall uncertainty on ε_K to 10%; a calculation at the 1% level would impact ε_K at the same level as \hat{B}_K . At first sight, the impact of $B \rightarrow \tau\nu$ seems irrelevant. Fortunately the nontrivial dependence of $\text{BR}(B \rightarrow \tau\nu)$ on ρ and η implies a certain degree of orthogonality between the constraints (15), as can be seen in Fig. 2. A numerical estimate of the impact of this constraint can be obtained by removing it from the fit and recalculating the overall p value: we obtain $p = 43\%$, meaning that no hint of NP is observed. The experimental uncertainty on $\text{BR}(B \rightarrow \tau\nu)$ is therefore an important ingredient of this analysis. Once the latter reaches 10%, improvements on f_B become relevant. We summarize this discussion in the following table:

δ_τ	δ_s	p_{SM}	$\theta_d \pm \delta\theta_d$	p_{θ_d}	$\theta_d/\delta\theta_d$
26%*	2.5%	3.3%	$-(9.2 \pm 3.5)^\circ$	98%	2.6 σ
10%	6.8%*	2%	$-(9.0 \pm 2.9)^\circ$	98%	3.1 σ
3%	6.8%*	0.08%	$-(9.0 \pm 2.3)^\circ$	98%	3.9 σ
10%	1%	0.004%	$-(9.0 \pm 2.0)^\circ$	98%	4.1 σ
3%	2.5%	0.004%	$-(9.0 \pm 2.0)^\circ$	98%	4.5 σ
3%	1%	0.00009%	$-(9.1 \pm 1.8)^\circ$	98%	4.9 σ

where $\delta_\tau = \delta\text{BR}(B \rightarrow \tau\nu)$, $\delta_s = \delta(f_{B_s} \hat{B}_s^{1/2})$, and * denotes the current uncertainties. The values $\delta_\tau = (10, 3)\%$ correspond to a super- B factory result with $(5, 50)\text{ab}^{-1}$ [28]. In the table we show the p value of the SM fit, the NP phase θ_d , the p value of the NP fit and its significance. We do not show the scenarios with NP in K mixing or

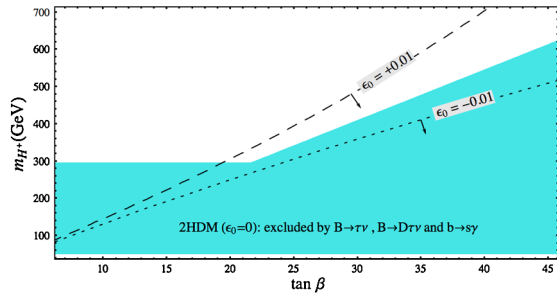


FIG. 3 (color online). 95% C.L. bounds in the $(\tan\beta, m_{H^+})$ plane. The shaded region is excluded in the 2HDM. The dotted (dashed) line shows how this region shifts in two MSSM scenarios with $\epsilon_0 = -0.01(0.01)$ (the arrows indicate the excluded region).

$B \rightarrow \tau\nu$ because they yield very low confidence levels and are, therefore, disfavored. From the inspection of the table, we conclude that even modest improvements in $f_{B_s} \hat{B}_s^{1/2}$ and/or $\text{BR}(B \rightarrow \tau\nu)$ will help enormously in isolating the presence of NP in the UT fit. For comparison, reducing the total uncertainty on $|V_{cb}|$ to the 1% level yields $p_{\text{SM}} = 2.8\%$ corresponding to 2.7σ effects in either θ_d or C_ϵ .

Conclusions.—The traditional fit of the UT within the SM displays a tension at the 2σ level. However, the $\sim 2\sigma$ discrepancies in the extraction of $|V_{cb}|$ and $|V_{ub}|$ between inclusive and exclusive semileptonic B decays tend to cast doubts on the reliability of this conclusion. We studied the removal of these constraints from the fit. In contrast with the generally accepted statement that information on $|V_{cb}|$ is required in order to make use of ϵ_K , we showed, for the first time, that the combination of ϵ_K , ΔM_{B_s} and $\text{BR}(B \rightarrow \tau\nu)$ provide a stringent constraint in the (ρ, η) plane.

After removing information from semileptonic decays, we find that the tension in the UT fit persists at the 1.9σ level and may be interpreted as possible NP in B_d (rather than in K) mixing. The preference of the fit for NP contributions to B_d mixing is caused by the $B \rightarrow \tau\nu$ constraint. This branching ratio is proportional to $|V_{ub}|^2$ and, as it can be seen from Fig. 2, points to a large value of $|V_{ub}/V_{cb}|$. This, in turn, favors a scenario with NP in the B_d mixing phase.

Even modest improvements on $\text{BR}(B \rightarrow \tau\nu)$ and/or $f_{B_s} \hat{B}_s^{1/2}$ may push this tension above the 3σ level; if errors on both constraints are reduced simultaneously, $\delta_\tau = (10, 3)\%$ and $\delta_s = (2.5, 1)\%$, the effect reaches $(4-5)\sigma$ (from the discussion above we see that an improved determination of f_B becomes relevant only for $\delta_\tau < 10\%$). Note that improvements on $B \rightarrow \tau\nu$ require a super- B factory [29–31] while the reduction of δ_s is a purely theoretical (i.e., lattice) issue. We stress that we are not suggesting abandoning the traditional approach with use of semileptonic decays, but rather in addition making concerted efforts towards improved lattice determination of $f_{B_s} \hat{B}_s^{1/2}$, and also of the $\text{BR}(B \rightarrow \tau\nu)$. These should pro-

vide valuable redundancy in our quest for NP through flavor studies even in the LHC era. Finally we would like to stress that the main focus of the present Letter is to propose a new clean strategy to implement simultaneously K and B_d mixing constraints on the (ρ, η) plane and that our projections on the reach of this method depend solely on improving δ_τ and δ_s and are quite insensitive to the rest of the inputs summarized in Table I, in particular, the assumed errors in lattice computations.

This research was supported in part by the U.S. DOE Contract No. DE-AC02-98CH10886(BNL).

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