# Magnetospheric Vortex Formation: Self-Organized Confinement of Charged Particles

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(Received 16 January 2010; published 10 June 2010)

A magnetospheric configuration gives rise to various peculiar plasma phenomena that pose conundrums to astrophysical studies; at the same time, innovative technologies may draw on the rich physics of magnetospheric plasmas. We have created a ''laboratory magnetosphere'' with a levitating superconducting ring magnet. Here we show that charged particles (electrons) self-organize a stable vortex, in which particles diffuse inward to steepen the density gradient. The rotating electron cloud is sustained for more than 300 s. Because of its simple geometry and self-organization, this system will have wide applications in confining single- and multispecies charged particles.

DOI: 10.1103/PhysRevLett.104.235004 PACS numbers: 52.27.Jt, 52.35.We, 52.72.+v

Magnetospheres are self-organized structures found commonly in the Universe. A dipole magnetic field sets the stage for charged particles to cause a variety of interesting phenomena [1–4]. The nontrivial nature of magnetospheres is primarily due to the strong inhomogeneity of the field strength. The motion of a single particle is not very complicated; it coils around an arching magnetic field line and bounces between the polar regions. Yet, the physics of magnetospheric *plasma* is very rich and even strange. For example, magnetized particles are normally diamagnetic, but the magnetosphere attracts some of the particles and produces a clump. These particles diffuse toward the higher density inner region and steepen the density gradient. Such a process is seemingly contradictory to the conventional idea of diffusion, which usually diminishes any gradient.

Hasegawa [5] explains the uniqueness of the magnetospheric configuration by comparing Cartesian and magnetic coordinates; the latter, consisting of the action variables pertaining to periodic motions of magnetized particles, are strongly distorted with respect to the former by the inhomogeneity of the field strength. The motions of magnetized particles, obeying the adiabatic constraints, are embedded in the magnetic-coordinate space. Hence, mapping from the magnetic-coordinate space onto Cartesian space (the laboratory frame) will produce strange images—the Jacobian weight will yield a strong heterogeneity in the Cartesian perspective [6].

Recently a ''laboratory magnetosphere'' has been created [7,8]; the RT-1 device levitates a superconducting ring magnet in a vacuum chamber and produces a magnetospheric configuration (Fig. 1). In this Letter, we report experimental evidence of inward diffusion, a peculiar but universal mechanism predicted by theory, which possibly plays an essential role in the self-organization of magnetospheric plasmas. This spontaneous process will have wide applications in confining various single- or multispecies charged particles (for example, highly charged ions, antimatter, or possibly high-temperature fusion plasmas) in a compact space [9–17].

We explore inward diffusion by producing a pure electron plasma. A single-species plasma has a strong internal electric field. Confinement is possible only if the electric field ($\boldsymbol{E}$) is balanced by an induction ($\boldsymbol{v} \times \boldsymbol{B}$) generated by a vortical motion ($\boldsymbol{v}$) in the magnetic field ($\boldsymbol{B}$). The flow and the electromagnetic field achieve stable coupling by

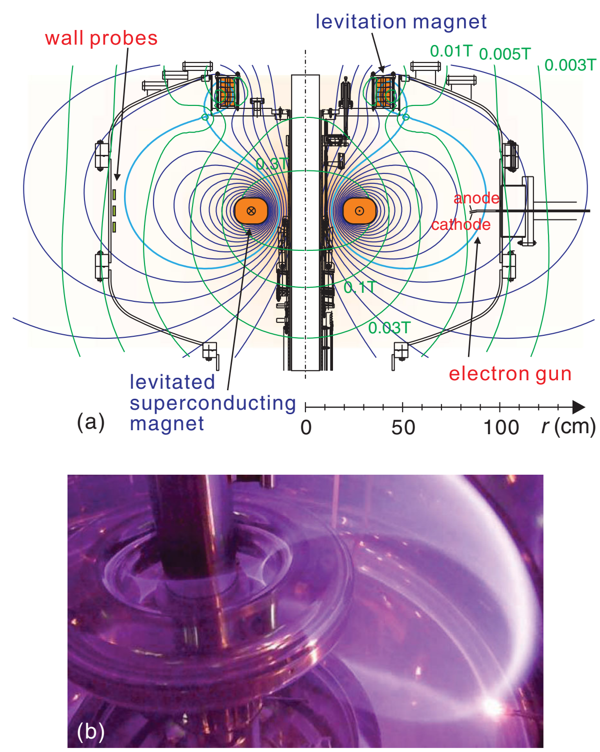


FIG. 1 (color). The RT-1 magnetospheric plasma system. (a) A dipole magnetic field is produced by the levitating superconducting ring magnet. The magnetic field lines (contours of the flux function $\psi$) and the field-strength contours are shown. The field strength in the confinement region varies from 0.5 to 0.01 T. (b) The magnetic surface is visualized by injecting electrons into hydrogen gas at a pressure $P_n = 1 \times 10^{-2}$ Pa from an electron gun located at $r = 0.70$ m on the midplane, producing weakly ionized plasma.

 

self-organizing a vortex [18–24]: Galaxies are close cousins, in which rotation creates a balance between gravitational and centrifugal forces.

The nontriviality of confinement in the magnetospheric topology may be highlighted by comparing its formation process with that of conventional traps. The most popular method of trapping a non-neutral plasma is to inject particles along a straight homogeneous magnetic field and then plug the entrance to the magnetic bottle by applying an electric field [18]. The injected particles form an axisymmetric rigidly rotating column [19,20,24]. This method (a so-called Penning-Malmberg trap) is widely used in the study of atomic or particle (especially antimatter) physics [25–27]. Magnetospheric non-neutral plasma is produced through an entirely different and simpler process. We simply emit particles (electrons from an electron gun) in a peripheral region of a static dipole magnetic field (see Fig. 1); we do not need to control the electric or magnetic field. Particles penetrate into the magnetosphere and self-organize a stable vortical structure.

Figure 2 shows the typical formation process. Soon after the start of injection (injection point $r = 0.8$ m, injection energy $eV_{\mathrm{acc}} = 175$ eV, beam current $I_{\mathrm{beam}} \sim 300$ $\mu$A), a charged cloud is created, which repels the beam and diminishes the current to about $10^{-5}$ A; see Fig. 2(a). Figure 2(b) shows the electrostatic fluctuation measured by a wall probe (a small piece of metallic plate facing the plasma), and Fig. 2(c) shows its frequency spectrum. The initial turbulent phase, in which the fluctuation reaches around 50% of the ambient electric field, quenches after about 0.05 s. As we show below, the turbulence drives particles inward. While the beam current is supplied, the fluctuation has a complex spectrum. When the beam current is stopped ($t = 0.32$ s), the plasma becomes turbulent and then relaxes into a quiescent state, in which the fluctuation level is less than 1%; through the transient turbulent phase, the driven state reorganizes into the confined state. By "triangulation," using an array of wall probes, we can estimate the location of the charged clump [28]. Electrons in the outer region (intersecting the electron gun) are lost immediately after the beam is stopped. The remaining particles (typically $10^{-8}$ C which amounts to about 40% of the total particles in the driven phase) move slowly inward and reside in a stronger field region. The quiescent phase continues for more than 300 s.

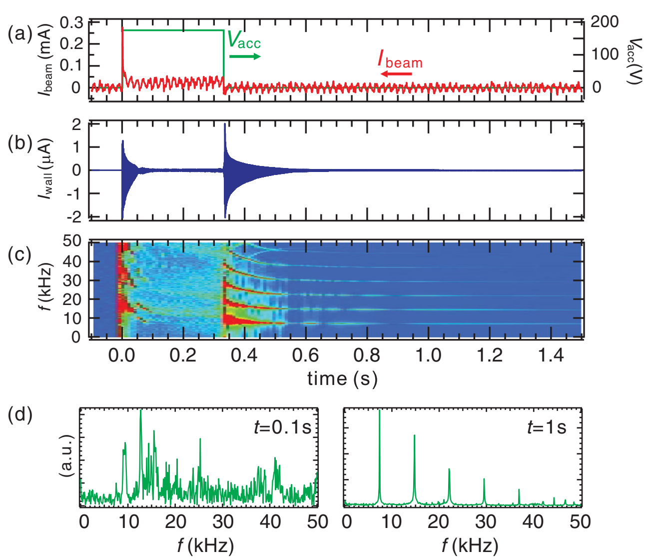


FIG. 2 (color). The formation and sustainment of a magnetospheric electron plasma: (a) The acceleration voltage $V_{\mathrm{acc}}$ and the beam current $I_{\mathrm{beam}}$ of the electron gun. (b) The electrostatic fluctuation measured by a wall probe. (c) The evolution of the frequency spectrum of the fluctuation. (d) In the driven phase, the fluctuation has a broad spectrum ($t = 0.1$ s). In the confined phase, the small-amplitude fluctuation has a discrete spectrum ($t = 1$ s).

Comparing Figs. 2(c) and 2(d), we find a remarkable difference between the driven and confined states. In the confined state, the frequency spectrum is sharply localized around 10 kHz and its higher harmonics. The oscillations are highly coherent and propagate in the toroidal direction; the mode number of the dominant component is 1. We observe a slow change in the spectrum (Fig. 3). First, the frequency increases slightly and then decreases; the frequency drops when the internal electric field diminishes or the clump shifts toward the stronger magnetic field side.

The lifetime $\tau^*$ of the confinement phase depends on the neutral gas pressure $P_n$ (Fig. 4). When $P_n > 7 \times 10^{-5}$ Pa, an appreciable amount of ions are produced during the injection phase [Fig. 1(b)], and $\tau^*$ becomes short. Long-term confinement is achieved in the regime $P_n < 7 \times 10^{-5}$ Pa, where we find the empirical relation $\tau^* \sim 2.4 \times P_n^{-0.35}$ s. The nonlinear relation ($\tau^* P_n \neq$ const) implies that electron-neutral collisions do not simply determine the lifetime. Confinement ends rather violently with the

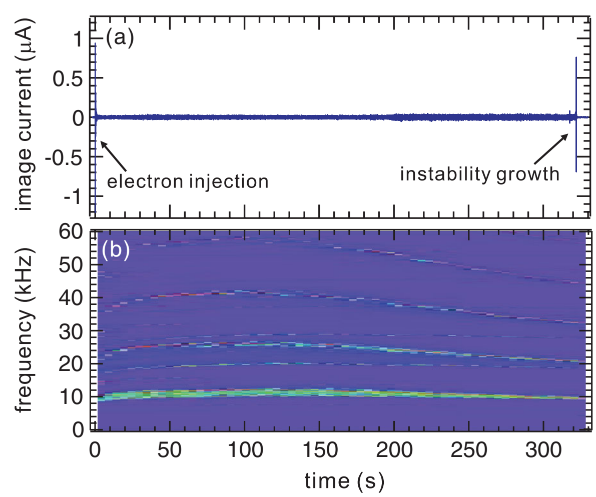


FIG. 3 (color). The long-term behavior of the electrostatic fluctuation. (a) Stable confinement continues for more than 300 s, and terminates with a rapid growth of the instability. (b) The evolution of the frequency spectrum. The dominant spectrum ($f \approx 10$ kHz) corresponds to the fundamental frequency of the toroidal rotation.

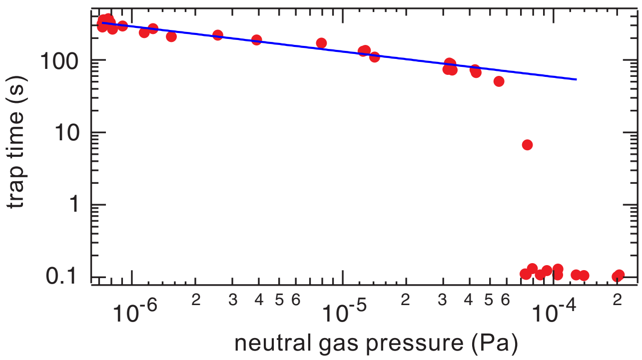


FIG. 4 (color online). The lifetime $\tau^*$ of the magnetospheric electron plasma as a function of the background pressure $P_n$ of the neutral gas (hydrogen). Stable long-term confinement was realized at $P_n < 7 \times 10^{-5}$ Pa.

onset of instability [Fig. 3(a)]; the instability grows exponentially with a typical time constant of 200 $\mu$s. Subsequently, the fluctuation amplitude decays and the trapped charge is lost with a typical time constant of 10 ms. The instability may be caused by ion resonance instability [29]; ionization of the residual gas at $P_n \sim 10^{-6}$ Pa may yield ions whose density is estimated to be less than $10^{-3}n$ ($n$: electron density $\sim 10^{11}$ m$^{-3}$).

Now, we discuss the mechanism that self-organizes the magnetospheric electron plasma. In a dipole magnetic field, a non-neutral plasma produces a vortical structure rotating around the polar axis. In a strong axisymmetric magnetic field ($\partial_\theta \boldsymbol{B} = 0$ in $r$-$\theta$-$z$ cylindrical coordinates), the orbit of a single electron cannot cross the magnetic surfaces (the level-sets of the flux function $\psi$ of the magnetic field $\boldsymbol{B} = \nabla\psi \times \nabla\theta$; as shown in Fig. 1, $\psi$ may be viewed as the radial coordinate measuring the distance from the ring magnet). This is due to conservation of the canonical angular momentum, which is approximately $-e\psi$ (omitting the small mechanical part $mrv_\theta$, where $e$ and $m$ are the change and mass of an electron). Inward diffusion (i.e., flattening of the particle distribution with respect to $\psi$) can occur if conservation of the canonical angular momentum is violated by breaking the symmetry with respect to the toroidal angle $\theta$ [5,30]. This is caused by the spontaneous electric field excited in the initial and transient turbulent phases (Fig. 2); the Kelvin-Helmholtz (KH) instability (or diocotron instability [24]), excited in shear flow (differential rotation), drives the turbulence.

We have yet to describe a more drastic consequence of inward diffusion: the particles are pushed further and create an inward density gradient. The driver of the turbulence, KH instability, is active as long as the rotation has a shear, so a turbulence-free quiescent state is realized only in a rigidly rotating vortex. If KH instability tends to self-organize a relaxed state and stabilize itself, turbulence-induced diffusion will cast the charge distribution into a specific profile that rotates rigidly; this does in fact happen, as we observe in the experiment, and the relaxed state turns out to be not flat. The angular frequency $\omega$, the flux function $\psi$, and the electric potential $\phi$ are related by $\omega = \nabla\phi/\nabla\psi$. For $\omega$ to be homogeneous, the density $n$ (by Poisson's equation, $n \propto \nabla^2\phi$) must have a profile such that $n \propto B$ [31]. The strength $B$ of a dipole magnetic field is strongly inhomogeneous, so the density of the relaxed state must have a steep inward gradient. This is in marked contrast to the homogeneous-density equilibrium of a Penning-Malmberg trap, which also rotates rigidly in a homogeneous magnetic field [20,24].

Interestingly, these two different equilibria are unified because the factor $B$, which dominates the density distribution, is the Jacobian weight for the map from the action $\mu = E_\perp/\omega_c$ to the perpendicular energy $E_\perp = mv_\perp^2/2$ ($\omega_c = eB/m$, $v_\perp$: perpendicular velocity with respect to the magnetic field). If the velocity distribution is a function of the action $\mu$ (instead of the energy $E_\perp$, as predicted by the adiabatic invariance theorem), the velocity-space integral $dv_\perp^2/2$ operates as $(\omega_c/m)d\mu$ to yield the laboratory-frame density $n$, including the Jacobian $\omega_c \propto B$ [9,32].

In a frame moving with the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity, the electric field vanishes, so particles are seemingly subject to no force. Hence, it might be thought that the density will become homogeneous. Indeed, it does so if $\boldsymbol{B}$ is homogeneous. However, particles are subject to magnetic force, and the equilibrium selects a different quantity, the particle number per unit magnetic flux tube, to homogenize [5,32].

Because the action $\mu$, which dictates the distorted phase space in which the distribution homogenizes, must be constant, particles are accelerated as they diffuse toward the larger-$B$ region (so-called betatron acceleration). In fact, by probing the charged cloud with an emissive Langmuir probe, we find that the space potential is higher than the initial energy imparted to the electrons by the gun, so the particles must be accelerated to climb the potential hill. The energy can be supplied by the turbulent electric field that drives the diffusion. Figure 5 shows the space potentials ($\phi_s$) measured at two different positions of the electron gun ($r_{\text{gun}} = 80$ and 70 cm). In both cases, the potential energy $-e\phi_s$ at the position of the electron gun agrees with the gun's acceleration voltage $V_{\text{acc}} = 500$ V, implying that the magnetosphere is fully charged. In the inner region ($r < r_{\text{gun}}$), $-e\phi_s$ increases beyond $V_{\text{acc}}$, clearly showing inward diffusion.

In summary, we have demonstrated long-term (exceeding 300 s) confinement of non-neutral plasma in the vicinity of a magnetic dipole. In our previous experiment on the Proto-RT device [14], which has a similar geometry except that the internal magnet is supported by rods, we had to bias the potential of the magnet to match the potential surfaces (level sets of $\phi$) and the magnetic surfaces (level sets of $\psi$). In the RT-1 device, however, an internal superconducting magnet is levitated, so it is electrically floating. Through the formation process, the magnet is charged by injected electrons up to the spontaneously selected potential. The structure is self-organized; the spontaneous process stabilizes KH instabilities by homog-

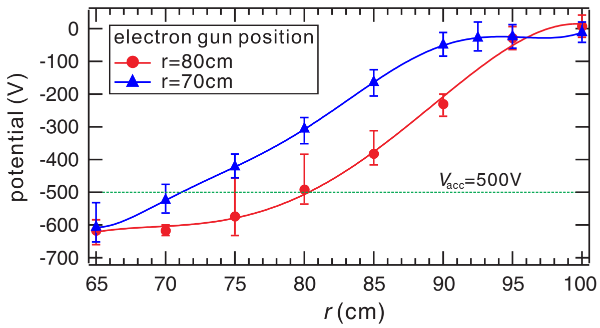


FIG. 5 (color online). The radial profile of the space potential $\phi_s$ measured by an emissive Langmuir probe. Two different gun positions are compared (circles: $r_{\rm gun} = 80$ cm, triangles: $r_{\rm gun} = 70$ cm). Electrons penetrate into the inner region ($r < r_{\rm gun}$), where $-e\phi_s > V_{\rm acc}$. The probe measurement can be performed only in the driven phase, because insertion of the probe damages the confinement.

enizing the vorticity (angular frequency of the rotation). We note that KH instability in a non-neutral plasma appears as an electrostatic mode (diocotron mode). We may expect a similar inward diffusion effect in quasineutral plasmas (for example, a planetary magnetosphere), in which KH modes will appear as the growth of perturbed vorticity in an ambient shear flow.

This work was supported by Grants-in-Aid for Scientific Research (14102033 and 19340170) from MEXT, Japan.

*Note added in proof.*—The authors thank the referee for bringing to their attention Ref. [33], published after the submission of their manuscript. This article describes turbulent inward diffusion in a neutral plasma of similar configuration. The turbulence is driven by interchange instabilities, and the self-organized state, which is marginally stable against the interchange modes, homogenizes the particle number per unit magnetic flux. While the underlying modes are different, the similarity of the systems speaks to the robustness of inward diffusion in magnetospheric geometry.

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