Spatial Frequency Clustering in Nonlinear Dust-Density Waves

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Self-excited density waves were studied in a strongly coupled dusty plasma of a radio-frequency discharge under microgravity conditions. The spatiotemporal evolution of the complicated three-dimensional wave field was investigated and analyzed for two different situations. The reconstructed instantaneous phase information of the wave field revealed a partial synchronization within multiple distinct domains. The boundaries of these regions coincide with the locations of topological defects.

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Synchronization, the mutual adjustment of rhythms of coupled independent oscillators, has enjoyed great popularity among all fields of natural sciences [1], since its first observation by Huygens [2]. Popular examples are swarms of fireflies [3], neural networks [4], and the interaction between the human cardiovascular and respiratory systems [5]. Synchronization is also observed in spatially extended systems consisting of a large number of constituents such as plasma discharges, where the entrainment of global oscillations [6–8] or waves [9,10] by external drivers are well-known features.

A unique laboratory for studying dynamic processes in many-body systems is strongly coupled dusty plasmas. These systems contain highly charged micrometer-sized particles that interact via a screened Coulomb (Yukawa) potential. Dusty plasmas are highly transparent, and their dynamics is slow with typical time scales of the order of a few ten Hz. Therefore, it is possible to track the motion of individual particles by means of video cameras. Since gravity leads to particle sedimentation into flat dust clouds, the investigation of large three-dimensional systems is preferentially done under weightlessness [11] or by compensating gravity by means of an additional force [12]. Damping and coupling between the particles can be adjusted over a wide range by changing the gas friction and the plasma parameters. By these means it is possible to produce dusty plasmas in crystalline, liquid, or gaseous states.

Wavelike instabilities have been observed in each of these states as propagating dust-density fluctuations [13–16] in one-, two-, and three-dimensional geometry. Waves can either be externally excited in otherwise stable systems by an additional, periodic force [17] or they can be self-excited if internal sources of free energy exist [13]. The latter case is attributed to a Buneman-type instability that arises from streaming ions relative to the dust particles [18]. The natural oscillation frequency of the individual particles is determined by the local plasma parameters and their interaction with neighboring particles. The manipulation of self-excited waves by external drivers is a well-established technique to get insight into the dispersion properties of the wave [19]. Further, the nonlinear interaction between the driver and the system was recently studied in detail [20]. Experiments on nonlinear phenomena concerned nonperiodic structures such as shock waves [21] and solitons [22], as well as wave breaking [23].

It is known from other fields of physics that nonlinear effects can significantly affect the macroscopic behavior of extended systems of coupled oscillators. A prominent example is the formation of so-called synchronization clusters or frequency plateaus, i.e., spatial regions in which the individual oscillators adjust to one collective frequency [24]. In such systems, distinct clusters of different oscillation frequencies may occur. This situation is called partial rather then global synchronization. This effect is well known from numerical calculations of chaotic Rössler oscillators [25], weakly nonlinear Ginzburg-Landau systems [26], Van der Pol oscillators [26,27], and the Luo-Rudy model for cardiac cells [28], when the natural frequencies of the individual oscillators have a spatial variation. Nevertheless, experimental observations of spatial frequency clustering are rare, especially in higher dimensions. In this Letter, we investigate self-excited dust-density waves (DDWs) in a nonuniform plasma background in order to study such nonlinear synchronization phenomena.

The experiments were performed in the IMPF-K2 parallel-plate radio-frequency (rf) reactor on parabolic flights under microgravity conditions. A vertical section through the device is shown in Fig. 1. It is identical to the chamber described in Ref. [15], except for a simplified electrode configuration. The discharge was operated with peak-to-peak voltages of $U_{\mathrm{rf}} = (40\text{–}70)\ \mathrm{V_{pp}}$ at 13.56 MHz in push-pull mode. Spherical monodisperse melamine-formaldehyde particles of $(9.55 \pm 0.13)\ \mu\mathrm{m}$ diameter were injected into an argon plasma. Self-excited dust-density waves emerge spontaneously in large dust clouds at neutral gas pressures below a critical value of $p_{\mathrm{crit}} \approx 30$ Pa.

A video microscope was used for the observation of the particle dynamics. A vertical sheet of light from a low-power laser diode illuminates a thin slice of the dust cloud

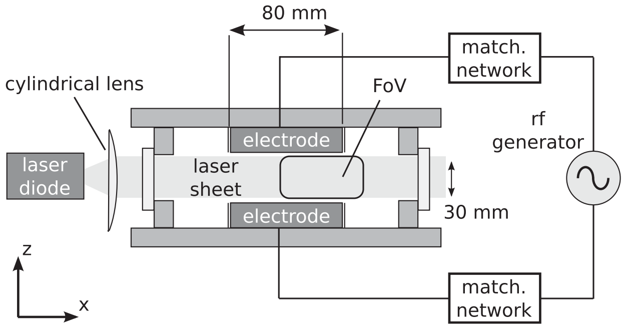

FIG. 1. Side view of the IMPF-K2 chamber. Particles are illuminated by laser diode. The camera's field of view (FoV) is marked with a rectangle. Both electrodes are fed by the same rf generator operating in push-pull mode via two matching networks.

inside the plasma. The scattered light of the particles is observed at a right angle by a camera operating at a frame rate of 100 frames per second. Aliasing effects can be excluded, since typical frequencies of the DDWs are below 20 Hz. The observed field of view has a spatial resolution of 400×910 pixels, which corresponds to an area of $(21.6 \times 47.3)\ \mathrm{mm}^2$ in the plane of the laser sheet.

A typical snapshot of a dust cloud at $U_{\mathrm{rf}} = 48\ \mathrm{V_{pp}}$ and $p = 15$ Pa is shown in Fig. 2(a). Obviously, the dust density is periodically modulated by self-excited DDWs, which propagate from the boundary of the central dust-free region (void) outwards with a phase velocity of $v_{\mathrm{ph}} = (10\text{–}30)\ \mathrm{mm\,s^{-1}}$. Although, the waves are driven by ions flowing from the center of the discharge outwards, the propagation direction of the wave is not necessarily parallel to the ion flow [15,16]. A first indication for a nonlinear character of the DDWs can be obtained directly from the snapshot: The wave has a strongly nonsinusoidal shape with very narrow crests and broad troughs, which contain a significantly reduced number of dust particles. The wavelength is not constant over the entire dust cloud. It varies from $\lambda_{\mathrm{DDW}} \approx 1.5$ mm close to the void to $\lambda_{\mathrm{DDW}} \approx 3$ mm in the outer regions of the cloud. Wave fronts are not strictly parallel, but they may split or merge. Such spots are known as topological defects, which are not fixed in the wave field, but move at (70–90)% of the local phase velocity of the wave.

The blurred intensity of a video frame yields direct information on the spatial dust-density distribution $n_d(x, z)$ as both quantities are assumed to be proportional to each other. For a detailed analysis of the temporal evolution of general wave parameters, n_d is transformed into its analytic signal $A(x, z, t) = n_d(x, z, t) + i \cdot \hat{n}_d(x, z, t)$ by means of a Hilbert transform, which is commonly used in digital signal processing. It allows us to define instantaneous wave properties such as the phase $\phi(t) = \arctan[\hat{n}_d(t)/n_d(t)]$ and the instantaneous frequency via $f_i = \partial\phi/\partial t$. With this method we have evaluated time series with a length of approximately 10 wave periods. In this way, it is possible to map the spatial phase

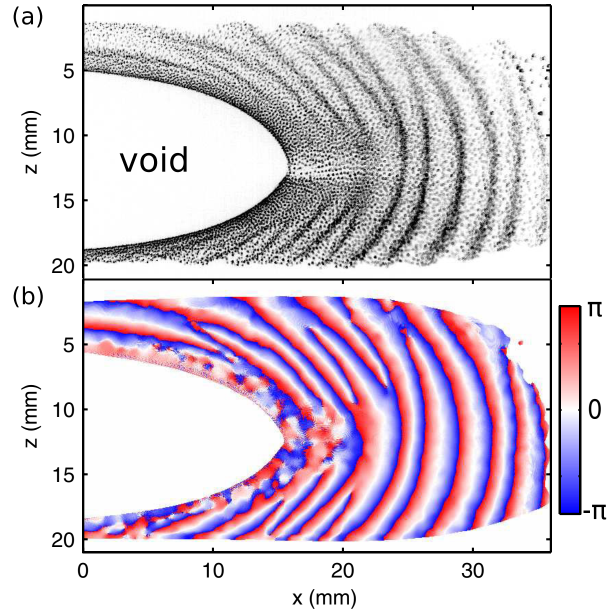

FIG. 2 (color online). (a) Typical image of the wave field in the discharge for a distinct time step. To pronounce the dominant features the image was inverted. Topological defects are merging points of two wave fronts. They develop in the bulk region of the dust cloud. Vertical asymmetries are due to residual gravity. (b) Calculated instantaneous phase for the same time step. Pixels with no wave activity were neglected.

information of the wave for any recorded video frame. Figure 2(b) shows a phase map for the same time step as the snapshot of Fig. 2(a). It becomes evident that the topology of the phase map perfectly matches the amplitude map.

The time derivative of the phase evolution represents the instantaneous frequency. The mean frequency at an arbitrary position in the wave field is defined as the average over the complete time series, $f_m = \langle f_i \rangle$. This spatial information can also be compiled in a map; see Fig. 3(a). One immediately recognizes that the frequency is not constant over the dust cloud but rather decreases from the central void to the discharge edge by a factor of roughly 0.6. Furthermore, the observed decrease is not monotonic. Instead, clusters of nearly the same frequency are found, a key result of this Letter. For the three largest clusters (around positions 1–3) frequencies of $f_1 = 5.7$ Hz, $f_2 = 6.5$ Hz, and $f_3 = 7.6$ Hz were determined, each with a standard deviation of 0.2 Hz.

To give further evidence for the formation of clusters, we have calculated the standard deviation of the frequencies in the immediate vicinity of each pixel coordinate. The resulting topology, displayed in Fig. 3(b), reveals the formation of sharp boundaries whereas only small deviations are found within one cluster. In addition, the positions of the topological defects are also marked in the map. They are obtained from the spatial phase information by calculating the topological charge, which is proportional to the curl of $\nabla\phi$. It is seen that the defects occur exclusively at the boundary between two clusters. The high amount of defects in the vicinity of the void is due to irregular wave motion. Note that the map shows all defects that appear in the evaluated time series. It does not give any information about their spatiotemporal evolution.

The described situation was exemplarily chosen from a set of measurements with nearly identical conditions and a similar clustering behavior. In order to examine the influence of external parameters on the cluster formation, we have inspected a dust cloud at a quite different set of discharge conditions ($p = 15$ Pa, $U_{\mathrm{rf}} = 70$ $\mathrm{V_{pp}}$, $d = 6.8$ μm), which was recorded in an earlier experiment [29]. The wave field had a much higher global coherence. The corresponding frequency map in Fig. 3(c) shows a large plateau of the dominant frequency with only small embedded clusters of different frequencies, and the number of defects is reduced.

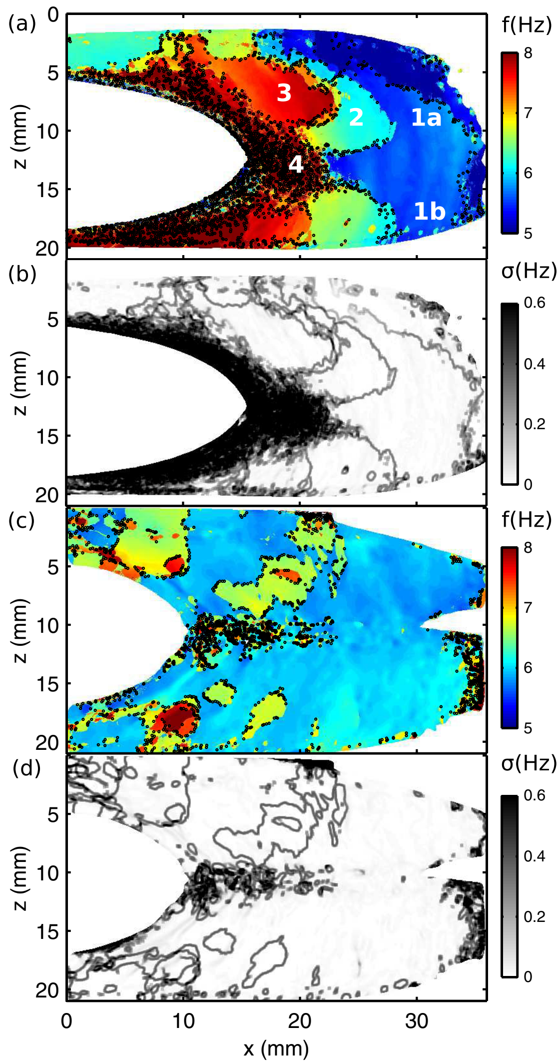

FIG. 3 (color online). (a),(c) Spatial frequency distribution for the same field of view as in Fig. 2 at two different sets of experimental parameters. Multiple regions of nearly constant frequencies arise. The topological defects (black dots) encircle single frequency clusters. (b),(d) Standard deviation of the frequency distribution in a 5×5 pixel vicinity of each spot. Pixels with no wave activity were removed from all maps.

A second way of studying the frequency clusters is focused on the relative phase evolution inside a single cluster or between different clusters, respectively. This method was introduced in Ref. [30] to analyze the interaction of nonlinear, noisy oscillators with potential frequency mismatches. The generalized phase difference (GPD) $\psi_{nm}(t) = n\phi_1(t) - m\phi_2(t)$ of two unwrapped phase series is calculated, where n:m is the ratio of their corresponding frequencies. This quantity should fulfill two conditions if synchronization is given: First, the GPD should be bounded, i.e., $|\psi_{nm}| <$ const. And second, the distribution of ψ_{nm} modulo 2π should be peaked. We have tested both conditions for four different situations: within one cluster (positions $1a/1b$), between clusters 1/2 and 2/3, and between $1a$ and position 4, which is located in a defect region. In order to get higher significance, we tested two groups of 16 pixels each, both located around the given positions.

It is found that within the selected cluster (n:$m = 1$) the GPD is bounded and its distribution is nearly Gaussian; see Fig. 4(a) and 4(b). For neighboring clusters the smallest rational frequency ratios within the measurement uncertainty are n:$m = 5$:6 [Figs. 4(c) and 4(d)] and n:$m = 4$:5 [Figs. 4(e) and 4(f)], respectively. The figures give no clear

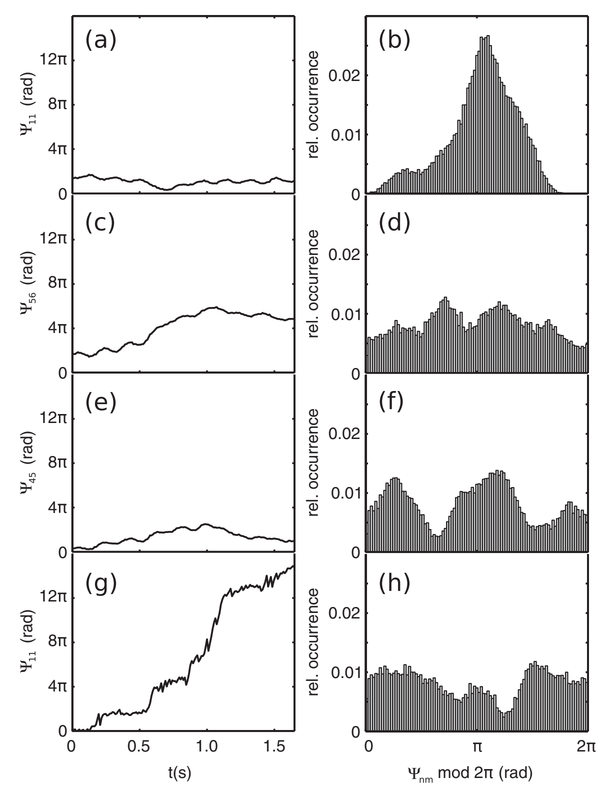

FIG. 4. Generalized phase difference ψ_{nm} mod2π and histograms of ψ_{nm} mod2π between positions $1a/1b$ (a),(b), between cluster 1/2 (c),(d) and 2/3 (e),(f), and for positions $1a$ and 4 (g), (h).

hint for synchronization. The GPDs behave similar to the unsynchronized situation of Figs. 4(g) and 4(h), where $n{:}m = 1$ was chosen. The temporal evolution is not bounded and the histogram is not unimodal but rather flat.

We explain the observed formation of spatial frequency clustering as follows: The high growth rate of the Buneman instability ensures that the fastest growing wave mode determines the local oscillation frequency in a small domain. Consequently, our dusty plasma can be considered as an ensemble of individual self-sustained oscillators with nonlinear damping. Such a system was often treated as Van der Pol-like, which is a well-known model for instabilities in plasma physics [9,31]. In our case the natural frequencies of the oscillators are defined by the local plasma parameters; i.e., they depend on the dust density n_d, particle charge q_d, and particle mass m_d, respectively. Because of nonuniform plasma conditions inside the discharge and a nonuniform dust distribution, that decreases from the void edge to the outer regions of the discharge, a gradient of natural frequencies is established in the ensemble.

Unlike linear waves excited by an external driver, which oscillate at the unique frequency of the driver, the present system can be understood as a multitude of individual oscillators which represent the most unstable wave mode at each position.

The situation in our experiment has similarities with the one-dimensional system described by Osipov and Sushchik [26]. It needs to be mentioned that the model in [26] assumes a diffusive, i.e., symmetric, coupling between the oscillators. In the present system, there is a unique wave propagation direction which may break this symmetry. Also, typical variations within one cluster and between different clusters are similar to those observed in Ref. [32]. The question of whether the system reaches a global synchronization or separates into clusters of different frequencies depends on the distribution of the natural oscillator frequencies, the coupling strength, and the degree of nonlinearity. The size of a frequency cluster increases with coupling strength and degree of nonlinearity and decreases for stronger frequency gradients. This is also true for our experiments. In the earlier experiment [29], the gradient of the particle density that determines the natural frequency within the system was approximately 0.7 times lower compared to the exposed experimental situation. In addition, the measured modulation depth, representing the degree of nonlinearity, was 1.4 times higher. Both trends would result in an increasing cluster size. The synchronization of the individual oscillators is not only limited to an adjustment of their frequencies but also of their phases.

In summary, we have observed the frequency clustering phenomenon in an extended field of self-excited waves. This behavior is known from systems of coupled oscillators [25,26], which also showed the formation of wave patterns [27,28]. Our experiments confirm that the degree of wave nonlinearity determines the cluster size. In future experiments this phenomenon will be studied for a large parameter set to explore this relationship in more detail and to identify the role of topological defects for frequency clustering.

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[1] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Science* (Cambridge University Press, Cambridge, England, 2001).
[2] C. Hugenii, *Horologium Oscillatorium* (Parisiis, France, 1673).
[3] S. H. Strogatz and I. Stewart, Sci. Am. **269**, 102 (1993).
[4] L. Glass, Nature (London) **410**, 277 (2001).
[5] C. Schäfer *et al.*, Nature (London) **392**, 239 (1998).
[6] C. Wilke, H. Deutsch, and R. W. Leven, Contrib. Plasma Phys. **30**, 659 (1990).
[7] M. E. Koepke and D. M. Hartley, Phys. Rev. A **44**, 6877 (1991).
[8] R. Timm and A. Piel, Contrib. Plasma Phys. **32**, 599 (1992).
[9] D. Block *et al.*, Phys. Rev. E **63**, 056401 (2001).
[10] M. E. Koepke *et al.*, Phys. Plasmas **3**, 4421 (1996).
[11] G. E. Morfill *et al.*, Phys. Rev. Lett. **83**, 1598 (1999).
[12] H. Rothermel *et al.*, Phys. Rev. Lett. **89**, 175001 (2002).
[13] A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[14] J. B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).
[15] A. Piel *et al.*, Phys. Rev. Lett. **97**, 205009 (2006); **99**, 209903 (2007).
[16] A. Piel *et al.*, Phys. Rev. E **77**, 026407 (2008).
[17] S. A. Khrapak *et al.*, Phys. Plasmas **10**, 1 (2003).
[18] M. Rosenberg, J. Vac. Sci. Technol. A **14**, 631 (1996).
[19] C. Thompson *et al.*, Phys. Plasmas **4**, 2331 (1997).
[20] I. Pilch, T. Reichstein, and A. Piel, Phys. Plasmas **16**, 123709 (2009).
[21] J. Heinrich, S.-H. Kim, and R. L. Merlino, Phys. Rev. Lett. **103**, 115002 (2009).
[22] R. Heidemann *et al.*, Phys. Rev. Lett. **102**, 135002 (2009).
[23] L.-W. Teng *et al.*, Phys. Rev. Lett. **103**, 245005 (2009).
[24] G. B. Ermentrout and N. Kopell, SIAM J. Math. Anal. **15**, 215 (1984).
[25] G. V. Osipov *et al.*, Phys. Rev. E **55**, 2353 (1997).
[26] G. V. Osipov and M. M. Sushchik, Phys. Rev. E **58**, 7198 (1998).
[27] A. K. Kryukov *et al.*, Phys. Rev. E **79**, 046209 (2009).
[28] O. I. Kanakov *et al.*, Chaos **17**, 015111 (2007).
[29] K. O. Menzel *et al.*, IEEE Trans. Plasma Sci. **38**, 838 (2010).
[30] P. Tass *et al.*, Phys. Rev. Lett. **81**, 3291 (1998).
[31] T. Klinger *et al.*, Phys. Lett. A **182**, 312 (1993).
[32] B. R. Noack, F. Ohle, and H. Eckelmann, J. Fluid Mech. **227**, 293 (1991).