# Nonperturbative Effects on Seven-Brane Yukawa Couplings

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We analyze nonperturbative corrections to the superpotential of seven-brane gauge theories on type IIB and $F$-theory warped Calabi-Yau compactifications. We show, in particular, that such corrections modify the holomorphic Yukawa couplings by an exponentially suppressed contribution, generically solving the Yukawa rank-one problem of certain $F$-theory local models. We provide explicit expressions for the nonperturbative correction to the seven-brane superpotential, and check that it is related to a non-commutative deformation to the tree-level superpotential via a Seiberg-Witten map.

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Within string theory, reproducing the standard model of particle physics (SM) or extensions thereof has proven to be a complex and challenging quest. This complexity is partly due to the different appearance of string vacua in diverse corners of the string landscape, providing not one but many possible paths to reproduce the SM as an effective theory. Rather than a drawback, this diversity of scenarios and the web of dualities relating them can be used to render the quest less challenging. Nevertheless, reproducing the qualitative and quantitative features of the SM still remains a nontrivial task.

A good example of the latter is given by the observed hierarchical fermion masses and mixing angles, which any realistic string model should reproduce via an appropriate set of Yukawa couplings. While in each corner of the string landscape the nature and characteristics of Yukawa couplings are quite different, in practice none of the scenarios built so far provides a scheme where a viable set of Yukawa couplings can be derived in a simple, natural way.

In this regard, an interesting arena where such a scheme could be developed is the local $F$-theory scenario recently introduced in [1], which realizes the idea of grand unification from a bottom-up approach. Indeed, as proposed in [2] (see also [3–5]), an appealing class of models would be those whose holomorphic Yukawa matrix has rank one, so that just one family of quarks and leptons develops a mass [6]. While this would be a promising starting point to reproduce the mass hierarchy between the third and first two families of SM fermions, in realistic models the lightest two families need to be massive as well. One then needs to find a source of Yukawa couplings for these two families, which should then provide a small correction to the rank-one piece. While such corrections were initially thought to be built-in within local $F$-theory constructions, it has been shown in [4] not to be the case, and so in order to solve this problem the tree-level seven-brane superpotential $W^{\rm tree}$ of [1] should be modified by external effects.

The aim of this note is to show that nonperturbative effects can address this rank-one Yukawa problem in the spirit of [2] in a rather natural way, by simply adding to $W^{\rm tree}$ a nonperturbatively generated contribution $W^{\rm np}$. In addition, we will argue that nonperturbative effects are the most important source of corrections to the holomorphic Yukawa couplings, at least in the context where the local $F$-theory models above were initially formulated.

The source of nonperturbative effects modifying the Yukawa couplings will be nothing but the $F$-theory analogues of type II D-brane instantons, whose effect on $4d$ effective theories has recently generated a lot of activity [8]. Indeed, that such a mechanism could work was proposed in [9], in the rather different context of the intersecting D6-brane models built in [10] and sharing the same rank-one problem. While this initial proposal does not seem to work for such D6-brane models, we will see that, when applied to $F$-theory grand unification theories, it provides a universal modification $W^{\rm tree} \to W^{\rm tree} + W^{\rm np}$. Moreover, as the actual expression for $W^{\rm np}$ turns out to be rather simple, this allows the computation of its effect in an explicit way, granting the necessary predictive power to the present proposal.

In order to motivate our expression for $W^{\rm np}$ let us first consider type IIB string theory on the warped background $\mathbb{R}^{1,3} \times_\omega \mathcal{M}_6$, with O3 and O7 planes, as in [11,12]. In addition, let us consider $n$ space-time filling D7 branes wrapping a four-cycle $\Sigma_4^{\rm np} \subset \mathcal{M}_6$, and such that their $4d$ effective field theory develops a gaugino condensate. If we now consider a D3-brane filling $\mathbb{R}^{1,3}$ and placed at a point $z_{\rm D3}$ (in complex coordinates) on $\mathcal{M}_6$, it will develop a superpotential of the form [13–15]

$$W^{\rm np}_{\rm D3} = \mu_3 \mathcal{A} e^{-T_\Sigma/n} f(z_{\rm D3})^{1/n}, \tag{1}$$

where $T_\Sigma = V_{\Sigma_4^{\rm np}} + i\int_{\Sigma_4^{\rm np}} C_4$ is the complexified Kähler modulus corresponding to the four-cycle $\Sigma_4^{\rm np}$, and $f$ stands for a holomorphic section of the divisor bundle specifying $\Sigma_4^{\rm np}$, with no poles and such that it vanishes on $\Sigma_4^{\rm np}$. In addition, $\mu_p = (2\pi)^{-p}\alpha'^{-(p+1)/2}$ is the tension of a D$p$-brane and $\mathcal{A}$ is a holomorphic function of the complex structure moduli of $\mathcal{M}_6$. This function can actually be considered a constant if supersymmetric three-form fluxes

 

$G_3 = F_3 + \tau H_3$ are introduced, since they lift the complex structure moduli of $\mathcal{M}_6$ via a closed string superpotential [16,17]. Note, however, that no extra superpotential is generated for the D3-brane in the presence of these fluxes, and so the result (1) remains unaffected. Alternatively, instead of a gaugino condensing D7-brane we may consider an isolated Euclidean D3-brane instanton wrapping the same four-cycle $\Sigma_4^{\rm np}$, provided that it contains the appropriate number of fermionic zero modes [18]. The superpotential generated by such an instanton is again given by Eq. (1), now with $n = 1$. For simplicity, in the following we will focus on this latter possibility.

In order to understand the superpotential (1) in terms of the $4d$ D3-brane gauge theory, we need to Taylor expand $f(z)$ around the D3-brane location $z_{\rm D3}$, and then express such expansion as D3-brane complex fields $\phi^i_{\rm D3} = \lambda(z^i - z_{\rm D3})$, with $\lambda = 2\pi\alpha'$. We thus obtain

$$W_{\rm D3}^{\rm np} = \mu_3 \mathcal{A} e^{-T_\Sigma}(f|_{z_{\rm D3}} + \lambda\phi^i[\partial_{z^i} f]_{z_{\rm D3}} + \ldots). \quad (2)$$

The same philosophy applies to a stack of $N$ D3 branes. The superpotential (1) naturally generalizes to

$$W_{\rm D3}^{\rm np} = \mu_3 \mathcal{A} e^{-T_\Sigma}[\det f(Z_{\rm D3})], \quad (3)$$

where $Z_{\rm D3}$ is now made up of $N \times N$ complex matrices. Expanding $Z^i_{\rm D3} = z^i_{\rm D3}\mathbb{1}_N + \lambda\phi^i$, we have

$$W_{\rm D3}^{\rm np} = \mu_3 \mathcal{A} e^{-T_\Sigma} f|^N_{z_{\rm D3}}\{1 + \lambda {\rm Tr}(\phi^i)[\partial_{z^i}\log f]_{z_{\rm D3}} + \ldots\}, \quad (4)$$

where $N$ is the D3-brane charge of the system.

Let us now replace the D3 branes at $z_{\rm D3}$ by a stack of D7 branes on $\mathbb{R}^{1,3} \times \mathcal{S}_4$, where $\mathcal{S}_4$ is a compact, complex four-cycle with no intersection with $\Sigma_4^{\rm np}$, and with a nontrivial world-volume flux $F = dA - \frac{i}{2}[A, A]$ along $\mathcal{S}_4$. Since $\mathcal{S}_4$ and $\Sigma_4^{\rm np}$ are physically separated, we can treat the four-cycle $\mathcal{S}_4$ as a smeared source of D3-brane, whose total charge is given by $N_{\rm D3} = \int_{\mathcal{S}_4}{\rm Tr}(F \wedge F)/8\pi^2 \in \mathbb{N}$. Such a smeared source will backreact on both the Ramond-Ramond (RR) potential $C_4$ and the warp factor, implying that the D3-instanton action (whose real part is the warped volume of $\Sigma_4^{\rm np}$) will depend on the D7-brane moduli. Adapting the analysis of [15], one is led to the conclusion that a D7-brane on $\mathcal{S}_4$ should develop a nonperturbative superpotential of the form

$$W_{\rm D7}^{\rm np} = \mu_3 \mathcal{A} e^{-T_\Sigma}\exp\left[\frac{1}{8\pi^2}\int_{\mathcal{S}_4}{\rm Str}(\log f F \wedge F)\right], \quad (5)$$

where Str indicates the symmetric trace. $W_{\rm D7}^{\rm np}$ is to be added to the tree-level superpotential [19]

$$W_{\rm D7}^{\rm tree} = 2\pi\alpha'\mu_7\int_{\Gamma_5}{\rm Str}\Omega \wedge F, \quad (6)$$

where $\Gamma_5$ is a 5-chain connecting $\mathcal{S}_4$ and a reference four-cycle, and $F$ is a proper extension of the D7-brane world-volume flux on $\Gamma_5$.

The non-Abelian expressions (5) and (6) can be made more precise by expanding them around a holomorphic embedding $\mathcal{S}_4$. Introducing local coordinates $(u, v, w)$ such that $\mathcal{S}_4$ is described by $w = 0$, we can expand $W_{\rm D7}^{\rm np}$ in the complex, non-Abelian field $\phi = 2\pi\alpha' w$. Since $f|_{\mathcal{S}_4}$ is a holomorphic function with no poles and zeros on the compact divisor $\mathcal{S}_4$, it is in fact a complex constant that can be pulled out of the integral, and so the first term of this expansion will be the constant. Then, up to second order terms in $\phi$, the nonperturbative superpotential (5) is given by

$$W_{\rm D7}^{\rm np} = \mu_3 \mathcal{A} e^{-T_\Sigma} f|^{N_{\rm D3}}_{\mathcal{S}_4} + \frac{\mu_3}{8\pi^2}\int_{\mathcal{S}_4}\theta{\rm Str}(\phi F \wedge F) + \ldots \quad (7)$$

with $\theta := \lambda\mathcal{A} e^{-T_\Sigma}(f^{N_{\rm D3}}\partial_w \log f)|_{\mathcal{S}_4}$. Adding up $W_{\rm D7}^{\rm tree}$ and $W_{\rm D7}^{\rm np}$ and neglecting constant contributions, we obtain

$$W_{\rm D7} = \frac{\mu_3}{4\pi^2}\int_{\mathcal{S}_4}\left[(\iota_w\Omega) \wedge {\rm Tr}(\phi F) + \frac{1}{2}\theta{\rm Str}(\phi F^2) + \ldots\right], \quad (8)$$

where we have kept only linear terms in $\phi$ [20].

Besides warping and $C_4$, a D7-brane also sources the dilaton and the RR potential $C_0$. Its backreaction is then more involved than $N_{\rm D3}$ smeared D3 branes, a fact which in principle could complicate the above analysis. However, such extra fields do not enter into the action of an instantonic D3-brane wrapping $\Sigma_4^{\rm np}$, whenever its world-volume flux $F$ vanishes. Hence, this additional backreaction does not change the computation above, and so the nonperturbative superpotential indeed reduces to (5). The same statement applies to $n$ condensing D7 branes with vanishing world-volume flux, for which (5) can be trivially extended.

As advanced, the D7-brane superpotential (8) splits as $W_{\rm D7}^{\rm tree} + W_{\rm D7}^{\rm np}$, the first piece being the superpotential considered in [1] and the second piece a nonperturbative correction. Note that $W_{\rm D7}^{\rm np}$, compared to $W_{\rm D7}^{\rm tree}$, contains an extra factor $\theta$ of dimension (length)$^2$. This compensates the higher dimensional integrand ${\rm Str}(\phi F \wedge F)$ from which we can extract a coupling of up to five fields. Corrections to the tree-level Yukawa couplings then arise from terms involving only three fluctuations, like

$$\int_{\mathcal{S}_4}\theta\epsilon^{i\bar{j}k\bar{l}}{\rm Str}(\phi D_i A_{\bar{j}} D_k A_{\bar{l}}) \quad (9)$$

($D_k = \partial_k + i\langle A_k\rangle\wedge$ not containing any fluctuation), as well as from deformations of the tree-level wave function profile induced by the presence of $W_{\rm D7}^{\rm np}$.

The corrected superpotential (8) admits an interesting interpretation, inspired by some observations made in [21]. There, it was proposed to encode nonperturbative corrections to D-brane superpotentials in terms of deformations of the bulk geometry as seen by D branes. In the case at hand, the correction would be encoded in a $\beta$ deformation of the internal complex structure. Indeed, in the type IIB and $F$-theory backgrounds of [11,12] there is an integrable complex structure specified by the holomorphic (3, 0)-form $\Omega$. A $\beta$ deformation replaces $\Omega$ by the more general pure spinor odd polyform $\mathcal{Z}$ [22,23]

$$\mathcal{Z} = \mathcal{Z}_1 + \mathcal{Z}_3 \quad \text{with} \quad \mathcal{Z}_3 = \Omega, \qquad \mathcal{Z}_1 = \beta\rfloor\Omega, \quad (10)$$

where $(\beta\rfloor\Omega)_k = \frac{1}{2}\beta^{ij}\Omega_{ijk}$. Here $\beta = \frac{1}{2}\beta^{ij}\partial_{z^i} \wedge \partial_{z^j}$ + c.c. is a (2, 0) + (0, 2) real bivector whose (2, 0) component is holomorphic, and defines a Poisson structure. Since integrability imposes that $d\mathcal{Z} = 0$, we can locally write $\mathcal{Z}_3 \equiv \Omega = \partial\chi_2$ and $\mathcal{Z}_1 = \partial\chi_0$. Hence, using the superpotential for Abelian D7 branes derived in [19] we obtain

$$W_{\mathrm{D7}} = 2\pi\alpha'\mu_7 \int_{\mathcal{S}_4} (\pi\alpha'\chi_0 F \wedge F + \chi_2 \wedge F) + \text{const.}, \quad (11)$$

which, appropriately choosing the additional constant, reproduces the Abelian version of (8) by simply taking $\chi_0 = \mathcal{A}e^{-T_\Sigma} f^{N_{\mathrm{D3}}}/N_{\mathrm{D3}}$ [24]. Now, as argued in [25,26], on a D-brane world-volume the effect of a $\beta$ deformation can be seen as a noncommutative deformation of the gauge theory. All this suggests that the nonperturbative correction to the D7-brane superpotential (5) should be equivalent to noncommutative deformation of the tree-level piece, via the standard Seiberg-Witten map [27].

Indeed, to connect our results with those in [25], let us consider the superpotential (11). Recalling that $d\chi_2 = \Omega$ and $d\chi_0 = \beta\rfloor\Omega$, the $F$-flatness conditions read

$$\eta|_{\mathcal{S}_4} \wedge F + \Omega|_{\mathcal{S}_4} = 0$$
$$(\iota_X\eta)|_{\mathcal{S}_4} F^2 + 2(\iota_X\Omega)|_{\mathcal{S}_4} \wedge F = 0 \quad \forall\ X \in TM|_{\mathcal{S}_4}, \quad (12)$$

where $\eta = 2\pi\alpha'\beta\rfloor\Omega$. The first $F$-flatness condition is automatically satisfied in our previous setup, since $\mathcal{S}_4$ was chosen to be a divisor (so that $\Omega|_{\mathcal{S}_4} = 0$) while $\chi_0$ was a constant function on $\mathcal{S}_4$, and thus $\eta|_{\mathcal{S}_4} = 0$. The second condition in (12) can be rewritten in a more explicit way by taking again the local system of coordinates $(u, v, w)$. In this system $\beta\rfloor\Omega|_{\mathcal{S}_4} = 0$ implies $\beta^{vw}|_{\mathcal{S}_4} = \beta^{uw}|_{\mathcal{S}_4} = 0$. Then, defining the bivector $\Theta \equiv 2\pi\alpha'\beta|_{\mathcal{S}_4}$, one can show that the second condition in (12) is equivalent to

$$FI + I^T F = -F(I\Theta + \Theta I^T)F \quad (13)$$

with $I$ the complex structure associated to $\Omega$. As shown in [25], Eq. (13) is nothing but the current-matching condition for a B-brane in the $\beta$-deformed topological theory.

It was shown in [25] that (13) is equivalent to $\hat{F}^{(0,2)} = 0$, $\hat{F}$ the noncommutative field-strength constructed via the Seiberg-Witten (SW) map [27]. We now show that this relation can be extended off shell, deriving from (8) the noncommutative superpotential used in [4,28].

Choosing again local coordinates such that $\Omega = du \wedge dv \wedge dw$, we have that $\Theta = \theta\partial_u \wedge \partial_v$ + c.c. We would then expect to arrive to a noncommutative superpotential of the form (omitting overall dimensionful constant factors)

$$\hat{W}_{\mathrm{D7}} = \int_{\mathcal{S}_4} \mathrm{Tr}(\hat{\varphi} \circledast \hat{F}), \quad (14)$$

where $\hat{\varphi} = \hat{\phi} du \wedge dv$, and $\circledast$ and $\hat{F}$ are noncommutative deformations of the ordinary wedge product and field strength, respectively (see below).

Let us start by assuming that we have a constant $\theta = \theta_0$, as in [25], so that the standard SW map of [27] can be applied. In this case $\circledast$ can be simply obtained from the ordinary wedge product by multiplying the components of forms using the ordinary Moyal $*$ product defined by the bivector $\Theta$, and $\hat{F}_{\alpha\beta} = \partial_\alpha\hat{A}_\beta - \partial_\beta\hat{A}_\alpha - i(\hat{A}_\alpha * \hat{A}_\beta - \hat{A}_\beta * \hat{A}_\alpha)$. We can now apply the non-Abelian SW map

$$\hat{F}_{\alpha\beta} = F_{\alpha\beta} + \Theta^{\gamma\delta}[\{F_{\alpha\gamma}, F_{\delta\beta}\} + \tfrac{1}{2}\{A_\gamma, (D_\delta + \partial_\delta)F_{\alpha\beta}\}] + \mathcal{O}(\theta^2), \quad (15)$$
$$\phi = \phi + \tfrac{1}{2}\Theta^{\alpha\beta}\{A_\alpha, (D_\beta + \partial_\beta)\phi)\} + \mathcal{O}(\theta^2)$$

with $D_\alpha F_{\beta\gamma} = \partial_\alpha F_{\beta\gamma} - i[A_\alpha, F_{\beta\gamma}]$, $D_\alpha\phi = \partial_\alpha\phi - i[A_\alpha, \phi]$, and where the action of the SW map on scalars $\phi$ can be obtained by consistency with $T$ duality. Plugging these definitions into (14), and keeping only terms up to order $\theta_0$ and $\bar{\theta}_0$, we indeed get the superpotential (8), providing the equivalence up to this order.

In order to allow for a nonconstant $\theta$, one can follow the strategy of [4], and choose a holomorphic frame $e_I = \{e_U, e_V\}$ (with $[e_I, e_J] = [e_I, e_{\bar{J}}] = 0$) in which $\Theta = \theta_0 e_U \wedge e_V$ + c.c., with $\theta_0$ again constant (see Appendix B of [4] for further details). Then the extension of the ordinary Moyal product is given by

$$f * g = f e^{(i\theta_0/2)\epsilon^{IJ}(\overleftarrow{e}_I \otimes \overrightarrow{e}_J)} e^{(i\bar{\theta}_0/2)\epsilon^{\bar{I}\bar{J}}(\overleftarrow{e}_{\bar{I}} \otimes \overrightarrow{e}_{\bar{J}})} g \quad (16)$$

and the noncommutative wedge product $\circledast$ by expanding the forms in the coframe $e^I$, $e^{\bar{I}}$ and applying the above $*$ product to the components. For instance, given a (1,0)-form $\alpha = \alpha_I e^I$ and a (0,1)-form $\beta = \beta_{\bar{J}} e^{\bar{J}}$, we have

$$\alpha \circledast \beta = (\alpha_I * \beta_{\bar{J}}) e^I \wedge e^{\bar{J}}. \quad (17)$$

Working in the basis $e_I$, $e_{\bar{I}}$, one can thus extend the above SW map to these cases with nonconstant $\theta$ and show that, up to first order in $\theta$, the noncommutative superpotential (14) is equivalent to (8).

Note that the above noncommutative products $*$ and $\circledast$ (when applied to nonholomorphic functions) do not coincide with the ones introduced in [4], denoted $*_h$ and $\circledast_h$ in the following. The key difference is that $*_h$ and $\circledast_h$ involve only the holomorphic (2, 0) component of $\Theta$, $\Theta^{2,0} = \theta\partial_u \wedge \partial_v$. For instance,

$$f *_h g = f e^{(i\theta_0/2)\epsilon^{IJ}(\overleftarrow{e}_I \otimes \overrightarrow{e}_J)} g. \quad (18)$$

In particular, [4] used the following superpotential

$$\tilde{W}_{\mathrm{D7}} = \int_{\mathcal{S}_4} \mathrm{Tr}(\tilde{\varphi} \circledast_h \tilde{F}) \quad (19)$$

with $\tilde{F}^{0,2} = \bar{\partial}A^{0,1} - \frac{i}{2}[A^{0,1}, A^{0,1}]_{*_h}$, in order to solve the rank-one Yukawa problem. It would thus seem that both deformations of the tree-level superpotential are unrelated. This is however not the case, since (14) and (19) are related by an antiholomorphic SW map. Let us explicitly discuss the case of constant $\bar{\theta} = \bar{\theta}_0$, the general case being analogous by the remarks of the previous paragraph. Then the antiholomorphic SW map, taking $\tilde{F}^{0,2}$ and $\tilde{\phi}$ into $\hat{F}^{0,2}$ and $\hat{\phi}$, is again of the form (15), with the substitutions $\Theta \to \Theta^{0,2} = \bar{\theta}_0 \partial_{\bar{u}} \wedge \partial_{\bar{v}}$, $F \to \tilde{F}^{0,2}$, $\phi \to \tilde{\phi}$ and all products built with $*_h$ on its right-hand side. This map indeed preserves the complexified gauge transformations $\hat{\delta}\hat{A}^{0,1} = \bar{\partial}\hat{\lambda} - i[\hat{A}^{0,1}, \hat{\lambda}]_{\circledast}$ and $\tilde{\delta}\tilde{A}^{0,1} = \bar{\partial}\tilde{\lambda} - i[\tilde{A}^{0,1}, \tilde{\lambda}]_{\circledast_h}$, which are symmetries of (14) and (19) respectively. One can then check that, up to first order in $\bar{\theta}_0$ and to all orders in $\theta_0$, (14) is indeed mapped to (19).

To summarize, we have provided evidence that nonperturbative effects generated by Euclidean D3 branes or gaugino condensing D7 branes produce a simple but interesting correction to the superpotential of D7-brane gauge theories. Moreover, by applying the approach of [15] to magnetized D7 branes, we have derived an explicit, general expression for such corrections. It would however be interesting to check this result by means of a direct conformal field theory computation, along the lines of [9,14].

Even if our discussion was carried in the type IIB context, it can be easily extended to $F$ theory. In particular, it can be applied to $F$-theory grand unification theory models with rank-one Yukawa couplings in order to lift their degeneracies. Note that nonperturbative corrections to the tree-level Yukawa matrix are exponentially suppressed, so they can be treated as a small correction to $Y^{\rm tree}_{ijk}$. Moreover, the simple expression obtained for $W^{\rm np}$ allows us to carry a systematic analysis of the textures that $W^{\rm np}$ may give rise to, a task that we leave for future work.

We have also identified via a SW map this nonperturbative correction with a noncommutative deformation of the initial, tree-level superpotential. We have, in particular, recovered, to first order in $\theta$, the noncommutative deformation considered in [4]. As pointed out there, such deformation generically solves the Yukawa rank-one problem in $F$ theory, in agreement with our expectations. In [4] the source for such noncommutative deformation was advocated to a tree-level effect due to the presence of background 3-form fluxes. This possibility is however excluded for the no-scale $F$-theory flux backgrounds of [12,29] where the models of [1,4] are formulated. Indeed, we have seen that the noncommutative deformation (19) is equivalent to a seven-brane superpotential piece of the form (7), and it is easy to convince oneself that [19]

$$W^{\rm tree}_{\rm D7} \supset \int_{\mathcal{S}_4} {\rm Str}(\chi_0 F \wedge F) \Leftrightarrow W^{\rm tree}_{\rm D3} = \chi_0, \qquad (20)$$

$\chi_0$ being a holomorphic function to be Taylor expanded. In [12,29], $W^{\rm tree}_{\rm D3} = 0$ and D3-brane superpotentials can only be generated at the nonperturbative level [15], so such noncommutative deformation can only have a nonperturbative origin, as we obtain from our setting.

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[1] R. Donagi and M. Wijnholt, arXiv:0802.2969; C. Beasley, J. J. Heckman, and C. Vafa, J. High Energy Phys. 01 (2009) 058; J. High Energy Phys. 01 (2009) 059.
[2] J. J. Heckman and C. Vafa, arXiv:0811.2417.
[3] A. Font and L. E. Ibáñez, J. High Energy Phys. 09 (2009) 036.
[4] S. Cecotti, M. C. N. Cheng, J. J. Heckman, and C. Vafa, arXiv:0910.0477.
[5] J. P. Conlon and E. Palti, arXiv:0910.2413.
[6] The rank-one scenario assumes that down- and up-type Yukawa couplings arise each from a single point of triple intersection of matter curves. Difficulties in building such a setting for the latter have been recently pointed out in [7].
[7] H. Hayashi, T. Kawano, Y. Tsuchiya, and T. Watari, arXiv:0910.2762; C. Cordova, arXiv:0910.2955.
[8] For a review see R. Blumenhagen, M. Cvetič, S. Kachru, and T. Weigand, arXiv:0902.3251.
[9] S. A. Abel and M. D. Goodsell, J. High Energy Phys. 10 (2007) 034.
[10] D. Cremades, L. E. Ibáñez, and F. Marchesano, arXiv:hep-ph/0212048; J. High Energy Phys. 07 (2003) 038.
[11] M. Graña and J. Polchinski, Phys. Rev. D **63**, 026001 (2000).
[12] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002).
[13] O. J. Ganor, Nucl. Phys. **B499**, 55 (1997).
[14] M. Berg, M. Haack, and B. Körs, Phys. Rev. D **71**, 026005 (2005).
[15] D. Baumann *et al.*, J. High Energy Phys. 11 (2006) 031.
[16] S. Gukov, C. Vafa, and E. Witten, Nucl. Phys. **B584**, 69 (2000); **B608**, 477(E) (2001).
[17] K. Dasgupta, G. Rajesh, and S. Sethi, J. High Energy Phys. 08 (1999) 023.
[18] E. Witten, Nucl. Phys. **B474**, 343 (1996).
[19] L. Martucci, J. High Energy Phys. 06 (2006) 033.
[20] Taking global properties of the holomorphic normal bundle into account, $W^{\rm tree}_{\rm D7}$ may also contain quadratic terms on $\phi$ whenever $F \neq 0$. These terms, not visible in our local analysis, can be easily incorporated into Eq. (8).
[21] P. Koerber and L. Martucci, J. High Energy Phys. 08 (2007) 059.
[22] M. Gualtieri, arXiv:math/0401221.
[23] This description of $\beta$ deformations should be understood as a deformation of the full type IIB supergravity background in which IASD 3-form fluxes are turned on, as in [11].
[24] Note that this description breaks down at $\Sigma_4^{\rm np}$, since $\partial\bar{\partial}\ln|\chi_0|^2$ is a $\delta$-function 2-form with support on $\Sigma_4^{\rm np}$.
[25] A. Kapustin, Int. J. Geom. Methods Mod. Phys. **1**, 49 (2004).
[26] V. Pestun, Adv. Theor. Math. Phys. **11**, 399 (2007).
[27] N. Seiberg and E. Witten, J. High Energy Phys. 09 (1999) 032.
[28] This derivation applies not only to the nonperturbative superpotential (8), but to any superpotential of the form (11) arising from general $\beta$-deformed complex spaces.
[29] K. Becker and M. Becker, Nucl. Phys. **B477**, 155 (1996).