Guess Your Neighbor's Input: A Multipartite Nonlocal Game with No Quantum Advantage

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We present a multipartite nonlocal game in which each player must guess the input received by his neighbor. We show that quantum correlations do not perform better than classical ones at this game, for any prior distribution of the inputs. There exist, however, input distributions for which general no-signaling correlations can outperform classical and quantum correlations. Some of the Bell inequalities associated with our construction correspond to facets of the local polytope. Thus our multipartite game identifies parts of the boundary between quantum and postquantum correlations of maximal dimension. These results suggest that quantum correlations might obey a generalization of the usual no-signaling conditions in a multipartite setting.

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In recent years, the study and understanding of quantum nonlocality—the fact that certain quantum correlations violate Bell inequalities [1]—has benefited from a cross-fertilization with information concepts.

On one hand, nonlocality has been identified as a key resource for quantum information processing. It allows, for instance, the reduction of communication complexity [2], and in the device-independent scenario, where one wants to achieve an information task without any assumption on the devices used in the protocol, it can be exploited for secure key distribution [3], state tomography [4], and randomness generation [5].

On the other hand, information concepts have provided a deeper understanding of the nature of quantum nonlocality. It is known, in particular, that the no-signaling principle (no arbitrarily fast communication between remote parties) is compatible with the existence of correlations more nonlocal than those allowed in quantum theory [6,7]. However, recent works have shown that the existence of such stronger-than-quantum correlations would have deep information-theoretic consequences: they would, for instance, collapse communication complexity [8] and allow perfect nonlocal computation [9]. In a related direction, it has been realized that quantum correlations actually obey a strengthened version of no-signaling, the principle of information causality [10].

Up to now, such questions have been almost exclusively considered in the bipartite scenario. Here our aim is to investigate the separation between quantum and no-signaling correlations in a multipartite scenario. For this, we introduce and study a simple multipartite nonlocal game, guess your neighbor's input (GYNI).

In GYNI, N distant players are arranged on a ring and each receive an input bit $x_i \in \{0, 1\}$ (see Fig. 1). The goal is that each participant provides an output bit $a_i \in \{0, 1\}$ equal to its right-hand neighbor's input bit:

$$a_i = x_{i+1} \quad \text{for all } i = 1, \ldots, N, \tag{1}$$

where $x_{N+1} \equiv x_1$. The 2^N possible input strings $\mathbf{x} = (x_1, \ldots, x_N)$ are chosen according to some prior distribution $q(\mathbf{x}) = q(x_1, \ldots, x_N)$, which is known to the parties. The figure of merit of the game is given by the average winning probability

$$\omega = \sum_{\mathbf{x}} q(\mathbf{x}) P(\mathbf{a}_i = \mathbf{x}_{i+1}|\mathbf{x}), \tag{2}$$

where $P(\mathbf{a}_i = \mathbf{x}_{i+1}|\mathbf{x}) = P(a_1 = x_2, \ldots, a_N = x_1|x_1, \ldots, x_N)$ denotes the probability of obtaining the correct outputs (1) when the players have received the input string $\mathbf{x}$. Of course, players are not allowed to communicate after the inputs are distributed. Thus, their performance depends only on the initially agreed-upon strategy and on the shared physical resources.

The GYNI game captures a particular notion of signaling: if the players were able to win with high probability, their output would reveal some information about their neighbor's input. We therefore expect that the nonlocal

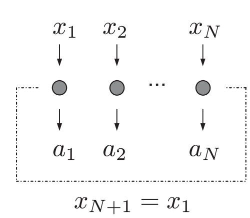

FIG. 1. Representation of the GYNI nonlocal game. The goal is that each party outputs its right-hand neighbor's input: $a_i = x_{i+1}$.

correlations of quantum theory cannot be exploited by noncommunicating observers to perform better at GYNI than using classical resources alone. We confirm this intuition and prove that, indeed, quantum correlations provide no advantage over classical correlations. Surprisingly, however, the no-signaling principle is not at the origin of the quantum limitation: for $N \geq 3$, there exist input distributions q for which no-signaling correlations provide an advantage over the best classical and quantum strategies. This suggests the possibility that in a multipartite scenario, quantum correlations obey a qualitatively stronger version of the usual no-signaling conditions.

Each of the input distributions q associated with a nontrivial no-signaling strategy defines a Bell inequality whose maximal classical and quantum values coincide, but whose no-signaling value is strictly larger. Interestingly, some of these inequalities define facets of the polytope of local correlations. We thus prove the existence of nontrivial facet Bell inequalities with no quantum violation, answering a question raised by Gill [11]. Moreover, since these Bell inequalities are facets, the GYNI game identifies a portion of the boundary of the set of quantum correlations of nonzero measure, in contrast with previous information-theoretic or physical limitations on nonlocality [8–10,12–14].

GYNI with classical and quantum resources.—We start by showing that the optimal classical and quantum winning strategies are identical for any prior distribution q of the inputs. Let us first show that there is a simple classical strategy achieving a winning probability

$$\omega_c = \max_{\mathbf{x}}[q(\mathbf{x}) + q(\bar{\mathbf{x}})], \tag{3}$$

where $\bar{\mathbf{x}}$ denotes the "negation" of the input string $\mathbf{x}$, $\bar{\mathbf{x}} = (\bar{x}_1, \ldots, \bar{x}_N)$ with $\bar{x}_i = x_i \oplus 1$, and $\oplus$ denotes addition modulo 2. This strategy is based on the following simple observation.

Let $\mathbf{y}$ be an arbitrary string. If $\mathbf{x} \neq \mathbf{y}, \bar{\mathbf{y}}$,

there exists an i such that $x_i = y_i$ and $x_{i+1} \neq y_{i+1}$. (4)

Indeed, if this was not the case, we would have that for any i, either $x_i \neq y_i$ or $x_{i+1} = y_{i+1}$. But this would in turn imply that either $\mathbf{x} = \mathbf{y}$ or $\mathbf{x} = \bar{\mathbf{y}}$, in contradiction with the hypothesis.

Consider now a classical strategy specified by the string $\mathbf{y}$, where each party outputs the bit $a_i = y_{i+1}$ if it received the input y_i, and outputs $a_i = \bar{y}_{i+1}$ if it received $\bar{y}_i$. It obviously follows that $P(\mathbf{a}_i = \mathbf{y}_{i+1}|\mathbf{y}) = 1$ and $P(\mathbf{a}_i = \bar{\mathbf{y}}_{i+1}|\bar{\mathbf{y}}) = 1$. On the other hand, $P(\mathbf{a}_i = \mathbf{x}_{i+1}|\mathbf{x}) = 0$ for all $\mathbf{x} \neq \mathbf{y}, \bar{\mathbf{y}}$. Indeed, from observation (4), there exists an i such that $x_i = y_i$, but for which $a_i = y_{i+1} \neq x_{i+1}$. The winning probability of this classical strategy is thus equal to $\omega = q(\mathbf{y}) + q(\bar{\mathbf{y}})$, which yields (3) if we take $\mathbf{y}$ to be $q(\mathbf{y}) + q(\bar{\mathbf{y}}) = \max_{\mathbf{x}}[q(\mathbf{x}) + q(\bar{\mathbf{x}})]$.

We now prove that there is no better quantum (hence classical) strategy. In the most general quantum protocol, the parties share an entangled state $|\psi\rangle$ and perform projective measurements on their subsystem dependent on their inputs x_i. They then output their measurement results a_i. Denoting $M_{a_i}^{x_i}$ the projection operator associated with the output a_i for the input x_i, the probability that the N players produce the correct output is thus given by

$$P(a_1 = x_2, \ldots, a_N = x_1|x_1, \ldots, x_N) = \langle M_{x_2}^{x_1} \otimes \ldots \otimes M_{x_1}^{x_n}\rangle,$$

and the average winning probability is

$$\omega = \sum_{\mathbf{x}} q(\mathbf{x})\langle M_{\mathbf{x}}\rangle, \tag{5}$$

where we have written $M_{\mathbf{x}} = M_{x_2}^{x_1} \otimes \cdots \otimes M_{x_1}^{x_n}$ for short. The operators $M_{\mathbf{x}}$ satisfy the following properties:

$$M_{\mathbf{x}}^2 = M_{\mathbf{x}}, \tag{6}$$

and

$$M_{\mathbf{x}} M_{\mathbf{y}} = 0 \quad \text{if } \mathbf{x} \neq \mathbf{y}, \bar{\mathbf{y}}. \tag{7}$$

The first property follows from the fact that the $M_{\mathbf{x}}$ are projection operators. The second property follows from the orthogonality relations $M_{a_i}^{x_i} M_{\bar{a}_i}^{x_i} = 0$ and observation (4). Note that protocols involving mixed states or general measurements can all be represented in the above form by expanding the dimensionality of the initial state.

We now show, using (6) and (7), that $\omega = \sum_{\mathbf{x}} q(\mathbf{x}) M_{\mathbf{x}} \leq \omega_c$, where $\leq$ should be understood as an operator inequality; i.e., $A \leq B$ means that $\langle A\rangle \leq \langle B\rangle$ for all $|\psi\rangle$. First note that $\sum_{\mathbf{x}} q(\mathbf{x}) M_{\mathbf{x}} \leq \sum_{\mathbf{x}} q'(\mathbf{x}) M_{\mathbf{x}}$, where $q'(\mathbf{x}) = q(\mathbf{x}) + [\omega_c - q(\mathbf{x}) - q(\bar{\mathbf{x}})]/2$, since by definition $\omega_c - q(\mathbf{x}) - q(\bar{\mathbf{x}}) \geq 0$. It is thus sufficient to consider weights q such that $q(\mathbf{x}) + q(\bar{\mathbf{x}}) = \omega_c$ for all $\mathbf{x}$. We can then write

$$\omega_c - \sum_{\mathbf{x}} q(\mathbf{x}) M_{\mathbf{x}} = \Big[\sqrt{\omega_c} - \sum_{\mathbf{x}} \alpha_{\mathbf{x}} M_{\mathbf{x}}\Big]^2 + \frac{1}{2}\sum_{\mathbf{x}}[\beta_{\mathbf{x}} M_{\mathbf{x}} - \beta_{\bar{\mathbf{x}}} M_{\bar{\mathbf{x}}}]^2, \tag{8}$$

where $\alpha_{\mathbf{x}} = \sqrt{\omega_c} - q(\bar{\mathbf{x}})/\sqrt{\omega_c}$ and $\beta_{\mathbf{x}} = \sqrt{q(\mathbf{x}) q(\bar{\mathbf{x}})/\omega_c}$. To verify this identity we only need to use (6) and (7) and the fact that $q(\mathbf{x}) + q(\bar{\mathbf{x}}) = \omega_c$. Note now that the right-hand side of (8) is ≥ 0, since it is a sum of square involving only Hermitian operators. This shows that $\sum_{\mathbf{x}} q(\mathbf{x}) M_{\mathbf{x}} \leq \omega_c$, as announced.

The inequality $\sum_{\mathbf{x}} q(\mathbf{x}) P(\mathbf{a}_i = \mathbf{x}_{i+1}|\mathbf{x}) \leq \omega_c$ can be interpreted as a Bell inequality whose local and quantum bound coincide. It is well known that in order to achieve a Bell violation in quantum theory one must perform measurements corresponding to noncommuting operators. The above proof, however, does not distinguish noncommuting operators from ordinary, commuting numbers: it is based on the algebraic identity (8) which follows only from Eqs. (6) and (7), regardless of whether the $M_{\mathbf{x}}$'s commute or not. This explains why the classical and quantum bounds are identical.

GYNI with no-signaling resources.—At first sight, it may seem that the quantum limitation on the GYNI game arises from the no-signaling principle: if the players were able to win with high probability, their output would somehow depend on their neighbor's input. This motivates us to look at how players constrained only by the no-signaling principle perform at GYNI.

Formally, the no-signaling principle states that the marginal distribution $P(a_{i_1}, \ldots, a_{i_k}|x_{i_1}, \ldots, x_{i_k})$ for any subset $\{i_1, \ldots, i_k\}$ of the n parties should be independent of the measurement settings of the remaining parties [7], i.e., that

$$P(a_{i_1}, \ldots, a_{i_k}|x_1, \ldots, x_N) = P(a_{i_1}, \ldots, a_{i_k}|x_{i_1}, \ldots, x_{i_k}).$$

This guarantees that any subset of the parties is unable to signal to the other by their choice of inputs.

We show in Appendix A [15] that players constrained only by no-signaling have a bounded winning probability $\omega_{\rm ns} \leq 2\omega_c$. They thus cannot win in general with unit probability at GYNI. Furthermore, for certain input distributions, such as the one where all input strings are chosen with equal weight $q(\mathbf{x}) = 1/2^N$, we show as expected that $\omega_{\rm ns} = \omega_c$. That is, for uniform and completely uncorrelated inputs, any resource performing better than a classical strategy is necessarily signaling.

Surprisingly, this property is not general. There exist distributions $q(\mathbf{x})$ for which no-signaling strategies outperform classical and quantum strategies. Consider for instance the following input distribution

$$q(\mathbf{x}) = \begin{cases} 1/2^{N-1} & \text{if } x_1 \oplus \cdots \oplus x_{\hat{N}} = 0 \\ 0 & \text{otherwise}, \end{cases} \tag{9}$$

where $\hat{N} = N$ if N is odd and $\hat{N} = N - 1$ if N is even. It easily follows from the previous analysis that for classical and quantum resources, $\omega_c = 1/2^{N-1}$. We now prove, however, that no-signaling resources can achieve $\omega_{\rm ns} = 4/3\omega_c$. Note that the distribution (9) can be interpreted as a promise that the sum of the inputs (modulo 2) is equal to zero. This prior knowledge does not yield any information to the parties about the value of their neighbor's input, yet it can be exploited by no-signaling correlations to outperform classical strategies.

We start by considering the case $N = 3$, for which

$$\omega = \tfrac{1}{4}[P(000|000) + P(110|011) + P(011|101) + P(101|110)], \tag{10}$$

where $P(000|000) = P(a_1 = 0, a_2 = 0, a_3 = 0|x_1 = 0, x_2 = 0, x_3 = 0)$, and so on. Consider the first three terms in (10). The no-signaling principle implies that

$$\begin{aligned} P(000|000) &\leq \sum_{a_3} P(00a_3|000) = \sum_{a_3} p(00a_3|001), \\ P(110|011) &\leq \sum_{a_2} P(1a_20|011) = \sum_{a_2} p(1a_20|001), \\ P(011|101) &\leq \sum_{a_1} P(a_111|101) = \sum_{a_1} p(a_111|001). \end{aligned} \tag{11}$$

By normalization of probabilities, the sum of the right-hand sides of Eqs. (11) is upper bounded by one, and thus $P(000|000) + P(110|011) + P(011|101) \leq 1$. Similar conditions are obtained for any of the four possible combinations of three terms in Eq. (10). Summing over these possibilities, we find $3[P(000|000) + P(110|011) + P(011|101) + P(101|110)] \leq 4$, or in other words $\omega_{\rm ns} \leq 4/3 \times 1/4 = 4/3\omega_c$. Furthermore, the inequality is saturated only if the four probabilities appearing in (10) are all equal to 1/3. It turns out that the remaining entries of the probability table $P(\mathbf{a}|\mathbf{x}) = P(a_1a_2a_3|x_1x_2x_3)$ can be completed in a way that is compatible with the no-signaling principle; i.e., the bound $\omega_{\rm ns} \leq 4/3\omega_c$ is achievable. Up to relabeling of inputs and outputs, there exist two inequivalent classes of extremal no-signaling correlations achieving this winning probability (see Appendix B in [15]). One of them takes the form $P(\mathbf{a}|\mathbf{x}) = 2/3g(\mathbf{a}, \mathbf{x}) + 1/3g'(\mathbf{a}, \mathbf{x})$, where g and g' are the following Boolean functions

$$\begin{aligned} g(\mathbf{a}, \mathbf{x}) &= a_1a_2a_3(1 \oplus x_1)(1 \oplus x_2)(1 \oplus x_3), \\ g'(\mathbf{a}, \mathbf{x}) &= (1 \oplus a_1)(1 \oplus a_2)(1 \oplus a_3) \oplus x_1a_2a_3 \\ &\quad \oplus a_1x_2a_3 \oplus a_1a_2x_3 \oplus x_1x_2x_3. \end{aligned} \tag{12}$$

From this definition, it is easy to verify that $P(a_1a_2a_3|x_1x_2x_3)$ satisfies the no-signaling conditions and achieves winning probability $\omega_{\rm ns} = 1/3 = 4/3\omega_c$.

The existence of no-signaling correlations achieving $\omega_{\rm ns} = 4/3\omega_c$ in the case $N = 3$ is enough to show that $\omega_{\rm ns} \geq 4/3\omega_c$ for any $N \geq 3$. This can be seen as follows. Consider the situation in which the first three parties use the optimal strategy for $N = 3$ while the remaining parties simply output their input. In this case, all the terms in ω vanish, except the four terms $P(000, 0\ldots0|000, 0\ldots0)$, $P(110, 0\ldots0|011, 0\ldots0)$, $P(011, 1\ldots1|101, 1\ldots1)$, and $P(101, 1\ldots1|110, 1\ldots1)$, which are all equal to 1/3.

Beyond these analytical results, we obtained the maximal no-signaling values of $\omega_{\rm ns}$ up to $N = 7$ players using linear programming. The ratios $\omega_{\rm ns}/\omega_c$ of no-signaling to classical winning probabilities are 4/3 for $N = 3, 4$, 16/11 for $N = 5, 6$, and 64/42 for $N = 7$, showing that for more parties there exist no-signaling correlations that can outperform the optimal no-signaling strategy for $N = 3$. (Note that it can be shown that the winning probability for an odd number N of parties is always equal to the winning probability for $N + 1$ players; see Appendix C in [15].)

GYNI Bell inequalities.—The GYNI Bell inequalities $\sum_{\mathbf{x}} q(\mathbf{x})P(\mathbf{a}_i = \mathbf{x}_{i+1}|\mathbf{x}_i) \leq \omega_c$ are not violated by quantum theory, but can be violated by more general no-signaling theories. In [11], Gill raised the question of whether there exist Bell inequalities which (i) feature this "no quantum advantage" property and (ii) define facets of the polytope of local correlations. Here we give a positive answer to this question. We have checked that the GYNI inequalities defined by the distribution (9) are facet defining for $N \leq 7$ players. More generally, we verified that the inequalities defined by the distribution $q(\mathbf{x})$ having uniform support on

$\oplus_{i=1}^{\hat{N}} x_i = 0$ are facet defining for all $N \leq 7$. We conjecture that they are facet defining for any number of parties. Note also that the polytope of local correlations for the case $N = 3$ (with binary inputs and outputs) was completely characterized in [16]; the inequality corresponding to (10) belongs to the class 10 of [16]. Geometrically, our result shows that the polytope of local correlations and the set of quantum correlations have in common faces of maximal dimension [we recall that a facet corresponds to a $(d-1)$-dimensional face of a d-dimensional polytope].

This also implies that GYNI is an information-theoretic game that identifies a portion of the boundary of quantum correlations which is of nonzero measure. To the best of our knowledge, all previously introduced information-theoretic or physical principles recovering part of the quantum boundary—including nonlocal computation [9], nonlocality swapping [12], information causality [10,13], and macroscopic locality [14]—only single out a portion of zero measure [17].

Discussion and open questions.—Our work raises plenty of new questions. First, it would be interesting to understand the structure of those input distributions q leading to a gap between no-signaling and classical or quantum correlations (see Appendix A in [15] for a class of distributions for which there is no gap). For instance, in the case of four parties, the distribution q having uniform support on $x_1 \oplus x_2 \oplus x_3 \oplus x_1x_2x_3 = 0$ leads to $\omega_{\rm ns} = 4/3\omega_c$. However, the corresponding Bell inequality is not a facet. Another question is thus to single out, among all relevant input distributions, those corresponding to facet Bell inequalities. For three parties, it follows from [16] that the distribution (9) is the unique possibility.

A further interesting problem is whether there exist facet Bell inequalities with no quantum advantage in the bipartite case. Note that our GYNI inequalities are nontrivial only for $N \geq 3$; for the case $N = 2$, the classical and no-signaling bounds are always equal. In Ref. [9], examples of bipartite Bell inequalities with no quantum advantage have been presented in the context of nonlocal computation. However, as mentioned earlier, none of the Bell inequalities associated with nonlocal computation have been proven to be facet defining. We studied this question here and could prove that none of the simplest inequalities from [9] (corresponding to the family of inequalities specified by the parameters $n = 2, 3$ in [9]) are facet inequalities. The proof uses a mapping from these inequalities to the space of correlation inequalities for n parties, two settings and two outcomes, which was fully characterized in Ref. [18]; see Appendix D in [15] for a detailed proof. We conjecture that none of the Bell inequalities introduced in [9] are facet defining.

Coming back to our original motivation, it would be interesting to get a deeper understanding of the structure and information-theoretic properties of the no-signaling correlations giving an advantage over classical or quantum correlations, for instance, those associated with inequality (10). In particular, it would be interesting to understand if they can be exploited for other information tasks. Finally, our results suggest that the quantum limitation on the GYNI game might originate from a generalization of the no-signaling principle in a multipartite setting. Can this intuition be made concrete? Are there more general information tasks with no quantum advantage?

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[1] J. S. Bell, Physics **1**, 195 (1964).
[2] H. Buhrman, R. Cleve, S. Massar, and R. de Wolf, arXiv:0907.3584 [Rev. Mod. Phys. (to be published)].
[3] A. Acin, N. Brunner, N. Gisin, S. Massar, S. Pironio, and V. Scarani, Phys. Rev. Lett. **98**, 230501 (2007).
[4] C.-E. Bardyn, T. C. H. Liew, S. Massar, M. McKague, and V. Scarani, Phys. Rev. A **80**, 062327 (2009).
[5] S. Pironio *et al.*, Nature (London) **464**, 1021 (2010).
[6] S. Popescu and R. Rohrlich, Found. Phys. **24**, 379 (1994).
[7] J. Barrett, N. Linden, S. Massar, S. Pironio, S. Popescu, and D. Roberts, Phys. Rev. A **71**, 022101 (2005).
[8] W. van Dam, arXiv:quant-ph/0501159v1; G. Brassard, H. Buhrman, N. Linden, A. A. Méthot, A. Tapp, and F. Unger, Phys. Rev. Lett. **96**, 250401 (2006); N. Brunner and P. Skrzypczyk, Phys. Rev. Lett. **102**, 160403 (2009).
[9] N. Linden, S. Popescu, A. J. Short, and A. Winter, Phys. Rev. Lett. **99**, 180502 (2007).
[10] M. Pawlowski, T. Paterek, D. Kaszlikowski, V. Scarani, A. Winter, and M. Zukowski, Nature (London) **461**, 1101 (2009).
[11] See Problem 26.b at http://www.imaph.tu-bs.de/qi/problems/26.html.
[12] P. Skrzypczyk, N. Brunner, and S. Popescu, Phys. Rev. Lett. **102**, 110402 (2009).
[13] J. Allcock, N. Brunner, M. Pawlowski, and V. Scarani, Phys. Rev. A **80**, 040103(R) (2009).
[14] M. Navascues and H. Wunderlich, Proc. R. Soc. A **466**, 881 (2010).
[15] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.104.230404 for detailed proofs.
[16] C. Sliwa, Phys. Lett. A **317**, 165 (2003).
[17] Note that Ref. [14] can actually recover the entire quantum boundary in the correlator space.
[18] R. F. Werner and M. M. Wolf, Phys. Rev. A **64**, 032112 (2001); M. Zukowski and C. Brukner, Phys. Rev. Lett. **88**, 210401 (2002).