Normal Phase of an Imbalanced Fermi Gas

Christophe Mora[1] and Frédéric Chevy[2]

[1]Laboratoire Pierre-Aigrain, École Normale Supérieure, CNRS and Université Paris 7 Diderot, 24 rue Lhomond, 75005 Paris, France
[2]Laboratoire Kastler-Brossel, École Normale Supérieure, CNRS and UPMC, 24 rue Lhomond, 75005 Paris, France

(Received 28 February 2010; revised manuscript received 17 April 2010; published 8 June 2010)

Recent experiments on imbalanced Fermi gases have raised interest in the physics of an impurity immersed in a Fermi sea, the so-called Fermi polaron. In this Letter, a simple theory is devised to describe dilute Fermi-polaron ensembles corresponding to the normal phase of an imbalanced Fermi gas. An exact formula is obtained for the dominant interaction between polarons, expressed solely in terms of a single-polaron parameter. The physics of this interaction is identified as a signature of the Pauli exclusion principle.

DOI: 10.1103/PhysRevLett.104.230402 PACS numbers: 05.30.Fk, 03.75.Ss, 34.50.−s, 67.85.−d

Quasiparticles are generic emergent properties of many-body systems that simplify the description of complex interacting ensembles of particles. This concept is probably one of the most important in quantum physics since it lies at the foundation of fields as diverse as chemistry of dilute solutions, dressed atom theory in atomic physics, and band theory in solid state physics. Recently, experiments on spin-imbalanced ultracold Fermi gases [1–3] have highlighted, once again, its importance by showing that the main features of the phase diagram of these systems could be understood quantitatively from the properties of an impurity immersed in a Fermi sea of spin-polarized atoms, the Fermi polaron [4–7]. It was shown, in particular, that the quasiparticle arising from the interaction between the impurity and the surrounding Fermi gas could be described with great accuracy by assuming that a single-particle-hole pair is excited [8,9]. The single-particle properties of the Fermi polaron have been characterized experimentally and theoretically and are now well understood. For instance, at unitarity, where the scattering length between the majority and minority spins is infinite, the chemical potential of the impurity is shifted by $\mu_p = A\mu_1$, where $A = -0.61$ and μ_1 is the chemical potential of the majority [4,5,9,10]. Similarly, the effective mass is found to be close to the bare mass m, with $m^* = 1.20$ m for recent experiments [3,11,12], close to the theoretical values obtained from variational or Monte Carlo calculations [4,6,8,9].

More generally, the Fermi polaron is a good description of an impurity immersed in a Fermi sea around unitarity and of "attractive" ($a < 0$) interactions where A and m^* have also been calculated with great accuracy [6,8,13]. Interestingly, the dressed impurity undergoes a transition from a fermionic polaron to a bosonic molecule at $1/k_{F1}a \sim 0.9$ [6,10,14–17], where $k_{F1} = (6\pi^2 n_1)$ is the Fermi wave vector of the majority species gas of density n_1. This transition modifies the collective behavior of an ensemble of impurities. In particular, in the fermionic sector $1/k_{F1}a < 0.9$, pioneering fixed node Monte Carlo simulations have shown that, for a small concentration of minority fermions, the equation of state of an imbalanced normal Fermi gas with two spin species denoted $\sigma = 1, 2$ and densities n_σ could be fitted by a Landau-Pomeranchuk law,

$$E = E_{\rm FG1}\left(1 + \frac{5A}{3}x + \frac{m}{m^*}x^{5/3} + Fx^2\right), \tag{1}$$

where $x = n_2/n_1$, $E_{\rm FG1}$ is the energy of a single-component (majority) Fermi gas with density n_1, and F describes interactions between polarons [4,18]. A, m^*, and F are functions of $1/k_{F1}a$. This Fermi liquid picture is supported by the absence of vortices in rotation experiments indicating a normal state [1]. By contrast, it was noted recently that experimental data could be fitted with great accuracy by a grand-canonical equation of state

$$P = \frac{1}{15\pi^2}\left[\left(\frac{2m}{\hbar^2}\right)^{3/2}\mu_1^{5/2} + \left(\frac{2m^*}{\hbar^2}\right)^{3/2}(\mu_2 - \mu_p)^{5/2}\right], \tag{2}$$

which apparently describes a mixture of two ideal Fermi gases of polarons and majority atoms [3,19]. However, the presence of a μ_1 dependence of μ_p in the polaron part of the equation of state implies a coupling between the two gases, and the two equations of state can be reconciled by noting that, expressed in the canonical ensemble, Eq. (2) indeed yields Eq. (1) with $F = 5A^2/9 \sim 0.2$ at unitarity, close to the Monte Carlo value $F \sim 0.14$ [4].

In this Letter, we show that the equation of state of the normal phase follows the phenomenological expansion (1). Moreover, we argue that the relationship between F and A is exact and can be generalized to the full Bose-Einstein condensate–Bardeen-Cooper-Schrieffer (BCS) crossover: Indeed, we will show that the parameter F is solely a function of the single-polaron chemical potential and is given by

$$F = \frac{5}{9}\left(\frac{d\mu_p}{dE_{F1}}\right)^2, \tag{3}$$

where μ_p is computed in the low impurity concentration limit where $\mu_1 = E_{F1} = \hbar^2 k_{F1}^2/2m$. Finally, the study of the BCS regime corresponding to small and negative values of a allows us to clarify the origin of the x^2 term in Eq. (1). We attribute it to a modification of the single-polaron properties due the Pauli blocking, created by the presence of the minority Fermi sea and overruling density-mediated polaronic interactions [20] which contribute to the higher-order $x^{7/3}$.

The starting point of our demonstration is the celebrated Luttinger sum rule, stating that if a many-body fermionic system can be analytically connected to an ideal Fermi gas [21], then it possesses a Fermi surface where the momentum distribution is discontinuous and which encloses a volume depending only on density [22,23]. More quantitatively, the Fermi surface is given by the wave-vector $\boldsymbol{k}_{F\sigma}$ solutions of the equation,

$$\xi_{k_F,\sigma} + \Sigma_\sigma(\omega = 0, \boldsymbol{k}_{F\sigma}; \mu_1, \mu_2) = 0, \tag{4}$$

where $\xi_{k\sigma} = \hbar^2 k^2/2m - \mu_\sigma$ and Σ_σ is the self-energy of spin σ particles. By definition, the single-polaron chemical potential μ_p is obtained for vanishingly small impurity densities n_2, and is thus the solution of the equation [8]

$$\mu_p = \Sigma_2(\omega = 0, \boldsymbol{k} = 0; \mu_1, \mu_2 = \mu_p), \tag{5}$$

and depends only on μ_1. Since we consider the situation of dilute polarons corresponding to a small concentration of impurities, the minority Fermi sea remains small and μ_2 can be expanded in the vicinity of μ_p. Let us assume for the moment that Σ_2 is analytic in μ_2 and $\boldsymbol{k}_{F2}$, or k_{F2}^2 by rotational invariance. Expanding Eq. (5) up to fourth order, we thus get [24]

$$\delta\mu_2 = \varepsilon_{k_{F2}} + k_{F2}^2 \frac{\partial \Sigma_2}{\partial k^2} + \delta\mu_2 \frac{\partial \Sigma_2}{\partial \mu_2} + \frac{k_{F2}^4}{2} \frac{\partial^2 \Sigma_2}{\partial k^4} + \delta\mu_2 k_F^2 \frac{\partial^2 \Sigma_2}{\partial \mu_2 \partial k^2} + \frac{\delta\mu_2^2}{2} \frac{\partial \Sigma_2}{\partial \mu_2^2} + \ldots, \tag{6}$$

with $\delta\mu_2 = \mu_2 - \mu_p$. The equation of state of the dilute impurity gas is obtained from the leading-order terms, i.e., the first three terms in Eq. (6),

$$\mu_2 = \mu_p + \frac{\hbar^2 k_{F2}^2}{2m^*}, \tag{7}$$

where

$$\frac{m^*}{m} = \frac{1 + 2m\partial_{k^2}\Sigma_2/\hbar^2}{1 - \partial_{\mu_2}\Sigma_2} \tag{8}$$

is the usual definition for the effective mass of a quasiparticle. Using the Luttinger sum rule we know that $k_{F2} = (6\pi^2 n_2)^{1/3}$, and combined with the Gibbs-Duhem relation $\partial_{\mu_i} P = n_i$, Eq. (7) leads to a pressure $P(\mu_i)$ identical to Eq. (2). To convert this equation of state in the canonical ensemble, we use the relationship

$$\mu_1 = E_{F1}\left(1 + x\frac{d\mu_p}{d\mu_1}\right)^{2/3}, \tag{9}$$

$$\mu_2 = \mu_p + E_{F2}, \tag{10}$$

where $E_{F2} = \hbar^2 k_{F2}^2/2m^*$ and we have neglected higher-order terms in n_2 that appear when taking the derivative of m^* with μ_1. Making use of the definition of the grand potential $-PV = E - \sum_i \mu_i N_i$, we finally get the Landau-Pomeranchuk law (1), with F given by (3).

We now verify that Eq. (3) is not altered when higher orders in Eq. (6) are included. Indeed, replacing $\delta\mu_2$ by its leading-order expression, terms neglected in Eq. (6) give rise to a k_{F2}^4 contribution to $\delta\mu_2(k_{F2})$. From the Gibbs-Duhem relation this gives rise to a term $\propto (\mu_2 - \mu_p)^{7/2}$ in Eq. (2), hence a $x^{7/3}$ contribution to the energy. For vanishing x, this term is therefore negligible against x^2 and does not contribute to the value of F; this argument proves that, provided analyticity conditions are fulfilled, Eq. (3) gives the *exact* value of F.

We now provide evidence for the analyticity of Σ_2. To do so, we make use of a time-ordered diagrammatic expansion of the self-energy illustrated in Fig. 1 [25]. For each diagram, a line going forward (backward) in time is associated with a $\theta(\xi_{\boldsymbol{k},\sigma})$ $[\theta(-\xi_{\boldsymbol{k},\sigma})]$, with θ the Heaviside step function, and contributes $\xi_{\boldsymbol{k},\sigma}$ to the energy denominator. In addition, the incoming (outgoing) minority line contributes to ω $(-\omega)$. The main point of the argument is the negativity of μ_p, and of μ_2 for a small impurity concentration. Indeed, in this case, $\xi_{\boldsymbol{k},2}$ is always positive, which implies that the Heaviside functions associated with impurities traveling backward in time vanish. As a consequence, diagrams containing an impurity loop do not contribute to the self-energy, and similarly, the inner part of the "main" impurity line cannot travel back in time. The denominators are therefore always strictly positive, and this absence of pole guarantees the analyticity of Σ_2. This can be interpreted physically by noting that the minority Fermi sea would be empty at these negative chemical potentials for a vanishing interaction. The creation of minority fermions is therefore only triggered by interaction processes with the majority component.

The above ideas are best illustrated by going to the BCS weak coupling limit $a \to 0^-$, where exact perturbative calculations can be performed. The gas of fermions with two spin species is described by the Hamiltonian

$$H = \sum_{\boldsymbol{k},\sigma} \varepsilon_{\boldsymbol{k}} c_{\boldsymbol{k},\sigma}^\dagger c_{\boldsymbol{k},\sigma} + \frac{g}{\mathcal{V}} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} c_{\boldsymbol{k}+\boldsymbol{q},1}^\dagger c_{\boldsymbol{k}'-\boldsymbol{q},2}^\dagger c_{\boldsymbol{k}',2} c_{\boldsymbol{k},1}, \tag{11}$$

where $\varepsilon_{\boldsymbol{k}} = \hbar^2 k^2/2m$, $\mathcal{V}$ is a quantization volume, and $c_{\boldsymbol{k},\sigma}$ annihilates a fermion of spin σ and momentum $\boldsymbol{k}$. The

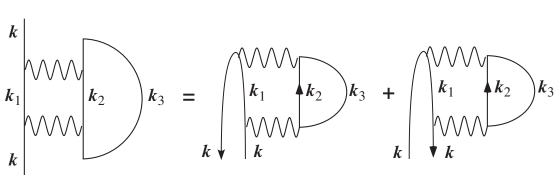

FIG. 1. Diagrammatic representation of second-order perturbation theory. Integration over frequencies allows one to decompose the leftmost diagram into two time-ordered diagrams. Since μ_2 is negative, inner minority lines traveling backward in time are forbidden, and the rightmost term vanishes.

zero-range interaction potential in Eq. (11) suffers from ultraviolet divergences that are cured by imposing a cutoff k_c in momentum space. The Lippmann-Schwinger formula then relates the bare coupling constant g to the scattering length,

$$\frac{1}{g} = \frac{m}{4\pi\hbar^2 a} - \frac{1}{\mathcal{V}}\sum_{\boldsymbol{k}} \frac{1}{2\varepsilon_{\boldsymbol{k}}}. \tag{12}$$

Building on the Luttinger equation (4) relating μ_2 and k_{F2} for the minority fermions, we wish to determine the equation of state $P(\mu_i)$ in the strongly imbalanced case with $\mu_1 > 0$ and $\mu_2 < 0$. The self-energy Σ_2 is calculated perturbatively in powers of g. In addition, Eq. (12) is used to expand the resulting expressions again in powers of a. The renormalizability of the model (11) imposes that ultraviolet divergences cancel out for each order in a, and the cutoff k_c is eventually taken to infinity.

The first order is given by the usual Hartree diagram, $\Sigma_2^{(1)}(\omega, \boldsymbol{q}) = (g/6\pi^2)(2m\mu_1/\hbar^2)^{3/2}$. We write the second order using the time-ordered diagrams displayed in Fig. 1,

$$\begin{aligned}\Sigma_2^{(2)}(\omega, \boldsymbol{q}) = {} & \frac{g^2}{\mathcal{V}^2}\sum_{\boldsymbol{k},\boldsymbol{q}'} \frac{\theta(\xi_{\boldsymbol{k},1})\theta(\xi_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k},2})\theta(-\xi_{\boldsymbol{q}',1})}{\omega - (\xi_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k},2} + \xi_{\boldsymbol{k},1} - \xi_{\boldsymbol{q}',1})} \\ & + \frac{g^2}{\mathcal{V}^2}\sum_{\boldsymbol{k},\boldsymbol{q}'} \frac{\theta(-\xi_{\boldsymbol{k},1})\theta(-\xi_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k},2})\theta(\xi_{\boldsymbol{q}',1})}{\omega - (\xi_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k},2} + \xi_{\boldsymbol{k},1} - \xi_{\boldsymbol{q}',1})},\end{aligned} \tag{13}$$

where the minority travels partially backward in time in the second term and always forward in the first one. As stated earlier, the negative minority chemical potential implies that $\xi_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k},2}$ is positive, and the second term of Eq. (13) thus vanishes in accordance with our general rule that backward travel is suppressed. Moreover, for the remaining first term in Eq. (13), the denominator does not vanish as long as $\omega < -\mu_2$, and the self-energy can be freely expanded with respect to μ_2 and $\boldsymbol{q}$ at $\omega = 0$.

Using the complete self-energy $\Sigma_2^{(1)} + \Sigma_2^{(2)}$, it is possible to calculate μ_p with the result

$$\mu_p = \frac{2a}{3\pi\hbar m}(2m\mu_1)^{3/2} + \frac{a^2}{\pi^2\hbar^2 m}(2m\mu_1)^2. \tag{14}$$

Using Eq. (3), we see that, including up to third order, the interaction parameter F should read

$$F = \frac{20}{9}\left(\frac{k_{F1}a}{\pi}\right)^2\left(1 + \frac{k_{F1}a}{\pi}\right) + \ldots. \tag{15}$$

It is illuminating to check the weak coupling prediction (15) for the interaction by a direct calculation of the ground state energy using the standard Rayleigh-Schrödinger perturbation theory. We first discuss the energy of a single polaron $E_{\rm pol}(\boldsymbol{q})$. The unperturbed state is then an impurity with momentum $\boldsymbol{q}$ immersed in a Fermi sea of majority atoms. The first-order correction to the energy is the mean-field correction gn_1, while the next-order correction involves the excitation of particle-hole pairs out of the majority Fermi sea. By definition of $E_{\rm pol}(\boldsymbol{q})$, the energy of the system is given by $E = E_{\rm FG1} + E_{\rm pol}(\boldsymbol{q})$ with

$$E_{\rm pol}(\boldsymbol{q}) = \frac{\hbar^2 q^2}{2m} + gn_1 + \frac{g^2}{\mathcal{V}^2}\sum_{\boldsymbol{k}',\boldsymbol{q}'} \frac{1}{\varepsilon_{\boldsymbol{q}'} + \varepsilon_{\boldsymbol{q}} - \varepsilon_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k}'} - \varepsilon_{\boldsymbol{k}'}}, \tag{16}$$

where the majority momenta $\boldsymbol{q}'$ and $\boldsymbol{k}'$ satisfy the conditions $q < k_{F1}$ (i) and $k > k_{F1}$ (ii) imposed by the Pauli exclusion principle.

We switch now to an ensemble of impurities, in which case two ideal Fermi gases with Fermi wave vectors k_{F1} and k_{F2} constitute the unperturbed ground state with energy $E_{\rm FG,1} + E_{\rm FG,2}$. The energy takes the form $E(n_1, n_2) = E_{\rm FG,1} + \tilde{E}$,

$$\begin{aligned}\tilde{E} = {} & E_{\rm FG,2} + \mathcal{V} g n_1 n_2 \\ & + \frac{g^2}{\mathcal{V}^2}\sum_{\boldsymbol{k}',\boldsymbol{q}',\boldsymbol{q}} \frac{1}{\varepsilon_{\boldsymbol{q}'} + \varepsilon_{\boldsymbol{q}} - \varepsilon_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k}'} - \varepsilon_{\boldsymbol{k}'}},\end{aligned} \tag{17}$$

with the previous restrictions (i) and (ii) complemented by $q < k_{F2}$ (iii) and $|\boldsymbol{q} + \boldsymbol{q}' - \boldsymbol{k}'| > k_{F2}$ (iv), where the last two conditions are imposed by the Pauli exclusion principle in the presence of the minority Fermi seas. Except for the constraint (iv), $\tilde{E}$ would simply be $\sum_{q<k_{F2}} E_{\rm pol}(\boldsymbol{q})$, which constitutes the energy of an ideal gas of polarons with a dispersion relation $E_{\rm pol}(\boldsymbol{q})$. However, we can recover this term explicitly by expressing (iv) in terms of its complementary domain (v) $|\boldsymbol{q} + \boldsymbol{q}' - \boldsymbol{k}| < k_{F2}$, in which case we can recast Eq. (17) as

$$\tilde{E} = \sum_{q<k_{F2}} E_{\rm pol}(\boldsymbol{q}) - \frac{g^2}{\mathcal{V}^2}\sum_{\substack{(i),(ii)\\(iii),(v)}} \frac{1}{\varepsilon_{\boldsymbol{q}'} + \varepsilon_{\boldsymbol{q}} - \varepsilon_{\boldsymbol{q}+\boldsymbol{q}'-\boldsymbol{k}'} - \varepsilon_{\boldsymbol{k}'}}. \tag{18}$$

The first term in Eq. (18) corresponds to an ideal gas of polarons and contributes to the x and $x^{5/3}$ scaling terms in Eq. (1), that is, to A and m^*. The second term describes the effect of Pauli blocking due to the minority Fermi sea on the formation of the polaron. A careful analysis of its behavior for low k_{F2} shows that it scales as x^2 and thus gives the effective interaction F between polarons.

The complete calculation of third-order corrections is lengthy but straightforward. In the limit $x \ll 1$, one finds again Eq. (1) for the ground state energy, together with an interaction parameter F arising again from Pauli blocking and identical to Eq. (15).

The argument presented above makes a strong case for a x^2 interaction between polarons. However, noticing that an s-wave interaction gives a x^2 scaling and a p-wave interaction gives a subleading $x^{7/3}$, this may seem to contradict the fermionic nature of polarons. On the other hand, Fermi liquid theory does not forbid alike particles to interact, and the corresponding interaction is in fact not necessarily short-ranged. This paradox can be solved by noting that polarons have fermionic statistics at large distances and are

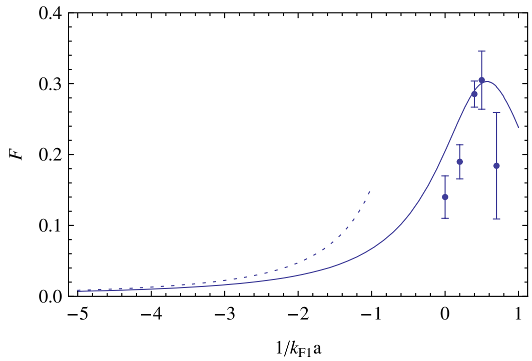

FIG. 2 (color online). Variations of F in the crossover and comparison with Monte Carlo simulations. Solid line: Eq. (3) with μ_p calculated variationally using [8]. Dotted line: third-order diagrammatic expansion [Eq. (15)]. Circles: Monte Carlo simulations [18].

composite objects at shorter distances. From this structure, they acquire an internal energy $\mu_p = A\mu_1$. This single-polaron energy is held fixed in the grand-canonical ensemble and is not modified by the presence of other impurities. By contrast, the internal energy depends on the minority concentration in the canonical ensemble through Pauli blocking, which yields the x^2 interaction in Eq. (1). Based on these arguments, it is probably not surprising to find that F is solely a function of the internal energy as given by Eq. (3).

Finally, in Fig. 2, we compare our prediction, Eq. (3), where μ_p is calculated using the variational scheme presented in [8], with the third-order expansion, Eq. (15), as well as Monte Carlo data [18]. As expected, we observe that the perturbative expansion and the nonperturbative result coincide for $a \to 0^-$. In the strongly interacting limit we observe that our result follows the same trend as the Monte Carlo simulation, with, in particular, the presence of a maximum of F close to $1/k_{F1}a \sim 0.5$.

In conclusion, we have demonstrated that in the low impurity concentration, the canonical equation of state of a spin-imbalanced system could be described by a Landau-Pomeranchuk energy. Quite surprisingly, we have shown that the interaction parameter F is related to single impurity properties. Several extensions of this Letter are worth exploring. From experimental data, it appears that Eq. (2) is valid in a wide range of impurity concentrations (up to $x = 0.5$ at unitarity). This surprisingly large validity domain remains to be understood by investigating higher orders or by making use of nonperturbative schemes. In fact, assuming further analyticity, the low density expansion performed here can, in principle, be extended to any order in x. The coefficients of the expansion are then expressed solely in terms of the single-polaron self-energy. Other open issues include the extension of our results to the one-dimensional situation [26,27] and to the case of repulsive interactions [28].

We acknowledge R. Combescot, S. Giraud, S. Giorgini, C. Lobo, S. Nascimbène, N. Navon, and A. Recati for stimulating discussions, and we thank G. Bertaina for providing us with the Monte Carlo data. F. C. acknowledges support from the EU (ERC Research Grant FERLODIM), Région Ile de France (IFRAF), and Institut Universitaire de France.

[1] M. Zwierlein, A. Schirotzek, C. Schunck, and W. Ketterle, Science **311**, 492 (2006).
[2] G. Partridge, W. Li, R. Kamar, Y. Liao, and R. Hulet, Science **311**, 503 (2006).
[3] S. Nascimbène, N. Navon, K. Jiang, F. Chevy, and C. Salomon, Nature (London) **463**, 1057 (2010).
[4] C. Lobo, A. Recati, S. Giorgini, and S. Stringari, Phys. Rev. Lett. **97**, 200403 (2006).
[5] F. Chevy, Phys. Rev. A **74**, 063628 (2006).
[6] N. Prokof'ev and B. Svistunov, Phys. Rev. B **77**, 020408 (R) (2008).
[7] A. Bulgac and M. McNeil Forbes, Phys. Rev. A **75**, 031605(R) (2007).
[8] R. Combescot, A. Recati, C. Lobo, and F. Chevy, Phys. Rev. Lett. **98**, 180402 (2007).
[9] R. Combescot and S. Giraud, Phys. Rev. Lett. **101**, 050404 (2008).
[10] A. Schirotzek, C.-H. Wu, A. Sommer, and M. W. Zwierlein, Phys. Rev. Lett. **102**, 230402 (2009).
[11] Y. Shin, Phys. Rev. A **77**, 041603(R) (2008).
[12] S. Nascimbene, N. Navon, K. J. Jiang, L. Tarruell, M. Teichmann, J. McKeever, F. Chevy, and C. Salomon, Phys. Rev. Lett. **103**, 170402 (2009).
[13] S. Pilati and S. Giorgini, Phys. Rev. Lett. **100**, 030401 (2008).
[14] M. Punk, P. T. Dumitrescu, and W. Zwerger, Phys. Rev. A **80**, 053605 (2009).
[15] C. Mora and F. Chevy, Phys. Rev. A **80**, 033607 (2009).
[16] R. Combescot, S. Giraud, and X. Leyronas, Europhys. Lett. **88**, 60007 (2009).
[17] F. Alzetto and X. Leyronas, Phys. Rev. A **81**, 043604 (2010).
[18] G. Bertaina and S. Giorgini, Phys. Rev. A **79**, 013616 (2009).
[19] N. Navon, S. Nascimbene, F. Chevy, and C. Salomon, Science Express 328, 729 (2010).
[20] L. Viverit, C. J. Pethick, and H. Smith, Phys. Rev. A **61**, 053605 (2000).
[21] We assume a not-too-small temperature such that pairing and its nonanalyticities can be neglected.
[22] J. Luttinger and J. Ward, Phys. Rev. **118**, 1417 (1960).
[23] S. Sachdev and K. Yang, Phys. Rev. B **73**, 174504 (2006).
[24] By analogy with the ideal Fermi gas, and anticipating the result of this Letter, we make the assumption that $\mu_2 - \mu_p \propto k_{F2}^2$.
[25] J. Luttinger, Phys. Rev. **121**, 942 (1961).
[26] G. Orso, Phys. Rev. Lett. **98**, 070402 (2007).
[27] H. Hu, X.-J. Liu, and P. D. Drummond, Phys. Rev. Lett. **98**, 070403 (2007).
[28] S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, arXiv:1004.1169.