PHYSICAL REVIEW LETTERS

Precision Physics of Simple Atoms and Constraints on a Light Boson with Ultraweak Coupling

S. G. Karshenboim*

D. I. Mendeleev Institute for Metrology, St. Petersburg, 190005, Russia
and Max-Planck-Institut für Quantenoptik, Garching, 85748, Germany

Constraint on spin-dependent and spin-independent Yukawa potential at atomic scale is developed. That covers constraints on a coupling constant of an additional photon γ^* and a pseudovector boson. The mass range considered is from 1 eV/c^2 to 1 MeV/c^2. The strongest constraint on a coupling constant α' is at the level of a few parts in 10^{13} (for γ^*) and below one part in 10^{16} (for a pseudovector) corresponding to mass below 1 keV/c^2. The constraints are derived from low-energy tests of quantum electrodynamics and are based on spectroscopic data on light hydrogenlike atoms and experiments with magnetic moments of leptons and light nuclei.

Introduction.—Precision physics of simple atoms is a field, which provides us with values of various fundamental constants with high accuracy [1] and enables us to perform low-energy tests of quantum electrodynamics (QED) [2]. Indeed, it is important to verify specific theoretical calculations, which are quite advanced and sophisticated [3], although we hardly have any doubts about QED as a universal framework to describe kinematics of photons, theory of electromagnetic interactions of pointlike particles, and phenomenology of electromagnetic interactions of structured objects, such as hadrons.

In the meantime, various unification theories suggest new particles, which have not yet been observed (see, e.g., [4,5]). One class of such particles deals with light (electrically) neutral particles with ultraweak coupling to conventional matter. Stable neutral particles of this kind are also a candidate for the dark matter [6]. This means that, while energetically such particles are reachable in existing particle-physics experiments, their ultraweak coupling makes their production and detection rate so low that their observation is in reality impossible.

The atomic physics allows one, on the contrary, very accurate measurements, and thus certain constraints within a keV/c^2 range of mass of an intermediate boson are in reach. Below we consider two basic options. At first, we constrain a kind of an additional photon γ^*, which interacts universally with all charged particles. The other kind of particles that we study is a pseudovector boson, which induces a spin-dependent interaction between atomic constituents.

Dealing with the effects at an atomic scale, it is more advantageous to apply the coordinate-space consideration and to constrain a certain long-range interaction in the form of the Yukawa-potential correction to the Coulomb law

$$-\frac{\alpha}{r} \rightarrow -\frac{\alpha_{\rm eff}(r)}{r} = -\frac{\alpha + \alpha' e^{-\lambda r}}{r}, \qquad (1)$$

$$-\frac{\alpha}{r} \rightarrow -\frac{\alpha + \alpha''(\mathbf{s}_1 \cdot \mathbf{s}_2)e^{-\lambda r}}{r}, \qquad (2)$$

which is multiplied by the nuclear charge Z if necessary. Here, the first line describes γ^*, while the second line is for an axial boson and **s** stands for the spin of a particle.

A requirement to constrain the Yukawa terms in a certain range of λ is quite simple: it is necessary to compare two experiments, which have different sensitivity to the correcting term. Here we compare experiments, which involve different distances. While in Secs. and we deal with distance scales, different by orders of magnitude, in the consideration in Sec. the compared distances are relatively close, but the sensitivity is enhanced.

Constraining an extra photon γ^*.—The most intensively studied atomic scale is at a few values of the Bohr radius, $a_0 \simeq 0.53 \times 10^{-10}$ m. For the Yukawa radius, equal to a_0, the related mass λ of the intermediate particle is 3.5 keV. (We apply here relativistic units, in which $\hbar = c = 1$.)

The fundamental constants related to this scale are

$$R_\infty = 10\,973\,731.568\,527(73) \text{ m}^{-1}, \qquad (3)$$

$$\alpha^{-1} = 137.035\,999\,59(53). \qquad (4)$$

Before applying both values to constrain α', we briefly explain their origin. Once we suggest a substitution (1), both results become related to $\alpha_{\rm eff}(r)$ at a certain effective value $r_{\rm eff} \sim (1 - 4)a_0$.

The value (3) is from the evaluation [1] of experimental and theoretical results, where a statistically dominant contribution involves data on the $1s$ and $2s$ states [7–9], while the Yukawa correction (1) to other excited levels also involved, is of marginal importance.

The value (4) is obtained by combining (3) with values of h/M for caesium [10] and rubidium [11] and with various results on auxiliary data (see, [1] for details).

In the meantime, certain results related to other mass-distance scales are also available. For longer distances one

can apply the value [12]

$$R_\infty = 10\,973\,731.568\,34(69) \text{ m}^{-1} \qquad (5)$$

obtained from a transition between the circular states (i.e., states where $l = n - 1$) with $n = 27$–30 in the hydrogen atom and thus related to distances of about $10^3 a_0$. The uncertainty above is tripled against its original value [12] because the result is a preliminary one. Nevertheless, as it was confirmed by authors of the experiment [13], that is rather an overconservative estimation. We have to remark that while this uncompleted experiment is of rather marginal importance in determination of the Rydberg constant (see, e.g., [1]), it is crucial to constrain various exotic effects at distances of 10^{-7}–10^{-6} m. There is no other compatible experimental data at this scale.

The fine-structure constant

$$\alpha_{g-2}^{-1} = 137.035\,999\,084(51), \qquad (6)$$

derived from the anomalous magnetic moment of an electron by combining the experimental result [14] with theory [15], represents physics of shorter distances, comparable with the Compton wavelength of the electron $\sim \lambdabar_C = a_0/\alpha = 1/m_e$. (Indeed, instead of a substitution (1), for the evaluation of the related correction one has to apply a complete propagator of γ^*.)

Comparison of $\alpha(a_0)$ with results at other scales delivers us constraints on α'. The results for asymptotic areas (where the strong inequalities on λ hold) are summarized in Table I, while the constraints in Fig. 1 are also applicable for an intermediate area $\lambda \sim 1$ keV, as well as, on the edges of the considered region (~ 1 eV and ~ 1 MeV). We also note that to saturate limits for the region, defined by a strong inequality, it is sufficient to have λ larger or smaller than the related value by a factor of 3–5.

To interpret constraints in Fig. 1, we note that the a constraint is related to a correction (1) for the Coulomb interaction of an electron and proton, while the b constraint is from a comparison of an electron-proton long-range interaction at atomic scale and electron-electron interaction at scale of $\lambdabar_C$.

TABLE I. The constraint on the deviation of the effective long-range interaction $\alpha(r)/r$ from the Coulomb exchange due to possible presence of γ^*. Here, $\alpha(r) = \alpha(\infty) + \alpha' \exp(-\lambda r)$ and r is a characteristic distance to be compared with a_0. The related distance range is $\lambda^{-1} = 0.5 \times 10^{-7}$ m (for $\lambda = 4$ eV), 0.5×10^{-10} m (for $\lambda = 4$ keV), and 0.4×10^{-12} m (for $\lambda = 0.5$ MeV). The values given in the lines identified by a and b correspond to the curves in Fig. 1.

	Mass range	r	$\alpha(r) - \alpha(a_0)$
a	4 eV $\ll \lambda \ll$ 1 keV	$10^3 a_0$	$(0.6 \pm 2.3) \times 10^{-13}$
b	4 keV $\ll \lambda \ll$ 0.5 MeV	$\lambdabar_C$	$(2.7 \pm 2.9) \times 10^{-11}$

In principle, one could have in mind not γ^*, but a light intermediate particle, which interacts in a different way with various charged particles. It is possible to proceed further for this case and this option will be explored in details elsewhere.

The constraints can be also applied for a nonvector intermediate particle coupled both to protons and electrons. A potential, which has a static spin-independent interaction (i.e., the Coulomb-like component), satisfies (1) and is directly applicable in atomic calculations for R_∞ at distances of both important scales, a_0 and $10^3 a_0$. That is related to true scalar, vector, tensor, etc., intermediate particles and the constraint a in Table I and Fig. 1 stands for them. On the contrary, pseudoscalars, vectors, etc., produce various spin-dependent potentials, that do not affect any determination of R_∞.

In the meantime, a calculation of corrections to $g_e - 2$ with all kinds of intermediate particles, coupled to electrons, produces results of the same order of magnitude. Suggesting that a coupling of the boson to electrons and to protons is the same, we can consider the b constraint as a rough estimation for true scalars. For pseudoscalars and vectors, this rough estimation is now for a pure boson-electron coupling and has an interval of applicability, extended to longer distances.

Constraining a pseudovector boson from the 1s hfs interval.—Until very recently, it was the 1s hyperfine structure (hfs) interval in the hydrogen atom that was the most accurately measured quantity in general and the most accurately measured quantity related to a simple atom, in particular. However, its application to fundamental problems used to be limited because of uncertainties related to the proton structure. In this section, in particular, we derive from the hydrogen hfs a constraint on a pseudoscalar particle, which is much stronger than a constraint on γ^*. Still a constraint from muonium hfs, which is free of problems with the nuclear structure, is even stronger.
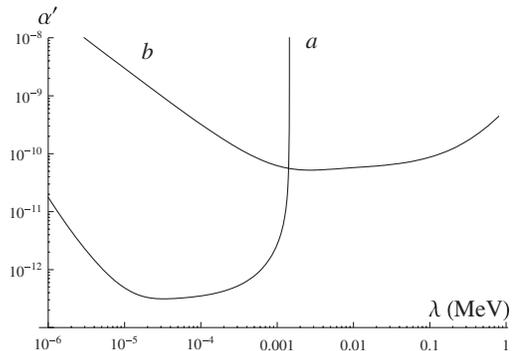
FIG. 1. Constraints on a long-range spin-independent interaction from hydrogen spectroscopy and $g_e - 2$, including a constraint from hydrogen spectroscopy with low and high ($n \simeq 30$) Rydberg states (curve a), a comparison of the low states and $g_e - 2$ (curve b). The lines are for the upper bound on $|\alpha'|$ and the confidence level corresponds to 1 standard deviation.

The hfs in two-body atoms opens a possibility to test a spin-dependent long-range interaction and to look for the Yukawa term in (2). The ground state hfs interval is known with high accuracy for a few light hydrogenlike atoms (see [2] for details and references).

A constraint here is based on the fact that the leading contribution to the hfs interval can be obtained from the known value of the nuclear magnetic moment. The latter is determined for a number of nuclei from experiments on behavior of a bound nuclear magnetic moment at a macroscopic magnetic field (see, [1] for details and references). In principle, only some of these measurements are sensitive to the Yukawa term in (2), and the involved macroscopic distances vary in a broad range. In summary, we can definitely conclude that for the Yukawa radius below, say, $l_0 = 1$ cm, the results for magnetic moments are not affected by the correcting term.

In the meantime, for $a_0 \ll \lambda^{-1} \ll l_0$ the hfs interval in muonium and hydrogen is shifted by $(-\alpha''/\alpha)Z^2 R_\infty$. The strongest constraint

$$\alpha'' = (1.6 \pm 6.0) \times 10^{-16} \qquad (7)$$

is derived from muonium physics [16], while the results, involving other light two-body atoms, such as $\alpha''(H) = \pm 1.6 \times 10^{-15}$ and $\alpha''(D) = \pm 8 \times 10^{-15}$, are somewhat weaker and, in fact, less reliable because of uncertainties in theoretical understanding of the nuclear contributions (see, e.g., [2,17]).

The constraints, obtained from all available precision data on the 1s hfs in light atoms are summarized in Fig. 2. They are extended there to shorter distances ($\lambda^{-1} < a_0$).

The constraints also include the one from positronium experiments, obtained in a different approach, through comparison of spin-dependent (hfs interval) and spin-independent (determination of e and m_e) physics.

Constraining a pseudo vector by comparing the 1s and 2s hfs intervals.—In summary, the constraints from the 1s hfs are limited either by the uncertainty of calculations of nuclear effects (for atoms with hadronic nuclei) or by experimental accuracy in leptonic atoms. Since both uncertainties in fractional units are larger than experimental uncertainty of the 1s hfs interval in hydrogen and some other atoms, the high accuracy of measuring the hfs intervals is not completely utilized and progress is possible.

As was shown in [17], the specific difference

$$D_{21} = 8E_{\text{hfs}}(2s) - E_{\text{hfs}}(1s), \qquad (8)$$

is essentially free of such a problem. The uncertainty of the nuclear effects is very much reduced and the accuracy of the 2s interval, being lower than for the 1s, is still higher than in experiments with unstable leptonic atoms, such as muonium and positronium.

To get use of the difference, accurate experimental data on the 2s hfs interval are required. Data with appropriate accuracy are available for few atoms: for hydrogen [18],
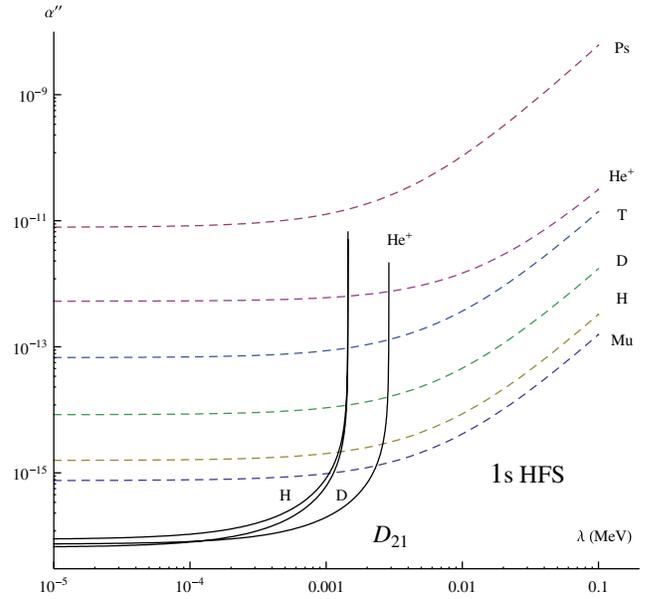
FIG. 2 (color online). Constraint on a long-range spin-dependent interaction from hfs intervals of two-body atoms. The dashed lines are for the upper bound on $|\alpha''|$ derived from the 1s hfs data for muonium, positronium, hydrogen, deuterium, tritium, and ^{3}He ion. The solid lines present constraints from data on D_{21} in hydrogen, deuterium, and ^{3}He ion. The confidence level corresponds to 1 standard deviation.

deuterium [19], and ^{3}He ion [20]. The related theory is reviewed in [21].

The Yukawa correction in (2) generates a correction for the hfs interval of the ns state, which is proportional to n^{-2} for $\lambda^{-1} \gg a_0$ and thus does not vanish in (8). This allows us to set a constraint on the spin-dependent Yukawa term (see, Table II and Fig. 2).

Comparing the hfs constraints in Fig. 2, we note that the D_{21} constraints are stronger for $\lambda \le 1$ keV, while the 1s hfs constraint for α'' is stronger in the case of a heavier mass range.

Summary.—In conclusion, we have demonstrated that precision physics of simple atoms is a powerful tool to constrain vector and pseudovector particles, coupled to leptons and nuclei, with a preferred mass range of keV/c^2. The latter in many cases can be extended into the 1-MeV domain. The stability of the intermediate par-

TABLE II. Comparison of experiment and theory for the D_{21} value in light hydrogenlike atoms. The constraint on α' is related to $\lambda \ll 1$ keV.

Atom	Experiment (kHz)	Theory (kHz)	α''
H	48.923(54)	48.953(3)	$(3.3 \pm 5.9) \times 10^{-17}$
D	11.280(56)	11.3125(5)	$(2.4 \pm 4.1) \times 10^{-17}$
^{3}He$^+$	$-1189.979(71)$	$-1190.08(15)$	$(-2.8 \pm 4.6) \times 10^{-17}$

ticle is not required and it is sufficient for our consideration that the width is substantially smaller than the mass ($\tau^{-1} \ll \lambda$).

The width of the boson, τ^{-1}, can be, in particular, induced by the interaction with the charged particles, which is constrained in this paper. However, that is not the only possible mechanism. On the contrary, it may happen that there is an interaction of the intermediate boson with the photons, which induces the interaction with charged particles. (The situation with π^0 and a_1 exchange, studied while calculating the hadronic contributions to the muonium hfs [22], is similar—the interaction of both mesons with photons couples them to electrons and muons.) That may provide smaller values of the lifetime of the intermediate particle.

Here, we have considered an option of a light boson with an ultraweak coupling, which has not been well explored in particle physics (see, e.g., [23]).

Various constraints on long-distance interactions of this range could also come from experiments, studying Casimir effect at small distances [24]. However, here, we consider somewhat smaller distances, which are not accessible in those experiments. Yukawa potentials, related to γ^* or to a pseudovector meson, are also outside of reach in those investigations, which involve a long-range spin-independent interaction of bulk neutral matter. So, our constraints are complementary to Casimir-effect studies.

The keV mass range can be explored by means of astrophysics and cosmology [6,25]. Those constraints involve additional details such as the lifetime and other couplings and they are also complementary to our constraint for $\alpha'(\lambda)$.

The novel method suggested here, based on a study of a few specific atomic transitions understood theoretically and experimentally with extremely high accuracy, covers an area of parameters (mass-coupling constant), not available by other methods (at least in a model-independent way). The details of our evaluations are to be published elsewhere. We expect that some other atomic data can be also useful for additional constraints.

---

*savely.karshenboim@mpq.mpg.de

[1] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008).
[2] S. G. Karshenboim, Phys. Rep. **422**, 1 (2005).
[3] M. I. Eides, H. Grotch, and V. A. Shelyuto, *Theory of Light Hydrogenic Bound States*, Springer Tracts Mod. Phys. Vol. 222 (Springer, Berlin, Heidelberg, 2007).
[4] B. Holdom, Phys. Lett. B **166**, 196 (1986).
[5] M. Pospelov, Phys. Rev. D **80**, 095002 (2009).
[6] M. Pospelov, A. Ritz, and M. Voloshin, Phys. Rev. D **78**, 115012 (2008).
[7] M. Fischer *et al.*, Phys. Rev. Lett. **92**, 230802 (2004).
[8] A. Huber *et al.*, Phys. Rev. Lett. **80**, 468 (1998).
[9] R. de Beauvoir *et al.*, Phys. Rev. Lett. **78**, 440 (1997); C. Schwob *et al.*, Phys. Rev. Lett. **82**, 4960 (1999); **86**, 4193(E) (2001).
[10] A. Wicht, E. Sarajlic, J. M. Hensley, and S. Chu, Phys. Rev. A **72**, 023602 (2005).
[11] M. Cadoret *et al.*, Phys. Rev. Lett. **101**, 230801 (2008).
[12] J. Ch. De Vries, Ph.D. thesis, MIT, 2001, available at http://dspace.mit.edu/handle/1721.1/4108.
[13] D. Kleppner (private communication).
[14] D. Hanneke, S. Fogwell, and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008).
[15] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. **99**, 110406 (2007).
[16] W. Liu *et al.*, Phys. Rev. Lett. **82**, 711 (1999).
[17] S. G. Karshenboim and V. G. Ivanov, Phys. Lett. B **524**, 259 (2002); Eur. Phys. J. D **19**, 13 (2002).
[18] N. Kolachevsky *et al.*, Phys. Rev. Lett. **102**, 213002 (2009).
[19] N. Kolachevsky, P. Fendel, S. G. Karshenboim, and T. W. Hänsch, Phys. Rev. A **70**, 062503 (2004).
[20] M. H. Prior and E. C. Wang, Phys. Rev. A **16**, 6 (1977).
[21] S. G. Karshenboim and V. G. Ivanov, Can. J. Phys. **83**, 1063 (2005); S. G. Karshenboim *et al.*, Nucl. Phys. B, Proc. Suppl. **162**, 260 (2006).
[22] S. G. Karshenboim, V. A. Shelyuto, and A. I. Vainshtein, Phys. Rev. D **78**, 065036 (2008).
[23] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[24] R. S. Decca *et al.*, Phys. Rev. D **75**, 077101 (2007).
[25] G. G. Raffelt, Annu. Rev. Nucl. Part. Sci. **49**, 163 (1999).