Multiple Colliding Electromagnetic Pulses: A Way to Lower the Threshold of e^+e^- Pair Production from Vacuum

S. S. Bulanov,[1,2] V. D. Mur,[3] N. B. Narozhny,[3] J. Nees,[1] and V. S. Popov[2]

[1]*FOCUS Center and Center for Ultrafast Optical Science, University of Michigan, Ann Arbor, Michigan 48109, USA*
[2]*Institute of Theoretical and Experimental Physics, Moscow 117218, Russia*
[3]*National Research Nuclear University MEPhI, 115409 Moscow, Russia*
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The scheme of a simultaneous multiple pulse focusing on one spot naturally arises from the structural features of projected new laser systems, such as the Extreme Light Infrastructure (ELI) and High Power laser Energy Research (HiPER). It is shown that the multiple pulse configuration is beneficial for observing e^+e^- pair production from a vacuum under the action of sufficiently strong electromagnetic fields. The field of focused pulses is described using a realistic three-dimensional model based on an exact solution of the Maxwell equations. The e^+e^- pair production threshold in terms of electromagnetic field energy can be substantially lowered if, instead of one or even two colliding pulses, multiple pulses are focused on one spot. The multiple pulse interaction geometry gives rise to subwavelength field features in the focal region. These features result in the production of extremely short e^+e^- bunches.

One of the most profound phenomena in the quantum electrodynamics of intense fields is the production of electron-positron (e^+e^-) pairs from a vacuum under the action of a strong electromagnetic (EM) field [1–4]. This nonlinear phenomenon attracts significant interest due to the fact that it lies beyond the scope of perturbation theory and sheds light on the nonlinear quantum electrodynamics properties of the vacuum. The e^+e^- production by strong EM fields in vacuum is crucial for understanding a number of astrophysical phenomena [5]. This process also places a natural physical limit on attainable laser pulse intensity due to EM pulse energy depletion [6,7]. Moreover the process of pair production was extensively discussed in a number of papers on the particle formation process in high energy hadronic interaction and the creation of quark-gluon plasmas [8]. The pair production process was first considered in a static electric field, then its theoretical description was extended to time-varying electric fields [9]. Until recently, these results were generally believed to be purely of theoretical interest since the value of the electric field strength needed to produce a noticeable quantity of e^+e^- pairs, the critical quantum electrodynamics field $E_S = m_e^2 c^3 / e\hbar = 1.32 \times 10^{16}$ V/cm (the corresponding intensity $I_S = E_S^2 / 4\pi = 4.65 \times 10^{29}$ W/cm²), seemed to be unreachable experimentally. However, the rapid development of laser technologies promises substantial growth of peak laser intensities. The intensity $I = 2 \times 10^{22}$ W/cm² is already available [10] and projects to achieve $I = 10^{26-28}$ W/cm² [11–13] are under way. Therefore various aspects of pair production by focused laser pulses are becoming urgent for experiments and are currently gaining much attention [6,14].

The way to obtain EM field strength close to E_S in the laboratory frame lies in generating very short and sharply focused laser pulses. Analytically, such pulses can be described by a realistic three-dimensional model developed in Ref. [15]. Unlike the case of a spatially homogeneous time-varying electric field [9], this model is based on an exact solution to the Maxwell equations and was successfully used in [6] for studying the effect of e^+e^- pair creation by focused circularly polarized laser pulses in vacuum. It was shown, in particular, that the effect becomes experimentally observable at intensities on the order of $I = 10^{28}$ W/cm² $\ll I_S$ for a single focused pulse. This is explained by a huge value of the preexponential factor in the formula for the number of created pairs which is of the order of the ratio of the effective laser pulse 4-volume, where pairs are effectively created, to the characteristic Compton 4-volume. It was also shown that the threshold intensity for the case of two head-on colliding laser pulses is much lower and is on the order of 10^{26} W/cm² $\ll I_S$. A similar result was demonstrated recently in Ref. [16], where the superposition of a focused optical pulse with an x-ray beam is shown to enhance the pair production.

In the present Letter we use this model [15] to consider the effect of e^+e^- pair creation in vacuum by several colliding coherent linearly polarized laser pulses. Such configurations are justified by the fact that the scheme of simultaneous multiple pulse focusing arises naturally from the structural features of projected new laser systems, such as Extreme Light Infrastructure (ELI) [12] and High Power laser Energy Research (HiPER) [13], and is implemented at the National Ignition Facility (NIF) [17]. We argue that superposition of four or more pulses essentially enhances the effect of pair production as compared with the case of a single or even two colliding pulses of the same total input energy. The total 4-volume of the resultant field decreases while the peak field grows. The number of created pairs depends on the peak field exponentially while the effective laser pulse 4-volume decreases as a power. This explains

the decrease of the threshold intensity for the case of a many-pulse collision. Moreover the interference of colliding waves generates a spotty temporal and spatial EM field structure in the focus that leads to the generation of ultra-short (tenths of a wavelength) electron and positron bunches, being another way to produce ultrashort electron bunches with intense focused EM pulses [18].

To calculate the number of e^+e^- pairs produced by focused pulses, the fact was used that the length of formation of the pair production process is determined by the Compton wavelength which is 6 orders of magnitude shorter than the typical laser radiation wavelength, i.e., $\lambda \gg l_C (= 3.86 \times 10^{-11}$ cm). At an arbitrary field point, which is characterized by the field invariants $\mathcal{F} = (\mathbf{E}^2 - \mathbf{H}^2)/2$ and $\mathcal{G} = \mathbf{EH}$, the number of pairs produced in a unit volume per unit time can be calculated by the formula for a constant EM field, and the total average number of particles produced is calculated as the following integral over volume V and time [6] ($\hbar = 1$, $c = 1$):

$$N = \frac{e^2 E_S^2}{4\pi^2} \int dV \int_{-\infty}^{\infty} dt \epsilon \eta \coth\frac{\pi\eta}{\epsilon} \exp\left(-\frac{\pi}{\epsilon}\right). \quad (1)$$

Here $\epsilon = \mathcal{E}/E_S$, $\eta = \mathcal{H}/E_S$, and $(\mathcal{E}, \mathcal{H}) = \sqrt{(\mathcal{F}^2 + \mathcal{G}^2)^{1/2} \pm \mathcal{F}}$ are the invariants that have the meaning of the electric and magnetic field strengths in the reference frame where they are parallel to each other.

In the general case, electric and magnetic fields of a focused pulse have longitudinal components, being super-positions of two waves: the e wave and the h wave that have either electric or magnetic transverse field components, respectively. There exists an exact solution to the Maxwell equations that describes the EM field of a linearly polarized focused pulse with focal spot radius R and Rayleigh length L [15]:

$$\mathbf{E}^e = iE_0 e^{-i\varphi}\{\alpha\mathbf{e}_x - \beta\mathbf{e}_y\},$$
$$\mathbf{H}^e = iE_0 e^{-i\varphi}\{(1 - i\Delta^2\partial_\chi)[\beta\mathbf{e}_x + \alpha\mathbf{e}_y] + 2i\Delta\gamma\mathbf{e}_z\}. \quad (2)$$

Here $\alpha = F_1 - F_2\cos2\phi$, $\beta = F_2\sin2\phi$, and $\gamma = \sin\phi\partial_\xi F_1$; x, y, and z are spatial coordinates; and $\varphi = \omega(t - z) + \tilde{\varphi}$, where $\tilde{\varphi}$ is the carrier-envelope phase, $\xi = \rho/R$, $\chi = z/L$, $\rho = \sqrt{x^2 + y^2}$, $\cos\phi = x/\rho$, $\sin\phi = y/\rho$, $\Delta \equiv 1/\omega R = \lambda/2\pi R$, $L = R/\Delta$. The electric and magnetic fields of the h wave are expressed via the fields of the e wave [19].

The exploited model admits different field configurations, which are determined by two functions, F_1, F_2. In particular, if $\Delta \ll 1$ they can be chosen in the form

$$F_1 = (1 + 2i\chi)^{-2}\left(1 - \frac{\xi^2}{1 + 2i\chi}\right)\exp\left(-\frac{\xi^2}{1 + 2i\chi}\right),$$
$$F_2 = -\xi^2(1 + 2i\chi)^{-3}\exp\left(-\frac{\xi^2}{1 + 2i\chi}\right), \quad (3)$$

(see Ref. [15]). We will work with expressions (3) for

functions F_1, F_2 throughout the Letter and consider pair production by e waves.

To describe a laser pulse with finite duration τ we introduce a temporal amplitude envelope $g[(t - z)/\tau]$ and make the following substitutions in Eq. (2) [15]: $\exp(-i\varphi) \rightarrow if'(\varphi)$, $\Delta\exp(-i\varphi) \rightarrow \Delta f(\varphi)$, where $f(\varphi) = g[(\varphi - \tilde{\varphi})/\omega\tau]\exp(-i\varphi)$. We assume $g[(\varphi - \tilde{\varphi})/\omega\tau] = 1$ at $\varphi - \tilde{\varphi} = 0$ and decreases exponentially at the periphery of the pulse for $|\varphi| \gg \omega\tau$. In this case the electric and magnetic fields of the model constitute an approximate solution of the Maxwell equations having second-order accuracy with respect to small parameters Δ and $\Delta' = 1/\omega\tau$, $\Delta' \lesssim \Delta \ll 1$.

The EM field invariants in the case of a single pulse have the following form,

$$\mathcal{F}_1^e = \Delta^2 E_0^2\{\Im(\beta e^{-i\varphi})\Re(\beta_\chi e^{-i\varphi}) + \Im(\alpha e^{-i\varphi})\Re(\alpha_\chi e^{-i\varphi}) - 2[\Re(\gamma e^{-i\varphi})] + O(\Delta^2)\},$$
$$\mathcal{G}_1^e = \Delta^2 E_0^2\{\Im(\beta e^{-i\varphi})\Re(\alpha_\chi e^{-i\varphi}) - \Im(\alpha e^{-i\varphi})\Re(\beta_\chi e^{-i\varphi}) + O(\Delta^2)\}, \quad (4)$$

where $(\alpha, \beta)_\chi = \partial_\chi(\alpha, \beta)$. Since $\mathcal{F}_1^e$ and $\mathcal{G}_1^e$ are proportional to $\Delta^2 E_0^2$, then the invariant fields are proportional to ΔE_0. Contrary to that, in the case of two colliding pulses with total energy equal to the energy of a single pulse, the invariants are no longer proportional to Δ:

$$\mathcal{F}_2^e = 2E_0^2\{(|\alpha e^{i\omega z}|^2 + |\beta e^{i\omega z}|^2)\sin^2\omega t - [\Im(\alpha e^{i\omega z})]^2 - [\Im(\beta e^{i\omega z})]^2 + O(\Delta^2)\},$$
$$\mathcal{G}_2^e = 2E_0^2\sin2\phi[\Im(\alpha\beta^*)\sin2\omega t + O(\Delta^2)]. \quad (5)$$

This is due to the fact that in the antinodes of standing light waves the electric fields sum up while the magnetic fields cancel; i.e., the pairs are mainly produced in the antinodes, whereas in the case of a single pulse the pairs are produced throughout the focal 4-volume. The invariant field for two colliding pulses is proportional to E_0. Since $\Delta \ll 1$, two colliding pulses produce many more pairs than a single pulse [19].

Further enhancement of the number of e^+e^- pairs (or lowering the threshold intensity) can be achieved with multiple colliding pulses. The use of such a configuration will not only lead to focusing of a larger part of the EM energy into a smaller volume but also to a redistribution of the focused energy in favor of the electric field. This will lead to an enhancement of the invariant electric field strength and thus to an increase in the number of pairs produced. The most beneficial setup will be arranged if the central axes of the pulses lie in one plane, with the pulses being linearly polarized in the direction perpendicular to this plane. The pulses are arranged in counter-propagating pairs so that at focus their magnetic fields cancel each other and electric fields sum up. Then the resulting peak electric field will be proportional to $\sqrt{n_p}$, where n_p is the total number of pulses. In the case, considered below, $n_p = 8$.
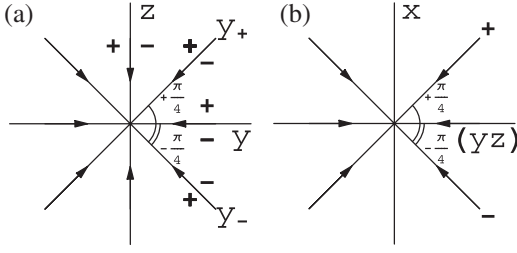
FIG. 1.   The principal scheme of multiple pulse focusing.

More pulses can be added, though with less efficiency, with their central axes being at some angle (θ) to the plane where the first eight are focused. In this case the resulting peak field in the focus will be proportional to $(n_{p_1} + n_{p_2} \cos\theta)/\sqrt{n_p}$, $n_{p_1} = 8$ and $n_{p_1} + n_{p_2} = n_p$.

In what follows we consider a configuration where up to 24 pulses are focused simultaneously to the same focal spot and the total EM energy is kept constant. In Fig. 1(a) we show how these pulses are focused. First eight pulses are focused in the (yz) plane in colliding pairs along the y and z axes and along two lines (y_+, y_-) which have angles $\pm\pi/4$ to y axis. Up to 24 pulses are introduced by adding pairs of pulses along the lines which do not lie in (yz) plane and have an angle of $+\pi/4$ or $-\pi/4$ with one of the 4 lines of pulse propagation in the (yz) plane. These angles are measured in the plane that goes through the line in (yz) plane and x axis [Fig. 1(b)].

In Fig. 2 we present the distribution of an invariant electric field in the (xy), (zy), and (zx) planes for different numbers of pulses focused. The duration of each pulse is 10 fs and $\Delta = 0.3$. As the number of pulses increases, so does the peak value of the invariant field. However, the volume where the invariant field is contained shrinks, forming a spiky structure with features of about a half wavelength in size. This can be expected from the significant increase in the effective numerical aperture of the focused light.
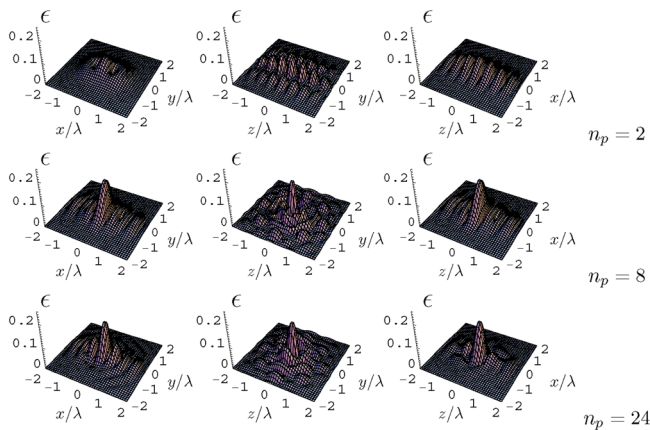
The time dependence of the invariant field shows a similar behavior. We present in Figs. 3(a) and 3(b) the evolution of the invariant electric field along the x, y, and z axes for the cases of two and 24 pulses. It can be seen from these figures that the field is localized in several sharp peaks. With the increase of the number of pulses the volume where the field is localized shrinks.

In what follows we present the results of numerical calculations of e^+e^- pair numbers produced by an EM field of multiple pulses. We use Eq. (1) and expressions (2) for the fields. The field configuration is described in Fig. 1. Each of the pulses has a wavelength of $\lambda = 1$ μm, a numerical aperture $\Delta = 0.3$, a duration $\tau = 10$ fs, and a focal spot of about λ, while the total EM energy of this multipulse configuration is a constant 10 kJ. The results are presented in Table I, where the number of pairs according to Eq. (1) is shown for different numbers of pulses. Two pulses are colliding along the z axis. Four pulses are two pairs of pulses colliding simultaneously along the z and y axes. Eight pulses are arranged in four colliding along the y, z, y_+, and y_- lines pairs. The 16 pulse configuration is constructed by adding to eight in-plane pulses four more pairs of pulses. These pulses collide along the lines that lie in the (y_+, x) and (y_-, x) planes and have an angle of $-\pi/4$ or $+\pi/4$ with an y_+ or y_- line, respectively. Twenty-four pulses represent the maximum number of pulses described in Fig. 1. We also show the threshold energy, i.e., the energy necessary to produce several ($1 < N_{e^+e^-} < 10$) e^+e^- pairs, for the different numbers of pulses. Such a definition is due to the extremely sensitive exponential dependence of the number of pairs on the field intensity.

According to our results, summarized in Table I, pair production exceeds the threshold when eight in-plane EM pulses are simultaneously focused on one spot. Doubling the number of pulses leads to the 3 orders of magnitude increase of the number of pairs. Tripling the number of pulses makes it possible to produce 6 orders of magnitude more pairs. The threshold energy drops from 40 kJ for two pulses to 5.1 kJ for 24 pulses. It clearly indicates that the multiple pulse configuration is much more favorable for
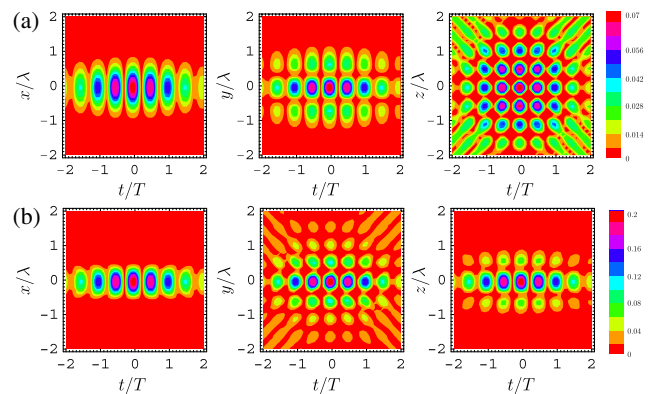

FIG. 2 (color online).   The distribution of an invariant electric field in (xy), (zy), and (zx) planes for 2, 8, and 24 pulses.

FIG. 3 (color online).   The evolution of the invariant electric field along the x, y, and z axes for the cases of 2 and 24 pulses.

TABLE I. The number of e^+e^- pairs ($N_{e^+e^-}$) produced by different numbers of pulses (the total energy is 10 kJ and $\Delta = 0.3$). The threshold value total energy needed to produce one e^+e^- pair is shown in the third column for different numbers of pulses.

n	$N_{e^+e^-}$ at $W = 10$ kJ	W_{th}, kJ ($N_{e^+e^-} \sim 1$)
2	0^a	40
4	0^b	20
8	4.0	10
16	1.8×10^3	8
24	4.2×10^6	5.1

[a]The average number of pairs is 9.0×10^{-19}.
[b]The average number of pairs is 3.0×10^{-9}.

e^+e^- pair production than a single pulse or even a superposition of two pulses.

The spiky temporal profile of the invariant electric field should lead to the production of very short electron and positron bunches with characteristic duration much smaller then the radiation period. The duration of the central bunch can be estimated as follows: first, we approximate the invariant electric field as $\epsilon = \epsilon_0(1 - \sum_i i^2/r_i^2)$, $i = x, y, z, t$. Here $r_t = T/4$, $r_x = \lambda/2$, $r_y = \lambda/4$, and $r_z = \lambda/4$, $\lambda/4$, and $\lambda/2$ for 2, 8, and 24 pulses, respectively. Then we set $\eta = 0$ and integrate (1) over space. We get the following expression for the dependence of the number of produced pairs on time:

$$n(t) = \frac{r_x r_y r_z}{4\pi^2 l_C^4}[\epsilon(t)]^{7/2} \exp\left(-\frac{\pi}{\epsilon(t)}\right). \tag{6}$$

Here $\epsilon(t) = \epsilon_0[1 - t^2/(r_t)^2]$. The duration of the bunch at FWHM is $\Delta t = (\ln 2)^{1/2}(7/2 + \pi/\epsilon_0)^{-1/2}T/2$. For $\epsilon_0 = 0.08$ (two pulses) the electron pulse duration is about 0.064 T (190 as for $T = 3$ fs). For $\epsilon_0 = 0.16$ (eight pulses) the electron pulse duration is about 0.086 T (260 as for $T = 3$ fs). For $\epsilon_0 = 0.21$ (24 pulses) the electron pulse duration is about 0.097 T (290 as for $T = 3$ fs). The results of numerical calculations of bunch duration agree with this estimate $\Delta t = 0.062$ T for 2 pulses, $\Delta t = 0.089$ T for 8 pulses, and $\Delta t = 0.1$ T for 24 pulses.

In conclusion, we have shown that the simultaneous focusing of multiple colliding pulses will lead to a significant reduction of the threshold energy needed for the pair production to become observable compared to the case of one or even two pulses. It is due to the localization of the EM energy in a smaller volume and to a redistribution of energy in favor of the electric field. According to the results presented in this Letter a system like ELI or HiPER with 10 kJ of energy in 8 pulses with a duration of about 10 fs will be able to observe the e^+e^- pair production from a vacuum by the direct action of the EM field. And for 24 pulses, the resulting intensity is more than adequate to produce a significant number of e^+e^- pairs.

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