Spin Drag in an Ultracold Fermi Gas on the Verge of Ferromagnetic Instability

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Recent experiments [Jo *et al.*, Science **325**, 1521 (2009)] have presented evidence of ferromagnetic correlations in a two-component ultracold Fermi gas with strong repulsive interactions. Motivated by these experiments we consider spin drag, i.e., frictional drag due to scattering of particles with opposite spin, in such systems. We show that when the ferromagnetic state is approached from the normal side, the spin drag relaxation rate is strongly enhanced near the critical point. We also determine the temperature dependence of the spin diffusion constant. In a trapped gas the spin drag relaxation rate determines the damping of the spin dipole mode, which therefore provides a precursor signal of the ferromagnetic phase transition that may be used to experimentally determine the proximity to the ferromagnetic phase.

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Introduction.—Arguably the two most important phase transitions in condensed matter physics are the superconducting and the ferromagnetic phase transition. The first occurs for attractive interactions between distinguishable fermions, causing formation of Cooper pairs that Bose-Einstein condense, according to Bardeen-Cooper-Schrieffer (BCS) theory. Within Stoner mean-field theory, ferromagnetism is the result of strong repulsive interactions between two spin species of fermions, that cause the system to spin polarize to save interaction energy. Over the past few years, the BCS transition has received a great deal of attention in the context of ultracold atomic Fermi gases [1,2], owing mainly to the use of Feshbach resonances to tune the interactions between the atoms [3]. Having explored the regime of attractive interactions, scientific interest has recently turned to the repulsive side of the Feshbach resonance where itinerant ferromagnetism is predicted to occur [4–8].

One of the most exciting developments is the very recent observation of ferromagnetic correlations in a two-component Fermi gas with strong repulsive interactions by Jo *et al.* [9]. The probes used in these experiments, i.e., enhancement of kinetic energy and reduction of atom-loss rate, are local probes and do not constitute conclusive evidence for a ferromagnetic phase with nonzero spontaneous spin polarization. In part as a result of this, there has been a lot of theoretical interest [10] in this experiment.

In this Letter we point out that spin drag [11,12], i.e., the friction between two different spin states due to interactions that was recently proposed in the context of semiconductors, provides a distinct experimental probe to determine the proximity to the ferromagnetic state. Spin drag effects lead to decay of spin currents, with a typical spin drag relaxation rate due to interaction effects. The spin drag relaxation rate determines the damping rate of the spin dipole mode in trapped cold-atom systems [13] and is thus accessible experimentally. Interestingly, an electronic analog of the spin dipole mode also exists [14]. Spin drag in cold-atom systems was proposed very recently both for fermionic atoms [13,15], and for bosonic ones [16]. In this Letter, we show that the spin drag relaxation rate will be strongly enhanced as the ferromagnetic state is approached from the normal side. This enhancement in spin drag is somewhat analogous to the enhancement of the Coulomb drag resistivity [17] in electron-hole bilayers as one approaches the exciton-condensed state [18,19]. Although conclusive evidence of exciton condensation in these sys-

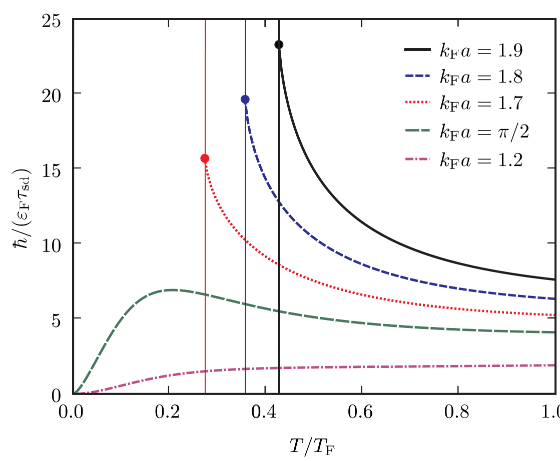

FIG. 1 (color online). Spin drag relaxation rate $1/\tau_{\rm sd}(T)$ as a function of temperature T, for various values of the interaction parameter $k_{\rm F}a$. The Fermi energy is denoted by $\varepsilon_{\rm F} = k_B T_{\rm F} = \hbar^2 k_{\rm F}^2/2m$. Note that for $k_{\rm F}a > \pi/2$, the spin drag relaxation rate shows a distinctive upturn when the critical temperatures, indicated by thin vertical lines, are approached from above. In particular, we have $T_c \simeq 0.43 T_{\rm F}$ for $k_{\rm F}a = 1.9$, $T_c \simeq 0.36 T_{\rm F}$ for $k_{\rm F}a = 1.8$, and $T_c \simeq 0.27 T_{\rm F}$ for $k_{\rm F}a = 1.7$. For $k_{\rm F}a < \pi/2$ no ferromagnetism occurs within mean-field theory. In that case $1/\tau_{\rm sd}(T)$ is smooth throughout the temperature range and exhibits the standard Fermi-liquid behavior $1/\tau_{\rm sd}(T) \propto T^2$ for $T \to 0$.

tems is still lacking, two experimental groups [20,21] have recently reported the observation of an upturn in the Coulomb drag resistivity as the temperature is lowered.

Our main findings are illustrated in Fig. 1. This plot shows the spin drag relaxation rate $1/\tau_{\rm sd}(T)$ as a function of temperature, for various interaction strengths determined by the product of the Fermi wave vector $k_{\rm F}$ and the scattering length a. The dramatic enhancement of the relaxation rate upon approaching the critical temperature for the ferromagnetic transition is clearly visible.

The enhancement of the spin drag relaxation rate as the ferromagnetic phase is approached serves as a precursor probe for ferromagnetism that is distinct from, and adds to, the experimental methods of Jo *et al.* [9], and is also interesting in its own right. In the following we present our calculations in detail, and present additional results and discussion.

Spin drag relaxation rate.—We consider a 3D homogeneous gas of fermionic atoms of mass m, with two hyperfine states denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$. The grand-canonical Hamiltonian that describes the system well at the temperatures and densities of interest is given by

$$\hat{H} = \int d^3\boldsymbol{x} \sum_{\alpha\in\{\uparrow,\downarrow\}} \hat{\psi}^\dagger_\alpha(\boldsymbol{x})\left(-\frac{\hbar^2\nabla^2_{\boldsymbol{x}}}{2m} - \mu\right)\hat{\psi}_\alpha(\boldsymbol{x}) + U\int d^3\boldsymbol{x}\,\hat{\psi}^\dagger_\uparrow(\boldsymbol{x})\hat{\psi}^\dagger_\downarrow(\boldsymbol{x})\hat{\psi}_\downarrow(\boldsymbol{x})\hat{\psi}_\uparrow(\boldsymbol{x}), \quad (1)$$

in terms of fermionic creation and annihilation operators $\hat{\psi}^\dagger_\alpha(\boldsymbol{x})$ and $\hat{\psi}_\alpha(\boldsymbol{x})$, respectively, and where μ is the chemical potential. At low temperatures s-wave scattering, described by $U = 4\pi a\hbar^2/m$, dominates, and we have therefore omitted other interaction terms from this Hamiltonian.

We first determine a frequency and momentum dependent scattering amplitude $A_{\uparrow\downarrow}(q,\omega)$ that takes into account many-body effects on the scattering of atoms with opposite spin. We use the generalized random-phase approximation that consists of summing all ''bubble'' diagram contributions to this effective interaction. This takes into account modifications of the interaction due to density and spin fluctuations in an approximate way that is sufficient for our purpose of illustrating the effect of the proximity of the ferromagnetic phase transition on the spin drag. Including spin fluctuations is essential as they are strongly enhanced close to the ferromagnetic phase transition. In terms of the noninteracting (Lindhard) response function at nonzero temperature

$$\chi_0(q,\omega) = 2\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{N_{\boldsymbol{q}+\boldsymbol{k}} - N_{\boldsymbol{k}}}{\varepsilon_{\boldsymbol{q}+\boldsymbol{k}} - \varepsilon_{\boldsymbol{k}} - \hbar\omega - i0}, \quad (2)$$

with $\varepsilon_{\boldsymbol{k}} = \hbar^2\boldsymbol{k}^2/2m$ and $N_{\boldsymbol{k}} = [e^{(\varepsilon_{\boldsymbol{k}}-\mu)/k_BT} + 1]^{-1}$ the Fermi-Dirac distribution function, the scattering amplitude reads

$$A_{\uparrow\downarrow}(q,\omega) = U + \frac{U^2}{4}\chi_{nn}(q,\omega) - \frac{3U^2}{4}\chi_{S_zS_z}(q,\omega), \quad (3)$$

where $\chi_{S_zS_z(nn)}(q,\omega) = \chi_0(q,\omega)/[1 + (-)U\chi_0(q,\omega)/2]$. In this notation $\chi_{nn}(q,\omega)$ is the density-density response function while $\chi_{S_zS_z}(q,\omega)$ describes the spin-spin response. The factor of 3 in the last term in the right-hand side of Eq. (3) comes about because longitudinal and transverse spin fluctuations are both taken into account.

Within Stoner mean-field theory, ferromagnetism occurs when $\chi_{S_zS_z}(0,0)$ diverges so that $1 + U\chi_0(0,0)/2 = 0$. This equation gives, together with the equation $n = 2\int d^3\boldsymbol{q} N_{\boldsymbol{q}}/(2\pi)^3$ for the total density determining the chemical potential, the critical temperature T_c as a function of $k_{\rm F}a$. For k_BT_c much smaller than the Fermi energy $\varepsilon_{\rm F} \equiv \hbar^2k_{\rm F}^2/2m = \hbar^2(3\pi^2n)^{2/3}/2m$, this gives a critical temperature

$$\frac{k_BT_c}{\varepsilon_{\rm F}} \simeq \frac{2\sqrt{3}}{\pi}\sqrt{\frac{U\nu(\varepsilon_{\rm F})}{2} - 1} = \frac{2\sqrt{3}}{\pi}\sqrt{\frac{2k_{\rm F}a}{\pi} - 1}, \quad (4)$$

where $\nu(\varepsilon_{\rm F}) = mk_{\rm F}/\pi^2\hbar^2$ is the density of states at the Fermi level. Note that one needs $k_{\rm F}a > \pi/2$ for the critical temperature to be nonzero, and that there is a quantum critical point when $k_{\rm F}a = \pi/2$ [4].

Our next step is to use the scattering amplitude $A_{\uparrow\downarrow}(q,\omega)$ in Eq. (3) in the well-known expression for the spin drag relaxation rate $1/\tau_{\rm sd}$, following from Boltzmann theory [11,13]. This yields the result

$$\frac{1}{\tau_{\rm sd}(T)} = \frac{\hbar^2}{4mnk_BT}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{q^2}{3} \times \int_{-\infty}^{+\infty}\frac{d\omega}{\pi}|A_{\uparrow\downarrow}(q,\omega)|^2\frac{[\Im m\chi_0(q,\omega)]^2}{\sinh^2[\hbar\omega/(2k_BT)]}, \quad (5)$$

which, together with Eq. (3), is the central result of this Letter and will be evaluated next. Before proceeding we note that our result is similar to the theory of spin diffusion in liquid He^3 [22], although the ferromagnetic phase transition was not considered in this context.

Results.—In Figs. 1 and 2 we present the results of a numerical evaluation of Eq. (5), both as a function of temperature for various interaction strengths $k_{\rm F}a$, and as a function of interaction strength for various temperatures. In experiments both dependencies can be explored using a Feshbach resonance to tune the interaction strength [3]. Both figures clearly show the strong enhancement of the spin drag relaxation rate as the ferromagnetic state is approached. The precise form of the enhancement is understood by keeping only the most divergent term in the scattering amplitude in Eq. (3) so that

$$A_{\uparrow\downarrow}(q,\omega) \simeq -\frac{3U^2\nu(\varepsilon_{\rm F})}{\frac{U\nu(\varepsilon_{\rm F})}{3}\left(\frac{q^2}{k_{\rm F}^2} - i\frac{6\pi m\omega}{\hbar k_{\rm F}q}\right) + 4\alpha(T)}, \quad (6)$$

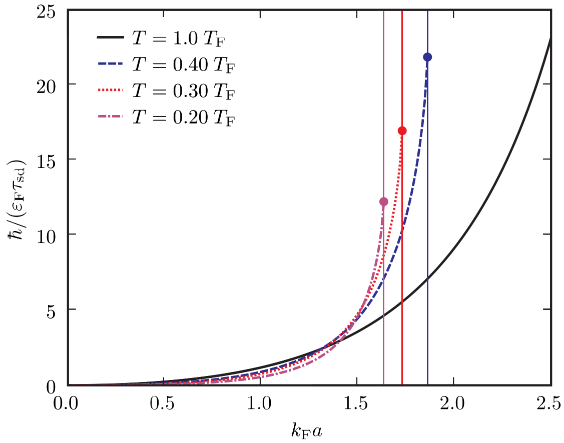

FIG. 2 (color online). Spin drag relaxation rate $1/\tau_{\rm sd}$ as a function of $k_{\rm F}a$ for various temperatures. The thin vertical lines indicate the critical values of the interaction parameter $k_{\rm F}a$ at which ferromagnetism occurs.

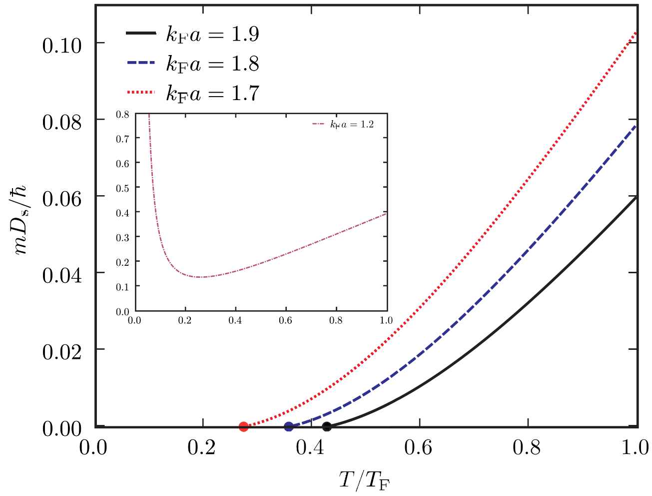

FIG. 3 (color online). Panel (a): spin diffusion constant D_s as a function of temperature T for various interaction strengths $k_{\rm F}a > \pi/2$. Panel (b): spin diffusion constant for $k_{\rm F}a = 1.2 < \pi/2$.

with $\alpha(T) = 1 + U\chi_0(0,0)/2 \simeq 1 - U\nu(\varepsilon_{\rm F}) + \pi(T/T_{\rm F})^2/12 + \ldots$ equal to zero at the phase transition and positive for T larger than the critical temperature. In obtaining this approximation we have expanded around the ferromagnetic singularity and used that $\Im m\chi_0(q,\omega) = -\pi\nu(\varepsilon_{\rm F})m\omega/2\hbar k_{\rm F}q$ for small ω/q. Using these results in Eq. (5), and expanding $1/\sinh^2(x) \simeq 1/x^2$, both the frequency integral and the momentum integral can be performed analytically if we use a cutoff of $2k_{\rm F}$ on the momentum integration that diverges because of the expansion of $1/\sinh^2(x)$. Ultimately we find in this manner that $1/\tau_{\rm sd}(T) - 1/\tau_{\rm sd}(T_c) \propto (T - T_c)\ln(T - T_c)$ for $T \downarrow T_c \ll T_{\rm F}$, which indeed accurately describes our numerical results near the critical temperature.

We also consider the spin diffusion constant, which from the Einstein relation is given by $D_s(T) = \sigma_s(T)/\chi_{S_zS_z}(0,0)$ and the "spin conductivity" $\sigma_s(T) = n\tau_{\rm sd}(T)/m$ [23]. In Fig. 3 we show this constant as a function of temperature for various values of the interaction strength. Near the critical temperature the spin diffusion constant vanishes as $D_s(T) \propto (T - T_c)^\kappa$ with an exponent $\kappa = 1$, because $\tau_{\rm sd}(T_c)$ remains finite and $\chi_{S_zS_z}(0,0)$ diverges as $1/(T - T_c)$ within our generalized random-phase approximation.

At this point it is important to realize that from the point of view of critical dynamics our findings are mean-field like. If the spin dynamics can be effectively described by an isotropic Heisenberg ferromagnet (model J [24]), the spin conductivity $\sigma_s(T)$ is expected to behave as $\xi^{(3-\eta)/2}$ very close to the transition, where the correlation length $\xi(T)$ diverges as $1/(T - T_c)^\nu$ and η and ν are the usual static critical exponents of the ferromagnetic transition [24]. Since $\chi_{S_zS_z}(0,0)$ diverges as $\xi^{2-\eta}$, we find that the spin diffusion constant goes to zero with an exponent $\kappa = (1 - \eta)\nu/2$. In view of this possibility, we have therefore made sure, using the Ginzburg criterion [25], that the upturn of the spin drag relaxation rate already takes place well outside the critical region where critical fluctuations can be neglected and our generalized random-phase approximation is appropriate.

Discussion and conclusions.—As we have mentioned in the introduction, the spin drag relaxation rate can be determined from the damping of the spin dipole mode [13,14] in a trapped gas. Since the Fermi energy is usually much larger than the level splitting in the trap, our results show that the spin dipole mode is typically strongly overdamped, which makes the experiment more challenging. Nevertheless, such a measurement, as well as measurements of the spin diffusion constant as a function of temperature, gives information on the proximity of the ferromagnetic phase transition. Although we have considered a homogeneous system, local-density approximations are generally valid for trapped Fermi systems, and in determining the damping of the spin dipole mode of a trapped two-component gas the homogeneous density should in first approximation be taken as the central density of the atomic cloud.

Within our present approach, we consider the transition to ferromagnetism within mean-field theory, which predicts it to be continuous. One interesting aspect is that taking into account correlation effects beyond mean-field theory [8] results in (i) an increase in the critical temperature for a given value of $k_{\rm F}a$ and (ii) a change in the character of the transition from second to first order at very low temperatures. We expect that these modifications will not qualitatively affect the upturn of $1/\tau_{\rm sd}$, as long as one remains outside the critical-fluctuation region. We note that the results presented in Fig. 1 for $k_{\rm F}a > \pi/2$ approximately collapse onto the same function when $\hbar/[(k_{\rm F}a)^2\varepsilon_{\rm F}\tau_{\rm sd}]$ is plotted as a function of T/T_c. Our theory should be interpreted as the simplest but nontrivial prediction for this function, and we expect that beyond-mean-field effects will not significantly alter this scaled result outside the regime of critical fluctuations. In this sense we believe that our findings do not only present a qualitative prediction for the behavior of spin drag near the ferromagnetic transition, but also contain quantitative information for future experiments.

In future work we intend to explore the effects of lower dimensionality, and the implications of critical and quantum critical fluctuations on the exponent κ that determines the behavior of the spin diffusion constant. Furthermore, as our approach is distinct from the work of Hu [18], who considered the upturn of the Coulomb drag resistivity as one approaches the exciton-condensed state in bilayers, we intend to study this system as well with our present approach.

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