Real-Time Dynamics of the Chiral Magnetic Effect

Kenji Fukushima,[1] Dmitri E. Kharzeev,[2] and Harmen J. Warringa[3]

[1]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

[2]*Department of Physics, Brookhaven National Laboratory, Upton New York 11973, USA*

[3]*Institut für Theoretische Physik, Goethe-Universität Frankfurt, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*

(Received 15 February 2010; published 27 May 2010)

In quantum chromodynamics, a gauge field configuration with nonzero topological charge generates a difference between the number of left- and right-handed quarks. When a (electromagnetic) magnetic field is added to this configuration, an electromagnetic current is induced along the magnetic field; this is called the chiral magnetic effect. We compute this current in the presence of a color-flux tube possessing topological charge, with a magnetic field applied perpendicular to it. We argue that this situation is realized at the early stage of relativistic heavy-ion collisions.

DOI: 10.1103/PhysRevLett.104.212001 PACS numbers: 12.38.Lg, 12.38.Aw, 12.38.Mh

Introduction.—The theory of the strong interactions, quantum chromodynamics (QCD), is an SU(3) Yang-Mills theory coupled to fermions (quarks). An intriguing aspect of SU(N) Yang-Mills theories is their relation to topology. This reveals itself in the existence of gauge field configurations carrying topological charge Q [1]. This charge is quantized as an integer if these configurations interpolate between two of the infinite number of degenerate vacua of the SU(N) Yang-Mills theory [2]. Expressed in terms of the field strength tensor $G_a^{\mu\nu}$ the topological charge reads $Q = \frac{g^2}{32\pi^2}\int d^4x G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}$; here g denotes the coupling constant and the dual field strength tensor equals $\tilde{G}^{\mu\nu a} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}$.

By interacting with fermions the $Q \neq 0$ fields induce parity ($\mathcal{P}$) and charge-parity ($\mathcal{CP}$) odd effects [3]. This can be seen by the following exact equation (valid for each quark flavor ψ separately) which is a result of the U(1) axial anomaly [4,5]: $\partial_\mu j_5^\mu = 2m\langle\bar{\psi}i\gamma^5\psi\rangle_A - \frac{g^2}{16\pi^2}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}$, where m is the quark mass, and $j_5^\mu = \langle\bar{\psi}\gamma^\mu\gamma^5\psi\rangle_A$ denotes the axial current density in the background of a gauge field configuration A^a_μ. Let us define the chirality density $n_5 = j_5^0$ and the chirality $N_5 = \int d^3x n_5$. Integrating the anomaly equation over space and time gives for massless quarks $\Delta N_5 = -2Q$, where ΔN_5 denotes the change in chirality over time. For massless quarks, the chirality N_5 is equal to the difference between the number of particles plus antiparticles with right-handed and left-handed helicity. Again for $m = 0$ right-handed helicity implies that spin and momentum are parallel whereas they are antiparallel for left-handed helicity.

The $Q \neq 0$ gauge fields are included in the path integral, and as a result they contribute to the amplitudes of physical processes. Experimental evidence for these configurations is, however, indirect. The clearest confirmation follows from the large mass of η' pseudoscalar meson compared to the π, K, and η mesons [3]. In this Letter we will discuss an alternative way in which topological configurations of gauge fields in QCD, i.e., gluon fields, could be studied in heavy-ion collisions.

Using high-energy heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) one can investigate the behavior of QCD at high-energy densities. Very strong color electric and color magnetic fields are produced during these collisions, whose strength is characterized by the gluon saturation momentum Q_s. In addition, extremely strong (electromagnetic) magnetic fields are present in noncentral collisions, albeit for a very short time. In gold-gold collisions at RHIC energies the magnitude of this magnetic field at the typical time scale $\sim Q_s^{-1} \approx 0.2$ fm$/c$ after the collision is of the order of $\sim 10^4$ MeV^2 which corresponds to $\sim 10^{18}$ G [6,7]. Such extremely strong magnetic fields are able to polarize to some degree the bulk of the produced quarks which have typical momenta of a few hundred MeV. More specifically, quarks with positive (negative) charge have a tendency to align their spins parallel (antiparallel) to the magnetic field. As a result, assuming the produced quarks can be treated as massless, a positively (negatively) charged quark with right-handed helicity will have its momentum parallel (antiparallel) to the magnetic field. For quarks with left-handed helicity this is exactly opposite. Hence a quark and antiquark both having the same helicity will move in opposite directions with respect to the magnetic field. This implies that an electromagnetic current is generated along the magnetic field if there is an imbalance in the helicity, i.e., a nonzero chirality. Because gauge fields with $Q \neq 0$ generate chirality, they will therefore induce an electromagnetic current along a magnetic field. This mechanism which signals $\mathcal{P}$- and $\mathcal{CP}$-odd interactions has been named the chiral magnetic effect [6,8]. In an extremely strong magnetic field $\boldsymbol{B}$, so strong that all quarks are fully polarized, it follows from the arguments presented above that for each quark flavor separately the induced current equals $\boldsymbol{J} = |q|N_5\boldsymbol{B}/|\boldsymbol{B}| = -2|q|Q\boldsymbol{B}/|\boldsymbol{B}|$, where q is the charge of the quark.

At the center of the collision, the magnetic field is pointing in a direction perpendicular to the reaction plane; the x-z plane in Fig. 1. As a result of the chiral magnetic effect the charge asymmetry between the two sides of the reaction plane will be generated. The sign of this asymmetry will fluctuate from collision to collision since (assuming the so-called θ angle vanishes and there is no global violation of parity) the probability of generating either positive Q or negative Q is equal.

Fluctuating charge asymmetries can be investigated using an event-averaged correlator $\langle \cos(\phi^- + \phi^+) \rangle$ [9]. Here $\phi^\pm$ denotes the angle between the momentum of a particle with charge $\pm$ and the reaction plane. The STAR Collaboration has analyzed this observable [10]. The results are qualitatively in agreement with the predictions of the chiral magnetic effect; the search for alternative explanations and additional manifestations of local parity violation is under way [11].

Several quantitative theoretical studies of the chiral magnetic effect have appeared in the literature [12–15]. Most of the analytic studies are based on introducing a chiral asymmetry by hand, after which the equilibrium response to a magnetic field is studied [12,14] (see also [16]). In this Letter we will for the first time investigate a situation in which the chirality is generated dynamically in real time in the presence of a magnetic field. For this we will take the simplest Yang-Mills gauge field configuration carrying topological charge, that is one which describes a color-flux tube having constant Abelian field strength, i.e., $G_a^{\mu\nu} = \mathcal{G}^{\mu\nu} n^a$ with $n_a n_a = 1$ and $\mathcal{G}^{\mu\nu}$ constant and homogeneous. Furthermore, we will take only the z components of the color electric ($\mathcal{E}_z = \mathcal{G}_{0z}$) and color magnetic ($\mathcal{B}_z = -\frac{1}{2}\epsilon_{zij}\mathcal{G}^{ij}$) field nonzero. Perpendicular to this field configuration we will apply an electromagnetic field B_y pointing in the y direction (see Fig. 1). Note that hereafter we write $\mathcal{B}$ to denote a color magnetic field and B for an electromagnetic one. Such color-flux tubes, which carry topological charge and are homogeneous over a spatial scale $\sim Q_s^{-1}$, naturally arise in the glasma [17], the dense gluonic state just after the collision, where $\mathcal{E}_z \sim \mathcal{B}_z \sim Q_s^2/g$. The induced current itself can generate electromagnetic and color fields, which can alter the dynamics. We will ignore this backreaction which can be justified as long as the induced current is small compared to the currents that create the external color and magnetic fields. This is likely to be the case in heavy-ion collisions due to the short-life time of the external magnetic field. Furthermore we will also ignore the production of gluons in the color-flux tube.

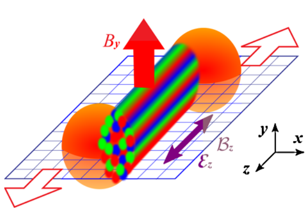

FIG. 1 (color online). Collision geometry and fields.

Calculation.—Using a color rotation we can choose only the third component of n^a nonvanishing. Since the generator $t^3 = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ of the SU(3) Lie algebra is diagonal, the different color components decouple. As a result for each quark flavor separately the problem is equivalent to a quantum electrodynamics (QED) calculation, in which the magnetic field $\boldsymbol{B} = (0, B_y, B_z)$ with $qB_z = \pm\frac{1}{2}g\mathcal{B}_z$ and the electric field $\boldsymbol{E} = (0, 0, E_z)$ with $qE_z = \pm\frac{1}{2}g\mathcal{E}_z$. Here $\pm$ labels the different color components. We will define K to be the coordinate frame in which the electromagnetic field has this form.

We hence need to compute the induced electromagnetic current density $j^\mu = q\langle\bar{\psi}\gamma^\mu\psi\rangle$ in K. To do this we will start in a different coordinate system K' in which $\boldsymbol{E} = (0, 0, E'_z)$ and $\boldsymbol{B} = (0, 0, B'_z)$. In this frame it is rather straightforward to do calculations. Then by applying a Lorentz transformation we can obtain the results in K as is illustrated in Fig. 2. We will switch on the electric field in K' uniformly at a time t'_i, i.e., $E'_z(t') = E'_z\theta(t' - t'_i)$. In this way the situation in K' is completely homogeneous.

In K' particle-antiparticle pairs are produced by the Schwinger process [4]. The rate per unit volume of this process equals [18] (see also [19,20])

$$\Gamma = \frac{q^2 E'_z B'_z}{4\pi^2} \coth\left(\frac{B'_z}{E'_z}\pi\right) \exp\left(-\frac{m^2\pi}{|qE'_z|}\right). \tag{1}$$

The production of pairs in K' gives rise to a homogeneous electromagnetic current density j'^μ. Because of symmetry reasons the only nonvanishing component of this current lies in the z direction. Furthermore, each time a pair is created the current will grow. Eventually when both components of the pair are accelerated by the electric field to (nearly) the speed of light, the net effect of the creation of one single pair will be that the total current has increased by two units of q. Therefore, sufficiently long after the switch-on, the change in current density in the z direction becomes $2q$ times the rate per unit volume of pair production, to be precise $\partial_{t'}\boldsymbol{j}' = 2q\Gamma\,\mathrm{sgn}(qE'_z)\boldsymbol{e}_z$. This equation has been verified explicitly both analytically [21,22] and numerically [23] even for $m \neq 0$.

Before we compute the induced currents in K let us point out that the rate Γ is consistent with the anomaly equation.

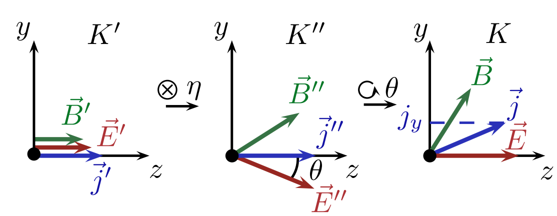

FIG. 2 (color online). Lorentz transformation from a frame K' in which the electric field ($\boldsymbol{E}$), magnetic field ($\boldsymbol{B}$), and the current density ($\boldsymbol{j}$) are parallel to each other, to a frame K in which $\boldsymbol{B}$ and $\boldsymbol{j}$ have a component perpendicular to $\boldsymbol{E}$.

In the limit of a very large magnetic field ($B_z' \gg E_z'$) all produced pairs will reside in the lowest Landau level causing maximal chiral asymmetry. Since each pair then produces two units of N_5, the pair-production rate should then be equal to half the chirality rate. Taking the limit $B_z' \gg E_z'$ in Eq. (1) gives

$$\Gamma \mathrm{sgn}(E_z' B_z') \approx \frac{q^2}{4\pi^2} E_z' B_z' \exp\left(-\frac{m^2\pi}{|qE_z'|}\right) = \tfrac{1}{2}\partial_{t'} n_5', \quad (2)$$

which is indeed in agreement with the anomaly equation (see Introduction) in the limit of $m = 0$, since the chiral current $\boldsymbol{j}_5$ vanishes because of homogeneity. It turns out that Eq. (2) also exactly gives the chirality rate for nonzero m and any E_z' and B_z' [22].

As is indicated in Fig. 2 we can go from frame K' to K'' by applying a boost with rapidity η in the x direction. In the new coordinate system K'' obtained by this boost, the electric and magnetic field, respectively, read $\boldsymbol{E}'' = -B_z' \sinh\eta \boldsymbol{e}_y + E_z' \cosh\eta \boldsymbol{e}_z$ and $\boldsymbol{B}'' = E_z' \sinh\eta \boldsymbol{e}_y + B_z' \cosh\eta \boldsymbol{e}_z$. Since j'^{μ} points in the z direction, the direction of j'^{μ} will not change after the boost in the x direction. However, because the boost implies that $t' = t'' \cosh\eta + x'' \sinh\eta$, the current density rate is modified to $\partial_{t''} \boldsymbol{j}'' = 2q\Gamma \mathrm{sgn}(qE_z') \cosh\eta \boldsymbol{e}_z$. The current density has now also obtained a gradient in the x direction ($\partial_{x''} \boldsymbol{j}'' \neq 0$). This and other inhomogeneities in K'' arise because the uniform switch-on of $\boldsymbol{E}'$ at t_i' implies an inhomogeneous switch-on of part of $\boldsymbol{E}''$ and $\boldsymbol{B}''$ at $t'' = t_i'/\cosh\eta - x'' \tanh\eta$.

To arrive in frame K we have to apply a rotation with angle θ around the x axis such that the electric field points in the z direction. The angle θ follows from Fig. 2 and satisfies $\sin\theta = -E_y''/E_z = B_z' \sinh(\eta)/E_z$ and $\cos\theta = E_z''/E_z = E_z' \cosh(\eta)/E_z$. The current density rate becomes

$$\partial_t \boldsymbol{j} = q\Gamma\left[\sinh(2\eta)\frac{B_z'}{E_z}\boldsymbol{e}_y + \cosh^2\eta \frac{2E_z'}{E_z}\boldsymbol{e}_z\right]\mathrm{sgn}(qE_z'). \quad (3)$$

We can eliminate η by expressing the above in terms of the fields in K. The magnetic field is $B_y = E_z' \sinh\eta \cos\theta + B_z' \cosh\eta \sin\theta$, implying that $\sinh(2\eta) = 2B_y E_z/(E_z'^2 + B_z'^2)$. Because both $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(B_y^2 + B_z^2 - E_z^2) = \frac{1}{2}(B_z'^2 - E_z'^2)$ and $\mathcal{H} = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = E_z B_z = E_z' B_z'$ are Lorentz invariant, one finds $a \equiv |E_z'| = (\sqrt{\mathcal{F}^2 + \mathcal{H}^2} - \mathcal{F})^{1/2}$, and $b \equiv |B_z'| = (\sqrt{\mathcal{F}^2 + \mathcal{H}^2} + \mathcal{F})^{1/2}$.

After summing over colors the z component of the current vanishes ($\partial_t j_z = 0$), implying that the only remaining component lies in the y direction. Using that $q\mathrm{sgn}(qE_z')B_z' = |q|\mathrm{sgn}(\mathcal{E}_z \mathcal{B}_z)b$ we obtain after summing over colors, for each quark flavor separately,

$$\partial_t j_y = \frac{q^2|q|B_y}{\pi^2}\frac{ab^2 \mathrm{sgn}(\mathcal{E}_z\mathcal{B}_z)}{a^2+b^2}\coth\left(\frac{\pi b}{a}\right)\exp\left(-\frac{m^2\pi}{|qa|}\right), \quad (4)$$

where a and b have dependence on $qE_z = \pm\frac{1}{2}g\mathcal{E}_z$ and $qB_z = \pm\frac{1}{2}g\mathcal{B}_z$. The rate of chirality production in K becomes $\partial_t n_5 = \cosh^2\eta \partial_{t'} n_5'$. Inserting Eq. (2) yields for the rate of current over chirality density generation

$$\frac{1}{|q|}\frac{\partial_t j_y}{\partial_t n_5} = \frac{2q^2 B_y b \coth(\pi b/a)}{q^2(a^2 + b^2 + B_y^2) + \frac{1}{4}g^2(\mathcal{E}_z^2 + \mathcal{B}_z^2)}. \quad (5)$$

Discussion.—Equation (4) clearly shows that an external magnetic field induces a current perpendicular to the color-flux tube. We display in Fig. 3 for three different values of $\xi = |\mathcal{B}_z/\mathcal{E}_z|$ the rate of generation of this current normalized to Eq. (5), the rate of chirality production. We will now analyze our results and show that $\partial_t j_y$ indeed behaves as the chiral magnetic effect predicts.

First of all let us take either $\mathcal{E}_z = 0$ or $\mathcal{B}_z = 0$, which implies that no chirality is generated. If $\mathcal{E}_z = 0$ then $a = 0$, for $\mathcal{B}_z = 0$ either $a = 0$ or $b = 0$. In all these cases $\partial_t j_y$ indeed vanishes as follows from Eq. (4). This is obvious when $a = 0$ since in that case no particles are produced as follows from Eq. (1). Also as expected $\partial_t j_y$ vanishes if there is no perpendicular magnetic field which can be seen from Fig. 3 as well.

Second, in the limit of $qB_y \gg g\mathcal{E}_z, g\mathcal{B}_z$, we have $b \simeq |B_y|$ so that from Eq. (5) it follows that $\partial_t j_y = |q|\mathrm{sgn}(B_y)\partial_t n_5$. This indicates that for large magnetic fields the current rate is indeed exactly given by the chirality rate in agreement with the prediction outlined in the introduction. Therefore the curves in Fig. 3 approach unity for when both $qB_y/g\mathcal{E}_z$ and $qB_y/(g\mathcal{E}_z\xi)$ are large.

A finite mass reduces the chirality and indeed also $\partial_t j_y$ as can be seen from Eq. (4). In fact Eq. (5) shows for any value of the mass the current is proportional to the chirality. Hence the curves displayed in Fig. 3 are independent of mass. Moreover, let us point out that the direction of the current is independent of the sign of the quark charge, but does depend on the direction of the magnetic field and the sign of the chirality, i.e., $\mathrm{sgn}(\mathcal{E}_z\mathcal{B}_z)$. For qB_y small compared to both $g\mathcal{E}_z$ and $g\mathcal{B}_z$, we have $a \simeq |\frac{g}{2q}\mathcal{E}_z|$ and

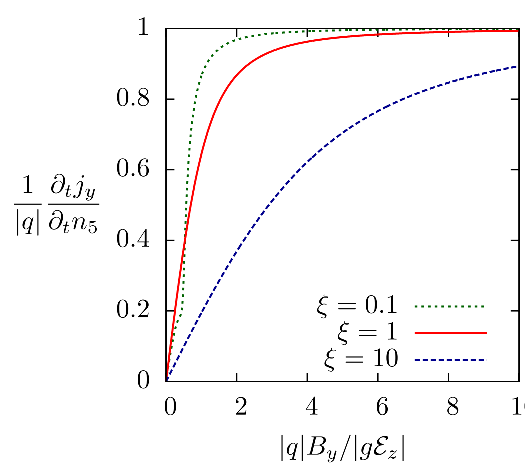

FIG. 3 (color online). Rate of current (j_y) over chirality density (n_5) generation in a color-flux tube, as a function of the perpendicular magnetic field B_y. The ratio $\xi = |\mathcal{B}_z/\mathcal{E}_z|$. The curves are valid for any value of the quark mass.

$b \simeq |\frac{g}{2q}\mathcal{B}_z|$ so that

$$\partial_t j_y \simeq \frac{q^2 B_y}{2\pi^2} \frac{g\mathcal{E}_z \mathcal{B}_z^2}{\mathcal{B}_z^2 + \mathcal{E}_z^2} \coth\left(\frac{\mathcal{B}_z}{|\mathcal{E}_z|}\pi\right) \exp\left(-\frac{2m^2\pi}{|g\mathcal{E}_z|}\right). \quad (6)$$

The linear dependence on B_y for small B_y is clearly visible in Fig. 3. The small kink at $qB_y/g\mathcal{E}_z \simeq 1/2$ and $\xi = 0.1$ is due to the fact that a and b vary rapidly around $\mathcal{F} = 0$ when $|\mathcal{H}|$ is small compared to $|\frac{g}{2q}\mathcal{B}_z|^2$, which is equivalent to $\xi \ll 1$.

The generation of a current by the transformation from frame K' to K is a very general result of Lorentz invariance, and is equivalent to the Lorentz force in frame K'. Therefore any charged colored particle that is present in the color-flux tube plus magnetic field background will experience a force in the y direction if $\mathcal{E}_z\mathcal{B}_z \neq 0$. To illustrate this we can consider the whole calculation for fictional colored and electrically charged scalar particles. In that case there is no anomaly so that no chirality is generated. The results for scalars can be obtained by replacing $\coth(\pi b/a)$ by $1/[2\sinh(\pi b/a)]$ in Eq. (1) [19] and in all subsequent equations. The ratio between the scalar and fermion current density rate becomes simply $1/[2\cosh(\pi b/a)]$, which is approximately $1/[2\cosh(\pi\xi)]$ for $qB_y \lesssim g\mathcal{E}_z/2$ and $1/\{2\cosh[\pi(2qB_y)^2/(g^2\mathcal{E}_z\mathcal{B}_z)]\}$ for $qB_y \gtrsim g\mathcal{E}_z/2$. Clearly scalar particles behave completely different from the predictions of the chiral magnetic effect, moreover the scalar contribution to j_y is always smaller than that of fermions and even exponentially suppressed for $qB_y \gtrsim g\mathcal{E}_z/2$.

Let us finally stress that our quantitative results are strictly speaking only valid for the rather special inhomogeneous switch-on of the fields in the color-flux tube. Nevertheless, as is the case at later times in heavy-ion collisions, if B_y is small compared to the color fields, the effects of the inhomogeneous switch-on are marginal. Therefore it is very likely that the result for small B_y, Eq. (6), is also correct for a homogeneous switch-on. To further address this issue one can start from an inhomogeneous switch-on in K' that becomes homogeneous in K''. However, this situation is more complicated, at present we are unfortunately unable to solve it exactly.

To conclude, we have shown by a dynamical calculation that if topological charge is present in a magnetic field, an electromagnetic current will be generated along this magnetic field. This very natural mechanism is called the chiral magnetic effect and signals $\mathcal{P}$- and $\mathcal{CP}$-odd interactions. As such it could be an explanation for the charge correlations in heavy-ion collisions observed by the STAR collaboration [10].

We are grateful to Antti Gynther, Larry McLerran, Anton Rebhan, and Andreas Schmitt for discussions. The work of H.J.W. was supported by the Alexander von Humboldt Foundation. K.F. was supported by the Japanese MEXT Grant No. 20740134 and by the Yukawa Program for Quark Hadron Sciences. This manuscript has been authored under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy.

[1] A. A. Belavin, A. M. Polyakov, A. S. Shvarts, and Yu. S. Tyupkin, Phys. Lett. B **59**, 85 (1975).
[2] R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976); C. G. Callan, R. F. Dashen, and D. J. Gross, Phys. Lett. B **63**, 334 (1976).
[3] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); Phys. Rev. D **14**, 3432 (1976).
[4] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).
[5] S. L. Adler, Phys. Rev. **177**, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento A **60**, 47 (1969).
[6] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. **A803**, 227 (2008).
[7] V. Skokov, A. Y. Illarionov, and V. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009).
[8] D. Kharzeev, Phys. Lett. B **633**, 260 (2006); D. Kharzeev and A. Zhitnitsky, Nucl. Phys. **A797**, 67 (2007);
[9] S. A. Voloshin, Phys. Rev. C **70**, 057901 (2004).
[10] B. I. Abelev *et al.* (STAR Collaboration), Phys. Rev. Lett. **103**, 251601 (2009); B. I. Abelev *et al.* (STAR Collaboration), arXiv:0909.1717.
[11] F. Wang, arXiv:0911.1482; R. Millo and E. Shuryak, arXiv:0912.4894; A. Bzdak, V. Koch, and J. Liao, Phys. Rev. C **81**, 031901(R) (2010).
[12] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008); D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **80**, 034028 (2009); K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Nucl. Phys. **A836**, 311 (2010).
[13] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya, and M. I. Polikarpov, Phys. Rev. D **80**, 054503 (2009); M. Abramczyk, T. Blum, G. Petropoulos, and R. Zhou, arXiv:0911.1348.
[14] H. U. Yee, J. High Energy Phys. 11 (2009) 085; A. Rebhan, A. Schmitt, and S. A. Stricker, J. High Energy Phys. 01 (2010) 026.
[15] S. i. Nam, Phys. Rev. D **80**, 114025 (2009).
[16] H. B. Nielsen and M. Ninomiya, Phys. Lett. B **130**, 389 (1983); A. Y. Alekseev, V. V. Cheianov, and J. Frohlich, Phys. Rev. Lett. **81**, 3503 (1998); M. A. Metlitski and A. R. Zhitnitsky, Phys. Rev. D **72**, 045011 (2005); G. M. Newman and D. T. Son, Phys. Rev. D **73**, 045006 (2006); J. Charbonneau and A. Zhitnitsky, arXiv:0903.4450; E. V. Gorbar, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. C **80**, 032801(R) (2009).
[17] D. Kharzeev, A. Krasnitz, and R. Venugopalan, Phys. Lett. B **545**, 298 (2002); T. Lappi and L. McLerran, Nucl. Phys. **A772**, 200 (2006); D. Kharzeev, E. Levin, and K. Tuchin, Phys. Rev. C **75**, 044903 (2007).
[18] A. I. Nikishov, Zh. Eksp. Teor. Fiz. **57**, 1210 (1969); F. V. Bunkin and I. I. Tugov, Sov. Phys. Dokl. **14**, 678 (1970).
[19] G. V. Dunne, arXiv:hep-th/0406216.
[20] T. D. Cohen and D. A. McGady, Phys. Rev. D **78**, 036008 (2008).
[21] S. P. Gavrilov and D. M. Gitman, Phys. Rev. D **78**, 045017 (2008).
[22] H. J. Warringa (to be published).
[23] N. Tanji, Ann. Phys. (N.Y.) **324**, 1691 (2009).