

New Correlated Model of Colossal Magnetoresistive Manganese Oxides

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A new minimal model is constructed for the doped manganese oxides which exhibit colossal magnetoresistance, involving a broad spin-majority conduction band as well as nearly localized spin-minority electron states. A simple mean-field analysis yields a temperature-dependent hybridized band structure with suppressed carrier weight at the Fermi level. Spin stiffness is complex, indicating strong spin-wave damping. Further investigations are needed to verify the relevance of the proposed model.

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The unusual properties of doped manganese oxides exhibiting colossal magnetoresistance (CMR) [1] are not yet understood theoretically, and the problem of formulating a suitable microscopic model remains open. These properties include the CMR phenomenon and metal-insulator transition, which are, in turn, intimately related to the temperature-induced variation of the electron density of states (“pseudogap” [2]) or of the effective carrier number [3]. This implies that, alongside double-exchange ferromagnetism, the effects of electron-electron interaction play a key role and, quite possibly, are responsible for CMR itself. Indeed, the band structure calculations [4] suggest that the on-site Hubbard repulsion U is the largest energy in the problem. Its effects are typically considered within the one- or two-orbital model whereby the strongly correlated behavior is induced by an interaction between the two spin-majority electronic e_g states on-site, or between the two spin components of a single e_g band [5].

It should be noted that the experimental data [6] indicate the presence of *spin-minority* electrons near the Fermi level even in the low-temperature ferromagnetic state. This agrees with the band structure calculations [4,7], suggesting that a narrow spin-minority band lies close to the Fermi energy. Since both the localized t_{2g} and itinerant e_g states originate from the same d shell of a Mn ion, and therefore are characterized by approximately the same value of Hund’s rule splitting J_H , it is clear [see Fig. 1(a)] that these spin-minority electrons populate the spin-down t_{2g} (localized) states [4]. This is further corroborated by the studies of $\text{La}_{1-x}\text{Ce}_x\text{MnO}_3$ (with $1+x$ conduction electrons per formula unit) [8], which show both the spin-minority character of the carriers and a large overall increase in the resistivity (in comparison with the usual $1-x$ electron case, e.g., $\text{La}_{1-x}\text{Ca}_x\text{MnO}_3$), consistent with the extra x electrons going into the spin-down t_{2g} states. Yet, while “two-fluid” models involving itinerant and localized states were suggested earlier by both experimentalists and theorists [9–11], the appealing possibility (mentioned in Ref. [12]) of these states having antialigned spins has not been addressed theoretically.

Furthermore, we note that electronic properties of a model where the orbital degree of freedom is taken into

account are strongly coupled to lattice dynamics via the Jahn-Teller effect [1], which results in additional splitting of both e_g and t_{2g} levels. However, CMR occurs in a broad class of bulk systems [three-dimensional (3D) perovskites and quasi-two-dimensional (quasi-2D) bilayered] and thin films of varying composition, and is presumably always due to the same physical mechanism. It is therefore worthwhile to consider a model with fewer orbitals, which still captures some of the important *intra-atomic* physics of a Mn ion, before pursuing more complicated (and probably more material-specific) options [13].

In the present Letter, we introduce such a simplified description and proceed with a simple mean-field analysis. While finer theoretical tools are required to gain a fuller picture, qualitatively our results for electron dispersion and magnetic properties appear very encouraging.

We consider a model involving a large spin \vec{S} and two conduction-electron orbitals (broadened and nearly localized) at each Mn site [cf. Figure 1(b)]:

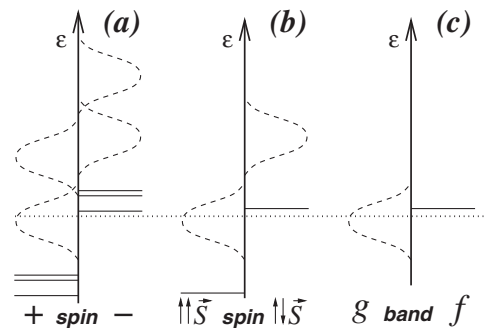


FIG. 1. (a) Crystal field splits the d -electron states of a Mn ion into t_{2g} (solid lines) and e_g (dashed lines); these split further due to a lattice distortion. Spin degeneracy is broken due to Hund’s rule, and the “spin-plus” t_{2g} electrons form the local spin $3/2$. Chemical potential (dotted line) lies within an e_g band, with a “spin-minus” t_{2g} level nearby. (b) A simplified model of Eq. (1), with the spin quantization axis fixed by a local spin S . (c) Relevant orbitals in the limit $J_H \rightarrow \infty$ (see text); adding an itinerant (localized) electron increases (reduces) the total on-site spin of $S + 1/2$ by $1/2$.

$$\begin{aligned}
\mathcal{H} = & -\frac{t}{2} \sum_{\langle i,j \rangle, \alpha} (c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha}) + E_d^{(0)} \sum_{i,\alpha} d_{i\alpha}^\dagger d_{i\alpha} \\
& - \frac{J_H}{S} \sum_i \vec{S}_i \vec{\sigma}_i + \frac{J}{S^2} \sum_{\langle i,j \rangle} \vec{S}_i \vec{S}_j - \frac{H}{S} \sum_i (S_i^z + \sigma_i^z) \\
& + U \sum_i \left(c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow} + d_{i\uparrow}^\dagger d_{i\downarrow}^\dagger d_{i\downarrow} d_{i\uparrow} + \sum_{\alpha,\beta} c_{i\alpha}^\dagger d_{i\beta}^\dagger d_{i\beta} c_{i\alpha} \right) \\
& - \frac{V}{2} \sum_{\langle i,j \rangle, \alpha} (c_{i\alpha}^\dagger d_{j\alpha} + c_{j\alpha}^\dagger d_{i\alpha} + d_{i\alpha}^\dagger c_{j\alpha} + d_{j\alpha}^\dagger c_{i\alpha}). \quad (1)
\end{aligned}$$

Here, the operators $c_{i\alpha}$ ($d_{i\alpha}$) annihilate an e_g (t_{2g}) conduction electron of spin $\alpha = \uparrow, \downarrow$ (in the laboratory frame) at a site i of a square (or simple cubic) lattice. Localized spins S_i originate from the remaining two t_{2g} electrons; hence, in reality $S = 1$. They interact via superexchange J and are also coupled to the spins of conduction electrons on-site, $\vec{\sigma}_i = \frac{1}{2} \sum_{\alpha,\beta} \vec{\sigma}_{\alpha\beta} (c_{i\alpha}^\dagger c_{i\beta} + d_{i\alpha}^\dagger d_{i\beta})$ (where $\vec{\sigma}_{\alpha\beta}$ are the Pauli matrices) via a strong ferromagnetic Hund's rule exchange J_H ; the external magnetic field H is applied along the z axis. Owing to the fact that the (electron) coordinate operator is not diagonal in the band index, there is a hybridization V between the t_{2g} and e_g states; $E_d^{(0)}$ is the (bare) energy of the t_{2g} electrons. Direct hopping between t_{2g} states on different sites is assumed to be negligible, while the nearest-neighbor hopping t between the e_g states will be used as an energy unit, $t = 1$.

We construct the spin-wave expansion for the model (1), keeping terms up to first order in $1/S$. In the spirit of Ref. [14], it is expedient to introduce a new basis of electron states on each site according to

$$\begin{aligned}
c_{\uparrow} & \approx g_{\uparrow} - \frac{1}{\sqrt{2S}} g_{\downarrow} \beta^\dagger - \frac{1}{4S} (g_{\uparrow} \beta^\dagger \beta + g_{\downarrow}^\dagger g_{\uparrow} g_{\uparrow} + f_{\downarrow}^\dagger g_{\downarrow} f_{\uparrow}), \\
c_{\downarrow} & \approx g_{\downarrow} + \frac{1}{\sqrt{2S}} g_{\uparrow} \beta - \frac{1}{4S} (g_{\downarrow} + g_{\uparrow} \beta^\dagger \beta - g_{\uparrow}^\dagger g_{\downarrow} g_{\downarrow} - f_{\uparrow}^\dagger g_{\uparrow} f_{\downarrow})
\end{aligned}$$

(expressions for d_{\uparrow} and d_{\downarrow} are obtained by substituting $g_\alpha \leftrightarrow f_\alpha$). Operators $g_{i\uparrow}$ and $f_{i\uparrow}$ ($g_{i\downarrow}$ and $f_{i\downarrow}$) correspond, respectively, to the e_g and t_{2g} electrons with a spin parallel (antiparallel) to the total spin on-site, $\vec{T}_i = \vec{S}_i + \vec{\sigma}_i$, whose vibrations are annihilated by a Holstein-Primakoff magnon operator β_i . The Hund's rule term takes form

$$\begin{aligned}
-\frac{J_H}{S} \vec{S}_i \vec{\sigma}_i = & -\frac{J_H}{2} \left\{ g_{i\uparrow}^\dagger g_{i\uparrow} + f_{i\uparrow}^\dagger f_{i\uparrow} - \frac{S+1}{S} (g_{i\downarrow}^\dagger g_{i\downarrow} + f_{i\downarrow}^\dagger f_{i\downarrow}) \right. \\
& \left. + \frac{1}{S} [g_{i\uparrow}^\dagger g_{i\downarrow}^\dagger g_{i\downarrow} g_{i\uparrow} - g_{i\uparrow}^\dagger f_{i\downarrow}^\dagger g_{i\downarrow} f_{i\uparrow} + (f \leftrightarrow g)] \right\}.
\end{aligned}$$

Finally, the Holstein-Primakoff operators a_i of the original spins \vec{S}_i are expressed as

$$a \approx \beta - \frac{g_{\uparrow}^\dagger g_{\downarrow} + f_{\uparrow}^\dagger f_{\downarrow}}{\sqrt{2S}} - \frac{g_{\uparrow}^\dagger g_{\uparrow} + f_{\uparrow}^\dagger f_{\uparrow} - g_{\downarrow}^\dagger g_{\downarrow} - f_{\downarrow}^\dagger f_{\downarrow}}{4S} \beta.$$

We next substitute these expressions into the Hamiltonian (1) and take the limit of $J_H \rightarrow \infty$ while keep-

ing $E_d = E_d^{(0)} + J_H$ constant. Now, if the chemical potential, denoted $\mu + U - J_H/2$, lies within the spin-up e_g band, then the spin-down e_g band is completely empty, and the spin-up t_{2g} band completely filled. This is precisely the case of interest to us, containing the effects of the Coulomb repulsion between the spin-up e_g and spin-down t_{2g} electrons in the presence of a localized spin $S + 1/2$ [Fig. 1(c)]. Henceforth, we drop all the terms containing the operators g_{\downarrow} and f_{\downarrow} , and suppress the spin index of remaining fermion operators.

Assuming the ferromagnetic ground state, the Hamiltonian takes the form $\mathcal{H}_e + \mathcal{H}_h + \mathcal{H}_m$ with the electronic and (magnon-assisted) hybridization terms,

$$\begin{aligned}
\mathcal{H}_e = & \sum_{\vec{k}} (\epsilon_{\vec{k}} - \mu) g_{\vec{k}}^\dagger g_{\vec{k}} + (E_d - \mu) \sum_j f_j^\dagger f_j \\
& + \frac{U}{N} \sum_{\vec{k}, \vec{k}', j} e^{i(\vec{k}' - \vec{k}) \vec{R}_j} g_{\vec{k}}^\dagger f_j^\dagger f_j g_{\vec{k}'}, \quad (2)
\end{aligned}$$

$$\mathcal{H}_h = -\frac{V}{\sqrt{2SN}} \sum_{\vec{k}, \vec{q}, j} (\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}}) e^{-i(\vec{k}+\vec{q}) \vec{R}_j} g_{\vec{k}}^\dagger f_j \beta_{\vec{q}}^\dagger + \text{H.c.} \quad (3)$$

Here, N is the number of lattice sites, $\epsilon_{\vec{k}} = -\cos k_x - \cos k_y (-\cos k_z)$ is the tight-binding dispersion law in two (three) dimensions, and \vec{R}_j is the radius vector of site j . It is assumed that the site basis is more appropriate for describing the narrow-band fermions f_j , easily localized by fluctuations or disorder. The term \mathcal{H}_m , which is of order $1/S$, contains Zeeman electron energy shifts, magnon dynamics, and double-exchange band-narrowing effects:

$$\begin{aligned}
\mathcal{H}_m = & \frac{H}{2S} \left(\sum_j f_j^\dagger f_j - \sum_{\vec{k}} g_{\vec{k}}^\dagger g_{\vec{k}} \right) + \frac{1}{S} \sum_{\vec{k}} [H - 2J(\epsilon_{\vec{k}} + d)] \\
& \times \beta_{\vec{k}}^\dagger \beta_{\vec{k}} + \frac{1}{4SN} \sum_{l=4}^l (2\epsilon_{2+l} - \epsilon_l - \epsilon_2) g_{\uparrow}^\dagger g_{2\beta_3}^\dagger \beta_4. \quad (4)
\end{aligned}$$

Here, momentum-conserving summation is denoted by Σ' , and d is the dimensionality of the system (2 or 3).

The electronic term, \mathcal{H}_e , is the familiar Falikov-Kimball model. The rich physics contained therein [15] crucially depends on the presence (and nature) of the interband hybridization. The form of our \mathcal{H}_h , Eq. (3), is dictated by spin conservation: electron transfer between the two opposite-spin bands must be accompanied by magnon creation or annihilation, $\beta_{\vec{q}}^\dagger$ or $\beta_{\vec{q}}$. Such transfers require misalignment of spins \vec{S}_i on neighboring sites; hence, the hybridization matrix element vanishes at $q \rightarrow 0$, underlining the importance of short-wavelength processes. The latter feature appears promising in the context of CMR compounds, where the unusual short-range correlations are reflected in the electronic and magnetic [16,17] properties. Importantly, magnetic field H affects the carriers both via double-exchange mechanism and by changing the energy difference between localized and itinerant states [see Eq. (4)].

We shall be interested in the regime characterized by nonzero values of *both* fermion occupancies n^g and n^f (the latter assumed independent on \vec{R}_i),

$$n^f = \langle f_i^\dagger f_i \rangle, \quad n^g = \frac{1}{N} \sum_{\vec{k}} n_{\vec{k}}^g, \quad n_{\vec{k}}^g = \langle g_{\vec{k}}^\dagger g_{\vec{k}} \rangle. \quad (5)$$

In addition, there also arises an off-diagonal average,

$$\langle f_j^\dagger g_{\vec{k}} \beta_{\vec{q}} \rangle \equiv \langle f_j^\dagger g_{\vec{k}} \beta_{\vec{q}} \rangle e^{-i(\vec{k}+\vec{q})\vec{R}_j} \quad (6)$$

(here and below, we omit the site indexes of operators f_j once the j -dependent exponent has been factored out).

In order to clarify the basic physics contained in our model, Eqs. (2)–(4), we will now proceed with a mean-field analysis of it. Here, we focus on the simplest self-consistent scheme, allowing only for average values (5) and (6) and for a nonzero magnon occupancy. While actual validity of this approach is probably restricted to the intermediate temperature range (on the scale of the Curie temperature) and moderate values of U (see below), it offers important guidance for future investigations. Mean-field equations can be found in a standard way by decoupling the equations of motion for the appropriate retarded Green's functions, expressing the latter as

$$\langle\langle f f^\dagger \rangle\rangle \equiv \langle\langle f_j f_j^\dagger \rangle\rangle = \left\{ \omega - \tilde{E}_d + \mu + i0 - \frac{V}{2SN^2} \sum_{\vec{k}, \vec{q}} \frac{V(1 + N_{\vec{q}} - n_{\vec{k}}^g)(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})^2 + U\Phi_{\vec{q}}(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})}{\omega + i0 - \tilde{\epsilon}_{\vec{k}} + \mu} \right\}^{-1}, \quad (7)$$

$$\langle\langle g_{\vec{k}} g_{\vec{k}}^\dagger \rangle\rangle = \left\{ \omega - \tilde{\epsilon}_{\vec{k}} + \mu + i0 - \frac{V}{2SN} \sum_{\vec{q}} [V(N_{\vec{q}} + n^f)(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})^2 + U\Phi_{\vec{q}}^*(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})] \right\}^{-1}, \quad (8)$$

$$\langle\langle \beta_{\vec{q}} \beta_{\vec{q}}^\dagger \rangle\rangle = \left\{ \omega - \omega_{\vec{q}}^0 + i0 + \frac{V^2}{2NS} \sum_{\vec{k}} \frac{(n^f - n_{\vec{k}}^g)(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})^2}{\omega + i0 - \tilde{E}_d + \tilde{\epsilon}_{\vec{k}}} + \frac{V^2 U}{2SN^2} \frac{[\sum_{\vec{k}} \frac{(n^f - n_{\vec{k}}^g)(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})^2}{\omega + i0 - \tilde{E}_d + \tilde{\epsilon}_{\vec{k}}}]^2}{1 - \frac{U}{N} \sum_{\vec{k}} \frac{n^f - n_{\vec{k}}^g}{\omega + i0 - \tilde{E}_d + \tilde{\epsilon}_{\vec{k}}}} \right\}^{-1}, \quad (9)$$

$$\Phi_{\vec{q}} = \Phi_{\vec{q}}^* \equiv \sqrt{2S} \sum_{\vec{k}} \langle g_{\vec{k}}^\dagger f \beta_{\vec{q}}^\dagger \rangle = -\frac{V}{\pi} \int \text{Im} \left\{ \frac{\frac{1}{N} \sum_{\vec{k}} \frac{(n^f - n_{\vec{k}}^g)(\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{q}})}{\omega + i0 - \tilde{E}_d + \tilde{\epsilon}_{\vec{k}}}}{1 - \frac{U}{N} \sum_{\vec{k}} \frac{n^f - n_{\vec{k}}^g}{\omega + i0 - \tilde{E}_d + \tilde{\epsilon}_{\vec{k}}}} \langle\langle \beta_{\vec{q}} \beta_{\vec{q}}^\dagger \rangle\rangle \right\} \frac{d\omega}{\exp(\omega/T) - 1}. \quad (10)$$

Here, the magnon occupancy is

$$\mathcal{N}_{\vec{q}} \equiv \langle \beta_{\vec{q}}^\dagger \beta_{\vec{q}} \rangle = -\frac{1}{\pi} \int \text{Im} \langle\langle \beta_{\vec{q}} \beta_{\vec{q}}^\dagger \rangle\rangle \frac{d\omega}{\exp(\omega/T) - 1} \quad (11)$$

($n_{\vec{k}}^g$ and n^f are expressed in a similar way). Hartree energies of magnons and those of e_g and t_{2g} electrons read

$$\omega_{\vec{q}}^0 = \frac{H}{S} - \frac{2J}{S} (\epsilon_{\vec{q}} + d) + \frac{1}{2NS} \sum_{\vec{k}} n_{\vec{k}}^g (\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}), \quad (12)$$

$$\tilde{\epsilon}_{\vec{k}} = \epsilon_{\vec{k}} + Un^f - \frac{H}{2S} + \frac{1}{2NS} \sum_{\vec{q}} \mathcal{N}_{\vec{q}} (\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}}), \quad (13)$$

$$\tilde{E}_d = E_d + \frac{U}{N} \sum_{\vec{k}} n_{\vec{k}}^g + \frac{H}{2S}. \quad (14)$$

The last terms in Eqs. (12) and (13) contain the double-exchange physics (ferromagnetic contribution to the spin-wave spectrum and the conduction band narrowing).

In analyzing Eqs. (7)–(10) we consider a 2D system; the 3D case can be expected to be similar. We replace all factors $(\epsilon_{\vec{k}} - \epsilon_{\vec{k}-\vec{q}})^2$ with their average values over the isoenergetic surfaces [18] $\epsilon = \epsilon_{\vec{k}}$ and $\epsilon = \epsilon_{\vec{q}}$. We arrive at a system of mean-field equations for μ , n_b , and four quantities $\int \Phi(\epsilon)(2 + \epsilon)\nu d\epsilon$, $\int \mathcal{N}(\epsilon)\epsilon\nu d\epsilon$, $\int \mathcal{N}(\epsilon) \times \langle v^2 \rangle_\epsilon \nu d\epsilon$, and $\int \mathcal{N}(\epsilon)(4 - \epsilon^2)\nu d\epsilon$, where $\nu(\epsilon)$ and $\langle v^2 \rangle_\epsilon$ are the tight-binding density of states and average velocity

square at a given energy ϵ . Solving these equations numerically, we observe the following:

(i) The localized band is broadened, and a temperature-dependent gap Δ (a new small *energy scale*) opens in the spectrum of itinerant electrons (fermions $g_{\vec{k}}$). Quasiparticle weight of itinerant electrons decreases when the energy approaches the gap from either side. The Fermi level lies below the gap, and the *quasiparticle weight* at the Fermi surface is strongly *suppressed* [Figs. 2(a) and 2(b)]. This behavior, which is already reminiscent of a T -dependent (pseudo)gap found experimentally [2], will be further modified in a more exact treatment (e.g., a finite relaxation time will arise in the second order in $1/S$). Equations (7) and (8) imply that in the present model, these spectral features are directly controlled by spin dynamics.

(ii) With only the spin-majority electrons contributing to the spin stiffness D , which in 2D or 3D is given by

$$DS = -\frac{1}{4dN} \sum_{\vec{k}} \epsilon_{\vec{k}} n_{\vec{k}}^g - J - \frac{V^2}{2dN} \sum_{\vec{k}} \frac{n^f - n_{\vec{k}}^g}{\tilde{\epsilon}_{\vec{k}} - \tilde{E}_d + i0} \left(\frac{\partial \epsilon_{\vec{k}}}{\partial \vec{k}} \right)^2,$$

the latter is suppressed in comparison with the usual double-exchange case [Fig. 2(c)]. An unusual feature of the present model is that the spin-flip continuum extends down to zero energy and momentum [19]. As a result, D also develops an imaginary part. This implies *strong magnon damping*, as observed experimentally [17].

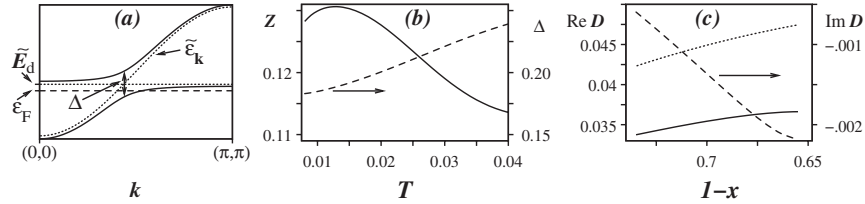


FIG. 2. Mean-field results for a 2D system with $E_d = -0.35$, $V = 0.2$, $U = 1.2$, and $J = 0$. (a) Schematic view of the itinerant fermion dispersion (solid lines); dotted lines show the unhybridized $\tilde{\epsilon}_k$ and \tilde{E}_d . (b) Temperature dependence of quasiparticle weight Z at the Fermi level (solid line) and the gap, $\Delta(T)$ (dashed line), for the electron density $1 - x = 0.7$. (c) Doping dependence of real (solid line) and imaginary (dashed) parts of spin stiffness D at $T = 0.0015$. Dotted line shows spin stiffness for a usual double-exchange model. t_{2g} occupancy increases from $n_f = 0.25$ at $1 - x = 0.65$ to $n_f = 0.43$ at $1 - x = 0.73$.

As for the diffusive central peak found in the inelastic neutron scattering [16], we expect it to arise once the magnon-assisted diffusive motion of t_{2g} electrons (neglected here) is taken into account. Experimentally, strongly damped magnons, central peak, and pseudogap in the density of states [2] (or optical Drude weight reduction [3]) are the key generic features of CMR manganates at the intermediate-to-high temperatures below T_C .

A relatively small value of U used in Fig. 2 is due to the reduced stability region for mean-field solutions with both n_f and n_g different from zero. This reduction is an expected artifact of a simplistic mean-field approach, mirroring, e.g., the greatly enhanced mean-field stability of ferromagnetism in the Hubbard model. This situation calls for further theoretical investigation, combining advanced mean-field schemes with numerical methods.

These future treatments will also have to address the issue of ferro- to paramagnetic transition and a possibility of charge ordering. We expect that any transition will be accompanied by a change of electron distribution between the two bands, thus changing the magnitude of the net spin \mathcal{T}_i on-site. Experimentally, the relevant quantity is the average total spin $\langle \mathcal{T} \rangle = (3 + n^g - n^f)/2$ of a Mn ion, which should show temperature and magnetic field dependence, especially in the region around and above T_C . In particular, this should lead to an unconventional longitudinal spin dynamics [20] and to a renormalization of the Curie-Weiss constant (cf. Ref. [21]). In principle, the value of $\langle \mathcal{T} \rangle$ should also be accessible more directly via muon spin rotation [22] and NMR [23] measurements. We suggest that these methods (combined with electron spectroscopy) should be used to measure the value of $\langle \mathcal{T} \rangle$. Its temperature dependence, especially if it correlates with (magneto)transport properties, would imply that a successful theoretical description of CMR compounds should indeed include spin-minority localized electrons.

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