Experimental Determination of the Statistics of Photons Emitted by a Tunnel Junction

Eva Zakka-Bajjani, J. Dufouleur,* N. Coulombel, P. Roche, D. C. Glattli,† and F. Portier‡
Service de Physique de l'Etat Condensé/IRAMIS/DSM (CNRS URA 2464), CEA Saclay, F-91191 Gif-sur-Yvette, France
(Received 22 December 2009; published 18 May 2010)

We report on an Hanbury Brown–Twiss experiment probing the statistics of microwave photons emitted by a tunnel junction in the shot-noise regime at low temperature. By measuring the cross correlation of the fluctuations of the occupation numbers of the photon modes of both detection branches, we show that while the statistics of electrons is Poissonian, the photons obey chaotic statistics. This is observed even for low photon occupation number when the voltage across the junction is close to $h\nu/e$.

DOI: 10.1103/PhysRevLett.104.206802 PACS numbers: 73.23.−b, 42.50.Ar, 73.50.Td

What is the statistics of microwave photons radiated by the current fluctuations of a quantum conductor? When the conductor is at equilibrium at temperature T, the statistics is that of blackbody radiation [1]. However, little is known about the statistics of photons emitted in the nonequilibrium case where the conductor is biased by a voltage $V \gg k_B T/e$ and the current fluctuations are due to quantum shot noise. An intriguing question is the link between the statistics of electrons and that of the emitted photons. This problem has recently attracted theoretical interest, and the full range of photon statistics, from chaotic to nonclassical, has been predicted. The result depends on the competition between the fermionic and bosonic statistics of electrons and photons, respectively [2]. On the one hand, the Pauli principle makes electrons emitted by a contact essentially noiseless, and the current noise only results from electron scattering with sub-Poissonian statistics. On the other hand, photons emitted by electrons may show a bunching effect transforming their initial statistics from sub-Poissonian to super-Poissonian. This rich physics relates to the problem of the electron full counting statistics [3], as the second moment of the photon noise directly links to a fourth moment of the current fluctuations [2,4,5].

In this work, we present the first measurements of the statistics of photons radiated by a quantum conductor in the shot-noise regime. For the simplest quantum conductor studied here, a tunnel junction, we show that although the statistics of electrons crossing the conductor is Poissonian, the photon statistics is chaotic. This is found even in the regime of vanishing electron shot noise where the voltage is close to the photon energy ($eV \gtrsim h\nu$) so that the photon population is small, in agreement with the prediction of [2]. As a by-product, our experimental method, based on Hanbury Brown–Twiss (HBT) microwave photon correlation, is found to provide a direct measurement of the nonsymmetrized current noise power, called emission noise. Here, the experiment linearly amplifies the field amplitude to a classical level and further detects the microwave power and its fluctuations. This contrasts with experiments measuring the average power with on-chip quantum detectors such as quantum dots or superconducting tunnel junctions [6].

The relation between current noise and photon emission can be understood following Nyquist's approach [1]. Consider a conductor of resistance R connected to a circuit made of a lossless transmission line with characteristic impedance Z_c and terminated by a matched resistor. For simplicity, let us assume that $Z_c \ll R$. When electrons in the conductor generate a current fluctuation $I(t)$, a voltage $V(t) = Z_c I(t)$ builds up at the input of the transmission line, exciting an electromagnetic mode that propagates and is finally absorbed in the resistive load. Introducing the spectral density of the current fluctuation $S_I(\nu) = 2\int_{-\infty}^{+\infty}\langle I(0)I(\tau)\rangle e^{i2\pi\nu\tau}d\tau$, we can express the electromagnetic power radiated by the conductor in frequency range ν, $\nu + d\nu$ as $dP = Z_c S_I(\nu)d\nu = N(\nu)h\nu d\nu$, where $N(\nu)$ is the mean photon population of the electromagnetic mode at frequency ν. This establishes a direct link between $S_I(\nu)$ and $N(\nu)$. Let us take one step further and consider the (low frequency) fluctuations δN^2 of the photon population.

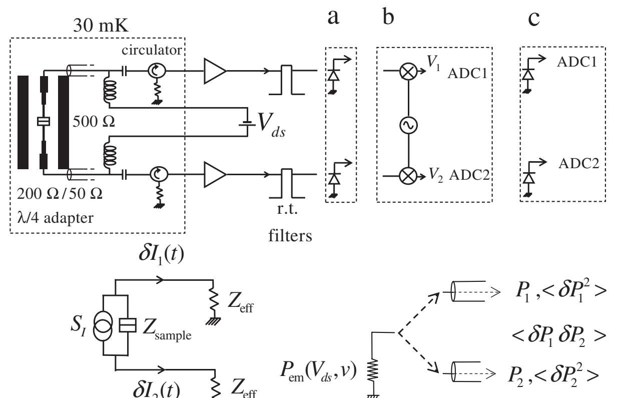

FIG. 1. Upper part: schematic diagram of the measurement setup corresponding to (a) the *first experiment*, dedicated to the measurement of finite frequency shot noise in both detection branches with quadratic detectors; (b) the *second experiment*, yielding the auto and cross-correlated finite current noises thanks to fast digitization of down-converted microwave signals; (c) the *third experiment*, probing the auto and cross-correlated power fluctuations thanks to fast digitization of the output voltage of the quadratic detectors. Lower part: equivalent microwave circuit.

They originate from the intrinsic fluctuations of the current noise in the conductor (the "noise" of the noise). However, the problem of the connection between the statistics of the photons and the electrons is complicated by the bunching effect occurring when several photons are simultaneously emitted into the transmission line. The photon distribution emitted by a classical current was first addressed by Glauber who showed that the photon statistics is Poissonian [7]. Solving the same problem in the case of quantum electronic shot noise requires a model that treats electrons and the detecting environment on the same quantum footing. Such a treatment was recently developed by Schomerus and Beenakker [2]. In particular, they have shown that a tunnel junction emits photons with chaotic statistics. This occurs even in the regime of small photon numbers when the voltage applied to the junction is close to $h\nu/e$. This is the regime addressed in this work.

The experimental setup, shown in Fig. 1, is similar to the one described in [8]. An $\mathrm{Al/AlO}_x\mathrm{/Al}$ tunnel junction of resistance $R_t = 502\ \Omega \pm 1\ \Omega$ was cooled to $\sim$30 mK in a dilution fridge. A 0.1 T magnetic field suppresses the Al superconductivity. The two sides of the tunnel junction are separately connected to 50 Ω coaxial transmission lines via two quarter wavelength impedance transformers, raising the effective input impedance of the detection lines to $Z_{\mathrm{eff}} = 200\ \Omega$ over a one octave bandwidth centered at 6 GHz. Two rf circulators, thermalized at mixing chamber temperature, ensure a circuit environment at base temperature.

We note $\delta I_{1,2}$ the fluctuating current in either detection branch resulting from the fluctuations of the current through the tunnel junction. S_{I_1}, S_{I_2}, $S_{I_1 I_2}$ stand for the autocorrelated and cross-correlated spectral densities. From the equivalent circuit represented in Fig. 1, one easily sees that

$$S_{I_1}(\nu, T, V_{ds}) = S_{I_2}(\nu, T, V_{ds}) = -S_{I_1 I_2}(\nu, T, V_{ds}) = \left(\frac{R_t}{2Z_{\mathrm{eff}} + R_t}\right)^2 S_I(\nu, T, V_{ds}),$$

where the two first equalities result from current conservation. The noise power detected in each detection line in a frequency range $\Delta\nu$ reads

$$P_{1,2} = Z_{\mathrm{eff}}\overline{\delta I_{1,2}^2} = \frac{4Z_{\mathrm{eff}}R_t}{(2Z_{\mathrm{eff}} + R_t)^2} P_{\mathrm{em}}, \tag{2}$$

where $P_{\mathrm{em}} = R_t S_I(\nu, T, V_{ds})\Delta\nu/4$ is the emitted power. This can be expressed as an excess noise temperature $\Delta T_{n1,2} = P_{1,2}/[k_B \Delta\nu_{1,2}]$.

The two emitted signals are then amplified by two cryogenic low noise amplifiers. Up to a calculable gain factor, the detected noise power contains the weak excess noise $\Delta T_{n1,2}$ on top of a large additional noise generated by the cryogenic amplifiers $T_{n1,2} \simeq 5$ K. After further room temperature amplification and eventually narrow bandpass filtering, current fluctuations are detected using three alternative techniques. First [Fig. 1(a)], we implemented the measurement scheme described in [8], using two calibrated quadratic detectors whose output voltage is proportional to noise power. Second [Fig. 1(b)], current fluctuations are digitized, after down conversion achieved by mixing with a suited local microwave signal, using an AP240 Acqiris Acquisition Card able to sample at $1G$ sample/s. A quantitative comparison with the well-established first method has validated this new method. The third method [Fig. 1(c)], dedicated to the study of photon noise, is a hybridization of the two previous ones: the outputs of the two quadratic detectors are digitized to perform the photon HBT cross and autocorrelations fluctuations of P_1 and P_2.

First experiment (Fig. 1(a)): mean photon occupation number.—We measure the increase in noise temperature due to the photon emission by shot noise, as a function of V_{ds} and the measuring frequency ν, using the quadratic detectors. In order to remove the background noise of the amplifiers, we measure the excess noise, $\Delta S_{I_{1,2}}(\nu, T, V_{ds}) = S_{I_{1,2}}(\nu, T, V_{ds}) - S_{I_{1,2}}(\nu, T, 0)$. Practically, this is done by applying a 93 Hz 0-V_{ds} square-wave bias voltage on the sample through the dc input of a bias-Tee, and detecting the first harmonic of the square-wave noise response of the detectors using lock-in techniques. The results are quite similar to the ones reported in Ref. [8], and lead to an electron temperature $T_e \sim 70$ mK. Although T_e is significantly higher than the mixing chamber temperature, it is low enough to make the thermal population of photons negligible in the 4–8 GHz frequency range of our experiments.

Second experiment (Fig. 1(b)): auto and cross-correlated electronic noise.—Here, we use two filters with bandpass $\Delta\nu = 200$ MHz, centered around the same frequency $\nu = 6$ GHz. After down conversion with a local oscillator at frequency ν, we get fluctuating signals in the $[0, \Delta\nu/2]$ frequency range. The corresponding signals are digitized using an optimum $1/\Delta\nu = 5$ ns sampling time. Here again, we eliminate background noise and parasitic correlation between the two inputs of the acquisition card by measuring excess fluctuations. $\Delta\overline{\delta V_1^2}$ and $\Delta\overline{\delta V_2^2}$ are proportional to the excess noise power,

$$\Delta\overline{\delta V_{1,2}^2} = G_{1,2} Z_0 P_{1,2} = G_{1,2} Z_0 Z_{\mathrm{eff}} \Delta S_{I_{1,2}} \Delta\nu,$$

where $Z_0 = 50\ \Omega$ is the input impedance of the acquisition card, and $G_{1,2}$ stands for the gain of chain 1,2. The benefit of this method is that it also gives access to the cross-correlation term

$$\Delta\overline{\delta V_1 \delta V_2} = \sqrt{G_1 G_2} Z_0 Z_{\mathrm{eff}} \int_{\Delta\nu} \cos(2\pi\nu\tau) \Delta S_{I_1 I_2} d\nu$$

where τ is the difference of propagation time of electromagnetic waves between the sample and detectors 1 and 2. One easily gets

$$\Delta\overline{\delta V_1 \delta V_2}_{\text{norm}} = \frac{\Delta\overline{\delta V_1 \delta V_2}}{\sqrt{\Delta\overline{\delta V_1^2}\Delta\overline{\delta V_2^2}}} = -\text{sinc}(\pi\Delta\nu\tau)\cos(2\pi\nu\tau) \simeq -\cos(2\pi\nu\tau) \quad \text{for } \Delta\nu\tau \ll 1. \quad (3)$$

Equation (3) expresses the anticorrelation of current fluctuations δI_1 and δI_2, modified by the phase difference of the microwave signals. It is experimentally illustrated by Fig. 2. Here, τ is varied using two calibrated phase shifters inserted in both detection lines, around a value τ_0 which is *a priori* not known.

As photons emitted at times differing by more than $\sim 1/\Delta\nu$ do not show correlations, one needs to minimize τ before measuring the cross-correlated power fluctuations $S_{P_1^{\text{out}}P_2^{\text{out}}}$. As $\Delta\nu \ll \nu$, the most sensitive way to do so is to ensure that $\Delta\overline{\delta V_1 \delta V_2}_{\text{norm}}$ is constant for various values of ν. The result of such an adjustment is shown in the inset of Fig. 2. Although small parasitic microwave reflections introduce extra modulations, $\Delta\overline{\delta V_1 \delta V_2}_{\text{norm}}$ does not change sign for 4 GHz $\leq \nu \leq$ 8 GHz. This implies that $\tau \leq 125$ ps so that $\Delta\nu\tau \ll 1$ and the delay between the two lines does not affect the power correlations.

Third experiment (Fig. 1(c)): auto and cross-correlated photon noise.—We now adopt the original HBT setup [9], using quadratic detectors connected to the digitizer. This gives access to $S_{P_i^{\text{out}}P_j^{\text{out}}}$ where $i, j = 1$ or 2. In the case of a tunnel junction consisting of many weakly transmitted electronic modes, one would expect [10] the emitted photons to follow a negative binomial distribution, as the emitted power results from the incoherent superposition of a large number of sources (the electronic modes of the tunnel junction). This is confirmed by the rigorous treatment of Ref. [2], where the photons emitted by the shot-noise of a tunnel junction are shown to have the same counting statistics as thermal photons, although their origin is quite different and the frequency dependence of photon occupation number $N(\nu)$ does not correspond to a Bose-Einstein thermal distribution. $S_{P_1^{\text{out}}P_2^{\text{out}}}$ is expected to be positive, reflecting the bosonic nature of the emitted photons, and proportional to the product of the power emitted in both detection branches, reflecting the chaotic nature of the emitted radiation. $S_{P_1^{\text{out}}P_1^{\text{out}}}$ and $S_{P_2^{\text{out}}P_2^{\text{out}}}$ are expected to be enhanced by the contribution of the noise of the amplifiers [11]. Let us note $\Delta\nu_{\text{min}}$ ($\Delta\nu_{\text{max}}$) the smaller (the bigger) of the bandpasses, both centered around the same frequency ν. One thus expects

$$S_{P_1^{\text{out}}P_1^{\text{out}}} = 2\Delta\nu_1[G_1^2 k_B^2 (T_{n1} + \Delta T_{n1})^2 + G_1 h\nu k_B (T_{n1} + \Delta T_{n1})] \simeq 2\Delta\nu_1 G_1^2 k_B^2 (T_{n1} + \Delta T_{n1})^2$$
$$S_{P_1^{\text{out}}P_2^{\text{out}}} = 2G_1 G_2 \Delta\nu_{\text{min}} k_B^2 \Delta T_{n1}\Delta T_{n2}. \quad (4)$$

Equation (4) shows the benefit of measuring HBT cross correlations: since it is not affected by the input noise of the amplifiers, it allows probing the statistics of the emitted photons. In order to get rid of imperfectly known gains and attenuations, we normalize the excess power fluctuations $\Delta S_{P_i^{\text{out}}P_i^{\text{out}}}$ by $S_{P_i^{\text{out}}P_i^{\text{out}}}(V_{ds} = 0)$. Figure 3 represents $\Delta S_{P_1^{\text{out}}P_1^{\text{out}}}$ normalized by $S_{P_1^{\text{out}}P_1^{\text{out}}}(V_{ds} = 0)$ as a function of $\Delta T_{n1}/T_{n1}$, measured at $\nu = 6.6$ GHz $\pm$ 115 MHz. The solid line represents the theoretical prediction

$$\frac{\Delta S_{P_1^{\text{out}}P_1^{\text{out}}}}{[S_{P_1^{\text{out}}P_1^{\text{out}}}]_{V_{ds}=0}} = \left(\frac{\Delta T_{n1}}{T_{n1}}\right)^2 + 2\frac{\Delta T_{n1}}{T_{n1}},$$

which agrees with the experimental observations within 0.5%. Measurements over the entire 4–8 GHz frequency range show similar agreement, with a maximum systematic deviation of roughly 3%. As shown by Fig. 4, the cross-

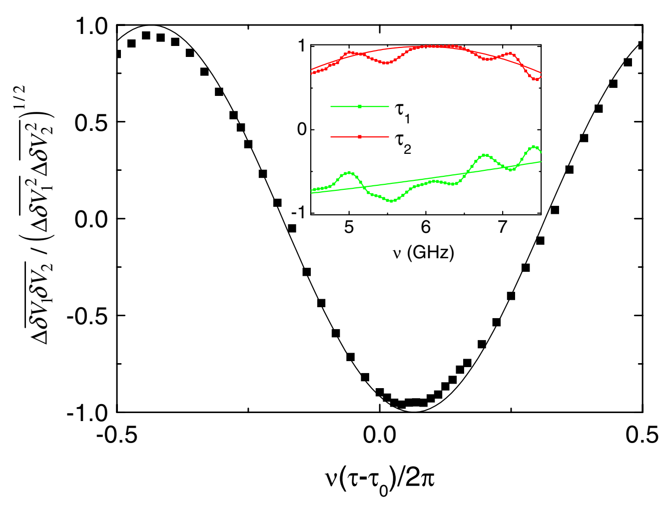

FIG. 2 (color online). Normalized cross-correlation spectrum $\Delta\overline{\delta V_1 \delta V_2}_{\text{norm}}$ measured at $\nu = 6$ GHz $\pm$ 100 MHz as a function of $\nu\Delta\tau$. The solid line represents the sinusoidal prediction of Eq. (2). Inset: $\Delta\overline{\delta V_1 \delta V_2}_{\text{norm}}$ averaged over 200 MHz, as a function of ν for two values of τ. The solid lines correspond to Eq. (2) for $\tau = 83$ ps (upper line) and $\tau = 12$ ps (lower line).

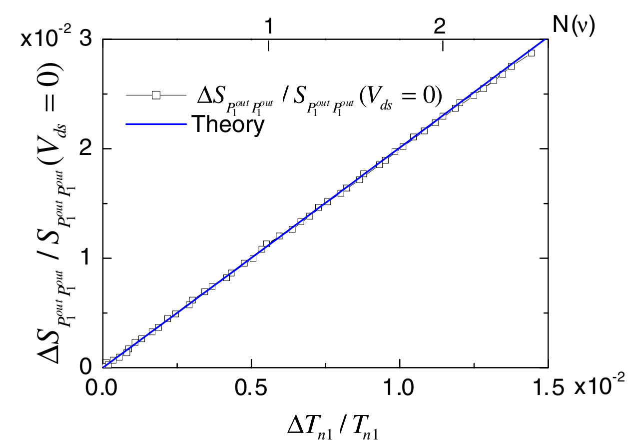

FIG. 3 (color online). Normalized autocorrelation spectrum of power fluctuations vs normalized excess noise temperature. Symbols: experimental data. Line: theoretical prediction. The top axis gives the corresponding $N(\nu)$ deduced from Eq. (2).

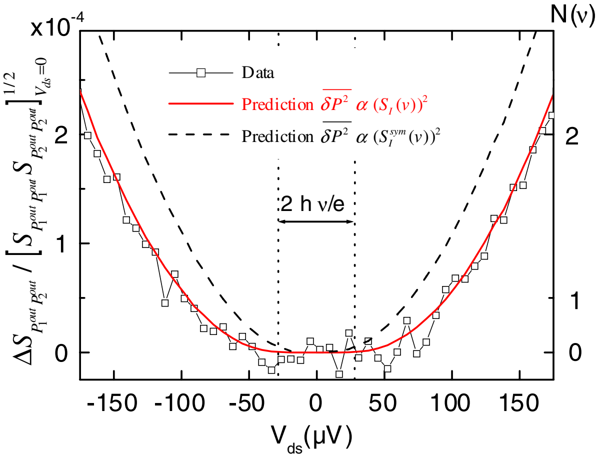

FIG. 4 (color online). Open squares: Normalized cross-correlation spectrum of power fluctuations. Solid line: theoretical prediction, assuming $S_{PP} \propto S_I^2$. Dashed line: theoretical prediction, assuming $S_{PP} \propto S_{I,\mathrm{sym}}^2$. The right axis gives the corresponding $N(\nu)$. The dotted line represents the onset of finite frequency excess shot noise [6,8,13].

correlated power fluctuations are positive, showing the bosonic character of the emitted excitations. On a quantitative level, one should observe

$$\frac{\Delta S_{P_1^{\mathrm{out}} P_2^{\mathrm{out}}}}{[S_{P_1^{\mathrm{out}} P_1^{\mathrm{out}}} S_{P_2^{\mathrm{out}} P_2^{\mathrm{out}}}]^{1/2}_{V_{ds}=0}} = \frac{\Delta T_{n1}}{T_{n1}} \frac{\Delta T_{n2}}{T_{n2}} \sqrt{\frac{\Delta \nu_{\min}}{\Delta \nu_{\max}}}. \qquad (5)$$

Figure 4 shows both terms of Eq. (5) as a function of V_{ds}. They are found to coincide within 4%. The fluctuations of the emitted power are thus found proportional to the square of the mean occupation number of the outgoing photon modes. This is characteristic of chaotic radiation. This remains true even in the "quantum regime," when this mean occupation number tends to zero (eV_{ds}, $k_B T \ll h\nu$). As the "photon noise" is related to a fourth order correlator of the electronic current [2,4], this constitutes, to the best of our knowledge, the first measurement of such a correlator in the quantum regime.

We would like to add a short note to the question of the symmetrization of the correlator involved in this experiment. The treatment of Ref. [2] yields power fluctuations proportional to the square of the electronic emission noise density [12] $S_I(\nu) = 2\int_{-\infty}^{+\infty} \langle I(0)I(\tau)\rangle e^{i2\pi\nu\tau} d\tau$. Using a symmetrized correlator $S_{I,\mathrm{sym}} = \frac{1}{2}[S_I(\nu) + S_I(-\nu)]$ increases the emitted power per unit bandwidth by $h\nu/2$. As the difference does not depend on bias voltage, excess noise measurements cannot distinguish between the two definitions. However, the quadratic dependence of the power fluctuations with the emitted power allows us to distinguish them. As shown by Fig. 4, assuming power fluctuations $S_{PP} \propto S_{I,\mathrm{sym}}^2$ yields a prediction $\Delta S_{PP} \propto [(\Delta S_I)^2 + 4G_{\mathrm{sample}} h\nu \Delta S_I]$, which is incompatible with our observations.

In conclusion, we have performed the first experiment probing the statistical properties of photons emitted by a phase coherent conductor. The data are found in perfect agreement with the predictions of Beenakker and Schomerus, showing that a biased low impedance tunnel junction emits chaotic radiation. The cross-correlated power fluctuations are found to be proportional to the square of the emission electronic noise. This opens the way to the investigation of the statistical properties of photons emitted by mesoscopic conductors where electronic correlations might have a stronger impact, such as quantum point contacts, for which the sub-Poissonian statistics of the electronic shot noise is expected to be "imprinted" onto the emitted photons [2,5].

It is a pleasure to acknowledge precious help from Q. Le Masne, P. Bertet, and D. Vion with the sample fabrication. We greatly benefited from help during the measurements from B. Dubost. This work was supported by the ANR Contract No. 2e-BQT and by the C'Nano Idf Contract No. QPC-SinPS.

*Present address: WSI TUMünchen Am Coulombwall 3, 85748 Garching, Germany.
†Also at LPA ENS, 24 rue Lhommond 75005, Paris, France.
‡fabien.portier@cea.fr

[1] H. Nyquist, Phys. Rev. **32**, 110 (1928).
[2] C. W. J. Beenakker and H. Schomerus, Phys. Rev. Lett. **86**, 700 (2001); Phys. Rev. Lett. **93**, 096801 (2004).
[3] L. Levitov and G. B. Lesovik, JETP Lett. **58**, 230 (1993); B. Reulet, J. Senzier, and D. E. Prober, Phys. Rev. Lett. **91**, 196601 (2003); S. Gustavsson *et al.*, Phys. Rev. B **75**, 075314 (2007); J. Salo, F. W. J. Hekking, and J. P. Pekola, Phys. Rev. B **74**, 125427 (2006); Phys. Rev. B **75**, 075314 (2007); G. Gershon *et al.*, Phys. Rev. Lett. **101**, 016803 (2008); A. Bednorz and W. Belzig, Phys. Rev. Lett. **101**, 206803 (2008); J. Gabelli and B. Reulet, J. Stat. Mech. (2008) 01049.
[4] J. Gabelli *et al.*, Phys. Rev. Lett. **93**, 056801 (2004).
[5] A. V. Lebedev, G. B. Lesovik, and G. Blatter, arXiv:0911.4676v1; I. C. Fulga, F. Hassler, and C. W. J. Beenakker, arXiv:1001.4389v1 (2010).
[6] E. Onac *et al.*, Phys. Rev. Lett. **96**, 176601 (2006); S. Gustavsson *et al.*, Phys. Rev. Lett. **99**, 206804 (2007); R. Deblock, E. Onac, and E. G. L. Kouvenhoven, Science **301**, 203 (2003).
[7] R. J. Glauber, Phys. Rev. **84**, 395 (1951).
[8] E. Zakka-Bajjani *et al.*, Phys. Rev. Lett. **99**, 236803 (2007).
[9] R. H. Brown and R. Twiss, Nature (London) **177**, 27 (1956).
[10] R. J. Glauber, Phys. Rev. **131**, 2766 (1963).
[11] C. M. Caves, Phys. Rev. D **26**, 1817 (1982).
[12] G. B. Lesovik and R. Loosen, JETP Lett. **65**, 295 (1997); G. B. Lesovik, Phys. Usp. **41**, 145 (1998); U. Gavish, Y. Levinson, and Y. Imry, Phys. Rev. B **62**, R10637 (2000); R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett. **84**, 1986 (2000).
[13] A. V. Lebedev, G. B. Lesovik, and G. Blatter, arXiv:0911.4676v1 (2009); I. C. Fulga, F. Hassler, and C. W. J. Beenakker, arXiv:1001.4389v1 (2010).