

## Nonlinear Magnetoresistance Oscillations in Intensely Irradiated Two-Dimensional Electron Systems Induced by Multiphoton Processes

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We report on magneto-oscillations in differential resistivity of a two-dimensional electron system subject to *intense* microwave radiation. The period of these oscillations is determined not only by microwave frequency but also by its intensity. A theoretical model based on quantum kinetics at high microwave power captures all important characteristics of this phenomenon which is strongly nonlinear in microwave intensity. Our results demonstrate a crucial role of the multiphoton processes near the cyclotron resonance and its harmonics in the presence of strong dc electric field and offer a unique way to reliably determine the intensity of microwaves acting on electrons.

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Over the past decade an array of remarkable effects was discovered in very high Landau levels of high-mobility two-dimensional electron systems (2DESs). Among these are microwave-induced resistance oscillations (MIROs) [1], phonon-induced resistance oscillations [2], Hall field-induced resistance oscillations [3,4], zero-resistance states [5], and zero-differential resistance states [6]. Theories of magnetoresistance oscillations are based on the quantum kinetic description and consider (1) the “displacement” mechanism originating from modification of impurity scattering by microwave (ac) or dc electric fields [7–9] and (2) the “inelastic” mechanism stepping from the nonequilibrium energy distribution [10]. MIROs are controlled by a parameter  $\epsilon_{ac} \equiv \omega/\omega_c$ , where  $\omega = 2\pi f$  is the microwave frequency,  $\omega_c = eB/m^*$  is the cyclotron frequency and  $m^*$  is the electron mass. Hall field-induced oscillations, which appear in differential resistivity, are governed by  $\epsilon_{dc} \equiv e\mathcal{E}_{dc}(2R_c)/\hbar\omega_c$ , where  $2R_c = 2v_F/\omega_c$  is the cyclotron diameter, and  $v_F$  is the Fermi velocity. The Hall field  $\mathcal{E}_{dc} = \rho_H I/w$ , where  $\rho_H$  is the Hall resistivity,  $I$  is the current and  $w$  is the sample width. Finally, in a 2DES subject to both ac and dc fields, the resulting oscillations were found to depend on simple combinations of ac and dc parameters, i.e.,  $\epsilon_{ac} \pm \epsilon_{dc}$  [11,12]. Such dependence indicates that the dominant scattering processes involved a single photon.

Importance of processes involving multiple microwave quanta was suggested by numerous experiments [13] reporting MIRO-like features in the vicinity of fractional values of  $\epsilon_{ac}$ . The most prominent series of, so called, fractional MIRO occurs near subharmonics of the cyclotron resonance,  $\epsilon_{ac} = 1/2, 1/3, 1/4, \dots$ . Theory considered both multiphoton [14,15] and single-photon [15,16] mechanisms to explain the response at fractional  $\epsilon_{ac}$ . On the other hand, relevance of multiphoton processes near the cyclotron resonance and its har-

monics was not considered either experimentally or theoretically.

In this Letter we report on a new class of magnetoresistance oscillations in a high-mobility 2DES exposed to high-power microwave radiation and strong dc electric field,  $2\pi\epsilon_{dc} \gg 1$ . These oscillations are manifested by a series of multiple maxima and minima all occurring in the close proximity to the cyclotron resonance and its harmonics. Furthermore, oscillatory differential magnetoresistance is strongly nonlinear not only in  $\epsilon_{dc}$  but also in the microwave power. This characteristic sensitivity to microwave intensity sets apart this phenomenon from all previously reported resistance oscillations.

To explain our experimental findings we propose a theoretical model based on quantum kinetics which captures all important characteristics of the phenomenon. As we will show, this unusual effect originates from the displacement mechanism owing to the quantum oscillations in the density of states (DOS) and a crucial role played by multiphoton processes. In the presence of radiation the electron states are split into Floquet subbands separated by  $\hbar\omega$  in a similar way as quasiparticle states split in the course of photon-assisted tunneling across a Josephson junction [17]. The disorder scattering rate is then controlled by the overlap of such subbands leading to oscillatory behavior in differential magnetoresistance.

Our experiment was performed on a  $w = 100 \mu\text{m}$  Hall bar etched from a GaAs/AlGaAs quantum well. After brief low-temperature illumination with visible light, density and mobility were  $n_e \approx 3.8 \times 10^{11} \text{ cm}^{-2}$  and  $\mu \approx 1.3 \times 10^7 \text{ cm}^2/\text{Vs}$ , respectively. All the data were recorded under continuous irradiation by  $f = 27 \text{ GHz}$  microwaves in a Faraday geometry at  $T \approx 1.5 \text{ K}$ . Differential resistivity,  $r \equiv dV/dI$ , was measured using a quasi-dc (a few hertz) lock-in technique.

As shown in Fig. 1(a), the magnetoresistivity measured under microwave irradiation in the absence of dc field exhibits sharp MIROs around the cyclotron resonance ( $\epsilon_{ac} = 1$ ) and its harmonics ( $\epsilon_{ac} = 2, 3$ ). In each case we observe exactly *one* minimum and *one* maximum positioned roughly symmetrically about  $\epsilon_{ac} = 1, 2$ .

In Fig. 1(b) we present differential resistivity  $r(B)$  measured at the same radiation frequency and intensity but at  $I$  from  $48 \mu\text{A}$  to  $78 \mu\text{A}$ . These currents correspond to  $2\pi\epsilon_{dc} \geq 10$  and  $\mathcal{E}_{dc} \geq 50 \text{ V/m}$  at  $B \approx 0.6 \text{ kG}$ . Remarkably, the data reveal *multiple* maxima and minima occurring in the proximity to the cyclotron resonance. For example, the  $I = 64 \mu\text{A}$  trace exhibits three maxima (cf.,  $\uparrow$ ) and three minima (cf.,  $\downarrow$ ). While not so pronounced, multiple peaks are also seen near  $\epsilon_{ac} = 2$ . These findings indicate that the observed magneto-oscillations are qualitatively different from all reported previously. As we show below, the phenomenon owes to a nontrivial role played by multiphoton processes near integer  $\epsilon_{ac}$  at strong microwave and dc electric fields.

We now present the expression for the differential resistivity  $\delta r = r - r_D$  which is our main theoretical result:

$$\frac{\delta r}{r_D} = \frac{(4\lambda)^2 \tau_{tr}}{\pi \tau_{\pi}} [\cos(2\pi\epsilon_{dc}) J_0(4\sqrt{\mathcal{P}_{\omega}} \sin(\pi\epsilon_{ac})) - \frac{2\epsilon_{ac}}{\epsilon_{dc}} \times \sqrt{\mathcal{P}_{\omega}} \sin(2\pi\epsilon_{dc}) \cos(\pi\epsilon_{ac}) J_1(4\sqrt{\mathcal{P}_{\omega}} \sin(\pi\epsilon_{ac}))]. \quad (1)$$

The magnetoresistance in Eq. (1) is nonlinear in  $\epsilon_{dc}$  and  $\mathcal{P}_{\omega}$ . Here,  $J_n(x)$  is the Bessel function of  $n$ th order,  $\tau_{tr}$  and

$\tau_{\pi}$  are transport scattering and backscattering times,  $r_D$  is the Drude resistivity,  $\lambda = \exp(-\pi/\omega_c \tau_q)$  is the Dingle factor, and  $\mathcal{P}_{\omega} = \mathcal{P}_{\omega}^+ + \mathcal{P}_{\omega}^-$  is the dimensionless power of unpolarized microwaves, where

$$\mathcal{P}_{\omega}^{\pm} = \frac{\mathcal{P}_{\omega}^{(0)}}{(\omega \pm \omega_c)^2 \tau_{em}^2 + 1}, \quad \mathcal{P}_{\omega}^{(0)} = \frac{e^2 \mathcal{E}_{ac}^2 v_F^2 \tau_{em}^2}{2\epsilon_{eff} \hbar^2 \omega^2}. \quad (2)$$

Here  $\mathcal{E}_{ac}$  is the electric field of incident microwave radiation,  $\tau_{em} = 2\epsilon_0 \sqrt{\epsilon_{eff}} m^* c / n_e e^2$ ,  $2\sqrt{\epsilon_{eff}} = \sqrt{\epsilon} + 1$ , [18] and  $\epsilon = 12.8$  is the dielectric constant of GaAs. In what follows we compare Eq. (1) with our experimental data and extract the only fitting parameter  $\mathcal{P}_{\omega}^{(0)}$ .

At  $\epsilon_{dc} = n + 1/4$  the first term in Eq. (1) vanishes and the position of the maximum closest to the cyclotron resonance is determined by the first maximum of the Bessel function  $J_1(x)$  which occurs at  $x = 1.84$ . In our experiment, such a maximum corresponding to  $\epsilon_{dc} \approx 1.25$  is found at  $\epsilon_{ac} \approx 0.95$ . From  $4\sqrt{\mathcal{P}_{\omega}} \sin(0.95\pi) \approx 1.84$  and using (2) we find  $\mathcal{P}_{\omega}^{(0)} \approx 7.7$ . Another way to obtain  $\mathcal{P}_{\omega}^{(0)}$  is to directly compare the experimental and theoretical curves. Figures 2(a) and 2(b) show our experimental data at  $I = 54 \mu\text{A}$  and the results of calculations using  $\mathcal{P}_{\omega}^{(0)} = 7.7$  and Eq. (1), respectively. Here we use  $\tau_q \approx 15 \text{ ps}$  and  $\tau_{tr}/\tau_{\pi} \approx 0.18$  [4]. We observe that the theoretical curve not only recreates the experimental data well but also captures the amplitude of the observed oscillations. Indeed, both experiment and theory show  $\delta r/r_D \approx 0.1$  which further reinforces the validity of our model.

We now examine the dependence on microwave intensity. In Fig. 3 we compare the results of our measurements (a), labeled by attenuation factors, with the calculations (b), marked by  $\mathcal{P}_{\omega}^{(0)}$ . The similarity is striking: *both* theory and experiment show that the oscillation period decreases with intensity and both reveal two (four) resistivity maxima at the lowest (highest) intensity.

The remaining part of the Letter is devoted to the derivation of Eq. (1). Our analysis is based on the quantum

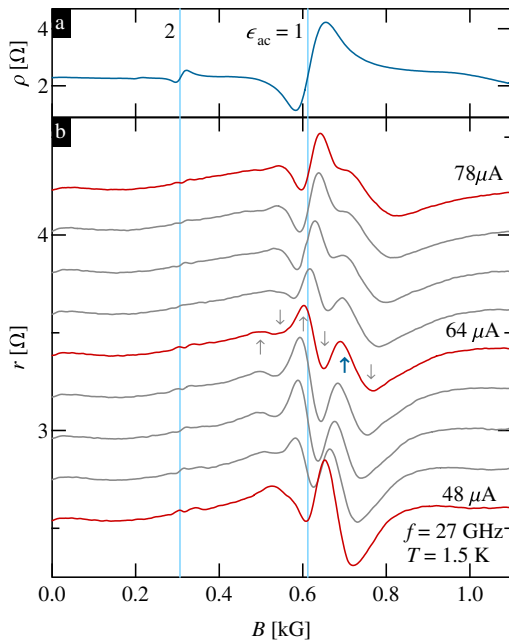


FIG. 1 (color online). (a) Magnetoresistivity  $\rho(B)$ . (b) Differential resistivity  $r(B)$  at currents from  $I = 48$  to  $78 \mu\text{A}$  in step of  $4 \mu\text{A}$ . All data are acquired under irradiation by  $f = 27 \text{ GHz}$  microwaves at  $T = 1.5 \text{ K}$ . Vertical lines mark positions of the cyclotron resonance and its second harmonic.

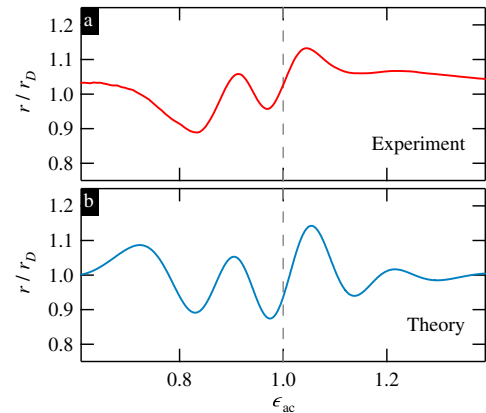


FIG. 2 (color online). (a),(b) Measured [calculated with  $\mathcal{P}_{\omega}^{(0)} = 7.7$ ] differential resistivity  $r(\epsilon_{ac})/r_D$  at  $I = 54 \mu\text{A}$ .

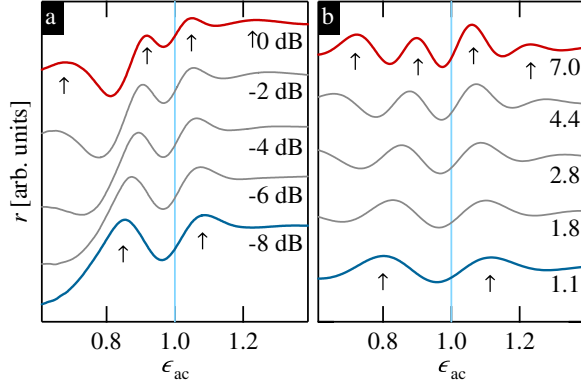


FIG. 3 (color online). (a),(b) Measured [calculated]  $r$  vs  $\epsilon_{ac}$  for  $I = 54 \mu\text{A}$  and different microwave intensities as marked by attenuation [ $\mathcal{P}_\omega^{(0)}$ ].

kinetic equation of electrons in the presence of in-plane electric fields in the limit of overlapping Landau levels. We calculate the distribution function  $f_{\epsilon;\varphi}$  which depends on the direction of the momentum  $\mathbf{p} = p_F \mathbf{n}_\varphi$  of an electron at the Fermi surface in the quasiclassical approximation,  $E_F \gg \hbar\omega_c$ . The current density is given by  $j = 2ev_F \int \cos\varphi \nu(\epsilon) f_{\epsilon;\varphi} d\epsilon d\varphi / 2\pi$ , where the DOS oscillates with a period  $\hbar\omega_c$  due to the Landau quantization [19],  $\nu(\epsilon) = \nu_0 [1 - 2\lambda \cos(2\pi\epsilon/\hbar\omega_c)]$  with  $\nu_0 = m^*/\hbar^2\pi$  being the zero-field DOS per spin. The distribution function satisfies the kinetic equation [8,12]

$$\omega_c \partial_\varphi f_{\epsilon;\varphi} = \text{St}\{f\}_{\epsilon;\varphi}. \quad (3)$$

with the collision term in the Floquet representation

$$\text{St}\{f\} = \sum_N \int \frac{d\varphi'}{2\pi} \Gamma_{\varphi\varphi'}^{(N)} [f_{\varphi'}(\epsilon + W_{\varphi\varphi'} + N\hbar\omega) - f_\varphi(\epsilon)]. \quad (4)$$

Equation (4) accounts for scattering events whereby the particle changes its momentum by  $\Delta\mathbf{q} = p_F(\mathbf{n}_\varphi - \mathbf{n}_{\varphi'})$  and absorbs ( $N > 0$ ) or emits ( $N < 0$ )  $|N|$  photons. The kinetic energy change includes the energy of absorbed or emitted photons,  $N\hbar\omega$ , and the work done by the dc electric field as a result of the shift of the guiding center,  $W_{\varphi\varphi'} = eER_c(\sin\varphi - \sin\varphi')$ . The scattering rate is

$$\Gamma_{\varphi\varphi'}^{(N)} = \frac{a_{\varphi-\varphi'}^{(N)}}{\tau_{\varphi-\varphi'}} \nu_0^{-1} \nu(\epsilon + W_{\varphi\varphi'} + N\hbar\omega), \quad (5)$$

where

$$a_\theta^{(N)} = J_N^2 \sqrt{2\mathcal{P}_\omega (1 - \cos\theta)}. \quad (6)$$

We note that a similar expression describes the rate of tunneling events with absorption or emission of several photons in irradiated Josephson junctions [17] or mesoscopic devices [20].

We look for the solution of Eq. (3) in the form  $f = f_T + \Delta f_{cl} + \lambda \Delta^{(1)} f$ , where  $f_T$  is the Fermi-Dirac distribution function,  $f_T + \Delta f_{cl}$  is the solution of the classical

kinetic equation, and

$$\Delta^{(1)} f(\epsilon, \varphi) = A_1 \cos\varphi \frac{\partial f_T(\epsilon)}{\partial \epsilon} \cos \frac{2\pi\epsilon}{\hbar\omega_c}, \quad (7)$$

which gives the oscillatory correction  $\delta j = \lambda^2 A_1 e v_F \nu_0$ . Upon substitution of Eq. (7) to Eqs. (3)–(5) we obtain

$$A_1 = \frac{4}{\omega_c} \sum_N \langle \sin\varphi K_{\varphi\varphi'}^{(N)} \rangle_{\varphi\varphi'}, \quad (8)$$

$$K_{\varphi\varphi'}^{(N)} = \frac{a_{\varphi-\varphi'}^{(N)}}{\tau_{\varphi-\varphi'}} (W_{\varphi\varphi'} + N\hbar\omega) \cos \left[ \frac{2\pi(W_{\varphi\varphi'} + N\hbar\omega)}{\hbar\omega_c} \right].$$

At  $\epsilon_{dc} \gtrsim \max\{\sqrt{2\mathcal{P}_\omega}, 1\}$  the angular averaging in Eq. (8) can be carried out using stationary phase approximation. The main contribution comes from narrow intervals centered at  $\varphi = \pm\pi/2$ ,  $\varphi' = \mp\pi/2$ , which corresponds to backscattering of electrons. Consequently, final expressions depend on  $\tau_\pi$  and  $\epsilon_{dc}$ . The remaining summation over  $N$  leads to Eq. (1).

To further clarify the physical origin of the phenomenon, we notice that the action of the microwaves amounts to a periodic phase change of the electron's wave function. As a result, energy levels are split into Floquet bands of levels [17,20] separated by  $\hbar\omega$ . The quantity  $a_\theta^{(N)}$ , Eq. (6), gives an overlap between initial and final Floquet bands in the inelastic scattering on angle  $\theta$  with absorption (emission) of  $|N|$  photons. In the considered case of backscattering,  $\theta = \pi$ ,  $a_\pi^{(N)} = J_N^2(2\sqrt{2\mathcal{P}_\omega})$ . Since  $J_N(x)$  is strongly suppressed for  $|N| > x$ , the number of absorbed (emitted) photons,  $|N| \leq N_m \approx 2\sqrt{2\mathcal{P}_\omega}$ . This is illustrated in Fig. 4(a) for  $\mathcal{P}_\omega = 4.4$ .

We now consider Landau levels tilted by a dc field, Figs. 4(b) and 4(c). The biggest differential resistance is obtained when rates of transitions along (opposite to) the dc field are maximized (minimized). These rates in turn are at maximum if total change of kinetic energy is a multiple of  $\hbar\omega_c$  (up to a phase). Namely,  $N$ -photon transitions give maximal resistance for integer  $N\epsilon_{ac} + \epsilon_{dc}$  and half-integer  $N\epsilon_{ac} - \epsilon_{dc}$ . One can therefore qualitatively explain the phenomenon by considering the  $N_m$ -photon transitions. The above conclusions also follow from the asymptotic form of Eq. (1),  $\delta\rho(j)/\rho_D \propto \sum_{\pm} [\epsilon_{dc} \pm 2\sqrt{2\mathcal{P}_\omega}\epsilon_{ac}] \cos[2\pi(2\sqrt{2\mathcal{P}_\omega}\epsilon_{ac} \pm \epsilon_{dc}) + \vartheta]$ , where  $\vartheta = \pi/4 - 4\pi\sqrt{2\mathcal{P}_\omega}$  in the regime  $\mathcal{P}_\omega \gtrsim 1$ . Notice, that close to the cyclotron resonance, once the resonance condition is met for  $N_m$ -photon scattering, it also holds for  $N < N_m$  as illustrated in Fig. 4(b). A similar condition for a minimum is illustrated in Fig. 4(c).

For typical experimental parameters, the main contribution comes from the transitions along dc field. As a result, close to the cyclotron resonance, oscillations are roughly periodic in  $2\sqrt{2\mathcal{P}_\omega}\epsilon_{ac} + \epsilon_{dc}$ . At fixed  $\omega$  and  $I$  the ratio  $\eta \equiv \epsilon_{dc}/\epsilon_{ac} \propto I/\omega$  does not change with the magnetic field and the differential resistivity oscillates with  $(2\sqrt{2\mathcal{P}_\omega} + \eta)\epsilon_{ac}$ . For the data presented in Fig. 2  $\mathcal{P}_\omega \approx 7.7$ ,  $\eta = 1.65$ , and



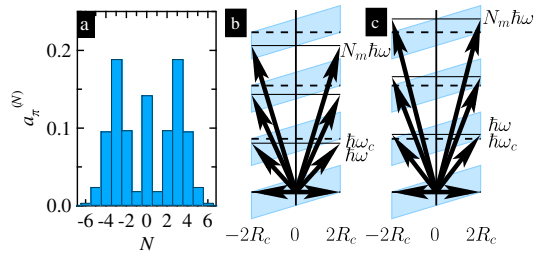


FIG. 4 (color online). (a)  $a_{\pi}^{(N)}$  is strongly suppressed at  $N > N_m = 3$  for  $\mathcal{P}_{\omega} = 4.4$ . (b),(c) Transitions (arrows) between Landau levels (tilted strips) due to absorption of  $N \leq N_m$  photons and backscattering off impurity leading to a maximum [minimum] in differential resistance just below,  $\omega < \omega_c$  [above,  $\omega > \omega_c$ ] of the cyclotron resonance. Notice that  $N$ -photon processes with  $N < N_m$  contribute constructively.

the separation between the peaks is  $1/(2\sqrt{\mathcal{P}_{\omega}} + \eta) \simeq 0.14$ , in good agreement with the experiment. This separation decreases with microwave power for a given current, as illustrated in Fig. 3.

In summary, we reported on a new class of magneto-resistance oscillations in high-mobility 2DESs driven away from equilibrium by microwave and dc electric fields. These oscillations occur only at intense microwave radiation and at strong dc electric fields. In intense microwave field, the energy spectrum consists of Floquet bands of levels separated by photon energy and a strong dc field ensures the dominant contribution of electron backscattering between those bands. Observed magnetoresistance oscillations occur near the cyclotron resonance and its harmonics, with positions of peaks and dips strongly dependent on intensity. Our calculations capture all important characteristics of the phenomenon—the oscillation period, the phase, and the amplitude are all in excellent agreement with experiment. Taken together, these results demonstrate the crucial role of the multiphoton processes near the cyclotron resonance and its harmonics in the presence of strong dc electric field and offer a unique way to reliably determine the microwave intensity seen by 2D electrons, which is hard to obtain by other methods. The knowledge of the microwave power offers an exclusive opportunity to finally test proposed theories of nonequilibrium transport in an irradiated 2DES against experiment without any fitting parameters.

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