Nucleon Mass from a Covariant Three-Quark Faddeev Equation

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We report the first study of the nucleon where the full Poincaré-covariant structure of the three-quark amplitude is implemented in the Faddeev equation. We employ an interaction kernel which is consistent with contemporary studies of meson properties and aspects of chiral symmetry and its dynamical breaking, thus yielding a comprehensive approach to hadron physics. The resulting current-mass evolution of the nucleon mass compares well with lattice data and deviates only by $\sim 5\%$ from the quark-diquark result obtained in previous studies.

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Experiments, and hereby especially electroweak probes at all energy scales, have provided detailed information about the structure of the nucleon. Nevertheless, understanding the nucleon's structure in terms of quarks and gluons, the elementary degrees of freedom of quantum chromodynamics (QCD), has remained a challenge in theoretical hadron physics.

Starting with the original work of Faddeev [1] a formalism has been developed to treat a relativistic three-body problem [2–4]. In its covariant form, the corresponding equation is the three-body analogue of the two-body Bethe-Salpeter equation (BSE) [5]. In the case of the nucleon, its solution is a covariant three-quark amplitude whose relativistic spin structure has been explored in [6,7] and in the light-front formalism in [8–11]. A complete classification according to the Lorentz group and the permutation group $\mathbb{S}_3$ was derived in [12] in terms of covariant three spinors.

The formalism of QCD's Dyson-Schwinger equations (for recent reviews, see, e.g., [13,14]) provides a way to embed the covariant three-quark equation in a consistent quantum-field theoretical setup. The dynamical ingredients in the equation (the dressed quark propagator and the three-quark kernel) can then be treated in perfect correspondence with studies of quark and meson properties as well as related aspects of QCD.

The biggest obstacle on the way to a direct numerical solution of the three-body bound-state equation is its complexity. Simplifications employed in the past implemented perturbative quark propagators [15,16], together with a three-body spectator approximation [17], or in a Salpeter-equation setup with instantaneous forces [4]. The corresponding equation of a scalar three-particle system with scalar two-body exchange was recently investigated and compared to the light-front approach [18]. Another kind of simplification can be achieved by considering diquark correlations (see, e.g., [19] for an overview). While maintaining full Poincaré covariance, the quark-diquark model neglects three-quark interactions and in addition traces the nucleon's binding to colored scalar- and axial-vector diquarks, thereby simplifying the three-quark equation to a quark-diquark BSE whose kernel describes a quark exchange between quark and diquark. This strategy has been applied so far to study nucleon and Δ properties [20–23].

In the present work, the bound-state problem is solved numerically and explicitly for the covariant three-quark amplitude. While the interaction kernel is truncated to a ladder dressed-gluon exchange between any two quarks upon neglecting explicit three-quark interactions, the quark-quark T matrix is for the first time no longer taken to be a separable sum of diquark poles. Thus, diquarks are eliminated as effective degrees of freedom. With this improvement and the fully covariant construction of the three-quark nucleon amplitude, the present work sets a new standard for such types of calculations.

In QCD baryons appear as poles in the three-quark scattering matrix. This allows one to derive a relativistic three-body bound-state equation:

$$\Psi = \tilde{K}_{(3)}\Psi, \qquad \tilde{K}_{(3)} = \tilde{K}_{(3)}^{\mathrm{irr}} + \sum_{a=1}^{3} \tilde{K}_{(2)}^{(a)}. \qquad (1)$$

Ψ is the bound-state amplitude defined on the baryon's mass shell. The full kernel $\tilde{K}_{(3)}$ comprises a three-quark irreducible contribution $\tilde{K}_{(3)}^{\mathrm{irr}}$ and the Faddeev kernel, defined as the sum of permuted two-quark kernels whose quark-antiquark analogues appear in a meson BSE, and the superscript a denotes the respective spectator quark.

The observation of a strong attraction in the $SU(3)_C$ antitriplet qq channel has been the guiding idea for the quark-diquark model, namely, that quark-quark correlations provide important binding structure in baryons. This motivates the omission of the three-body irreducible contribution from the full three-quark kernel. The resulting equation is commonly referred to as the covariant Faddeev equation and includes only the Faddeev kernel (cf. Fig. 1):

$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_{a=1}^{3} \int_k \tilde{K}^{(a)}_{\alpha\alpha'\beta\beta'\gamma\gamma'} \Psi_{\alpha'\beta'\gamma'\delta}(p^{(a)}, q^{(a)}, P), \qquad (2)$$

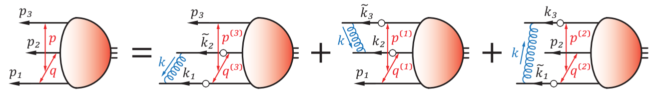

FIG. 1 (color online). Covariant Faddeev equation (2) with ladder-truncated Faddeev kernel. The internal relative momenta $p^{(a)}$, $q^{(a)}$ and quark momenta k_a, $\tilde{k}_a$ are defined implicitly in Eqs. (2) and (3).

where $\tilde{K}^{(a)}$ denotes the renormalization-group invariant products of a qq kernel and two dressed quark propagators:

$$\tilde{K}^{(a)}_{\alpha\alpha'\beta\beta'\gamma\gamma'} = \delta_{\alpha\alpha'}\mathcal{K}_{\beta\beta''\gamma\gamma''}S_{\beta''\beta'}(k_b)S_{\gamma''\gamma'}(\tilde{k}_c). \qquad (3)$$

$\{a, b, c\}$ is an even permutation of $\{1, 2, 3\}$ and linked to the respective Dirac index pairs.

The spin-momentum part of $\Psi_{\alpha\beta\gamma\delta}(p, q, P)$ is a spin-1/2 four-point function with positive parity and positive energy: it carries three spinor indices $\{\alpha, \beta, \gamma\}$ for the involved valence quarks and one index δ for the spin-1/2 nucleon. The amplitude depends on the total momentum P and two relative Jacobi momenta p and q, where $P^2 = -M^2$ is fixed. It can be decomposed into 64 Dirac structures:

$$\Psi_{\alpha\beta\gamma\delta}(p, q, P) = \sum_{k=1}^{64} f_k \tau^k_{\alpha\beta\gamma\delta}(p, q, P), \qquad (4)$$

where the amplitude dressing functions f_k depend on the five Lorentz-invariant combinations

$$p^2, \quad q^2, \quad z_0 = \hat{p}_T \cdot \hat{q}_T, \quad z_1 = \hat{p} \cdot \hat{P}, \quad z_2 = \hat{q} \cdot \hat{P}. \qquad (5)$$

Here, a hat denotes a normalized 4-vector and $p^\mu_T = T^{\mu\nu}_P p^\nu$ a transverse projection with $T^{\mu\nu}_P = \delta^{\mu\nu} - \hat{P}^\mu \hat{P}^\nu$.

A general spinor four-point function which depends on three independent momenta involves 128 independent components of positive parity. An orthogonal basis $\{\tau^k\}$ for the 64-dimensional subspace of a positive-parity and positive-energy nucleon is given by the set

$$\begin{pmatrix} \mathsf{S}^r_{ij} \\ \mathsf{P}^r_{ij} \end{pmatrix} := \begin{pmatrix} \mathbb{1} \otimes \mathbb{1} \\ \gamma^5 \otimes \gamma^5 \end{pmatrix} (\Gamma_i \otimes \Gamma_j)(\Lambda^r \gamma_5 C \otimes \Lambda^+), \qquad (6)$$

where $C = \gamma^4\gamma^2$ is the charge-conjugation matrix, $r = \pm$ refers to the positive- and negative-energy projectors $\Lambda^\pm(P) = (\mathbb{1} \pm \hat{\not{P}})/2$, and the tensor product is understood as $(A \otimes B)_{\alpha\beta\gamma\delta} = A_{\alpha\beta}B_{\gamma\delta}$. The relative-momentum dependence of the basis elements is carried by the Γ_i, $i = 1, 2, 3, 4$, defined by

$$\Gamma_i(p, q, P) = \left\{\mathbb{1}, \frac{1}{2}[\hat{\not{p}}_T, \hat{\not{q}}_t], \hat{\not{p}}_T, \hat{\not{q}}_t\right\}. \qquad (7)$$

The momenta $\{\hat{p}_T, \hat{q}_t, \hat{P}\}$ were conveniently chosen to be orthonormal with respect to the Euclidean metric via

$$p^\mu_T := T^{\mu\nu}_P p^\nu, \qquad q^\mu_t := T^{\mu\nu}_{p_T} T^{\nu\lambda}_P q^\lambda = T^{\mu\nu}_{p_T} q^\nu_T. \qquad (8)$$

A partial-wave decomposition leads to linear combinations of the $\{\mathsf{S}^r_{ij}, \mathsf{P}^r_{ij}\}$ as eigenstates of quark-spin and orbital angular momentum operators $\boldsymbol{S}^2$ and $\boldsymbol{L}^2$ in the nucleon rest frame. The 64 basis covariants (32 each for total quark spin $s = 1/2$ and $s = 3/2$, respectively) can be arranged into sets of 8 s waves ($l = 0$), 36 p waves ($l = 1$), and 20 d waves ($l = 2$). For instance, the dominant contributions to the Faddeev amplitude are given by the s waves

$$\begin{aligned} \gamma_5 C \otimes \Lambda^+ &= \sum_{r=\pm} \mathsf{S}^r_{11}, \\ \gamma^\mu_T C \otimes \gamma^\mu_T \gamma_5 \Lambda^+ &= \sum_{r=\pm} (r\mathsf{S}^r_{22} + \mathsf{P}^r_{33} + \mathsf{P}^r_{44}), \end{aligned} \qquad (9)$$

with $\gamma^\mu_T = T^{\mu\nu}_P \gamma^\nu$. In the quark-diquark model, these correspond to scalar-scalar and axial-vector–axial-vector combinations of diquark and quark-diquark amplitudes for either of the three diagrams appearing in the Faddeev equation.

The basis elements can be expressed in terms of quark three-spinors frequently used in the literature, e.g., Ref. [12]. In this context the elements $\mathsf{S}^+_{11} = \Lambda^+\gamma_5 C \otimes \Lambda^+$ and $\mathsf{A}^+_{11} := \Lambda^+\gamma^\mu_T C \otimes \gamma^\mu_T \gamma_5 \Lambda^+$ read

$$\begin{aligned} -\mathsf{S}^+_{11} U^\uparrow &= (U^\uparrow U^\downarrow - U^\downarrow U^\uparrow)U^\uparrow, \\ \mathsf{A}^+_{11} U^\uparrow &= (U^\uparrow U^\downarrow - U^\downarrow U^\uparrow)U^\uparrow - 2U^\uparrow U^\uparrow U^\downarrow, \end{aligned} \qquad (10)$$

where the $U^\sigma(P)$ are eigenspinors of Λ^+ and therefore satisfy the free Dirac equation for a spin-1/2 particle.

The Pauli principle requires the Faddeev amplitude to be antisymmetric under exchange of any two quarks. The Faddeev kernel is invariant under the permutation group $\mathbb{S}_3$. Its eigenstates can hence be arranged into irreducible $\mathbb{S}_3$ multiplets

$$\Psi_{\mathcal{S}}, \qquad \Psi_{\mathcal{A}}, \qquad \begin{pmatrix} \Psi_{\mathcal{M}_\mathcal{A}} \\ \Psi_{\mathcal{M}_\mathcal{S}} \end{pmatrix}, \qquad (11)$$

of which the first two (totally symmetric or antisymmetric) solutions are unphysical while the mixed-symmetry doublet constitutes the Dirac part of the nucleon amplitude. Taking into account the flavor and color structure, the full Dirac–flavor–color amplitude reads

$$\Psi(p, q, P) = \{\Psi_{\mathcal{M}_\mathcal{A}} \mathsf{T}_{\mathcal{M}_\mathcal{A}} + \Psi_{\mathcal{M}_\mathcal{S}} \mathsf{T}_{\mathcal{M}_\mathcal{S}}\} \frac{\varepsilon_{ABC}}{\sqrt{6}}, \qquad (12)$$

where $\mathsf{T}_{\mathcal{M}_\mathcal{A}}$, $\mathsf{T}_{\mathcal{M}_\mathcal{S}}$ denote the isospin-1/2 flavor tensors for proton and neutron and ε_{ABC} the antisymmetric color-singlet wave function. A flavor-dependent kernel in the Faddeev equation will mix $\Psi_{\mathcal{M}_\mathcal{A}}$ and $\Psi_{\mathcal{M}_\mathcal{S}}$ whose dominant contributions are given by S^+_{11} and A^+_{11}, respectively. Similarly to the analogous case of a diquark amplitude, the symmetry does, however, not reduce the number of Dirac covariants since the dressing functions f_k transform under the permutation group as well.

To proceed with the numerical solution of the covariant Faddeev Eq. (2), we need to specify the quark-quark kernel $\mathcal{K}$ and the dressed quark propagator $S(p)$ which appear in Eq. (3). This is achieved via the axial-vector Ward-

Takahashi identity which encodes the properties of chiral symmetry in connection with QCD. Its satisfaction by the interaction kernels in related equations guarantees the correct implementation of chiral symmetry and its dynamical breaking, leading, e.g., to a generalized Gell-Mann–Oakes–Renner relation valid for all pseudoscalar mesons and all current-quark masses [24,25]. In particular the pion, being the Goldstone boson related to dynamical chiral symmetry breaking, becomes massless in the chiral limit, independent of the details of the interaction. Specifically, we describe the qq kernel by a ladder dressed-gluon exchange:

$$\mathcal{K}_{\alpha\alpha'\beta\beta'}(k) = Z_2^2 \frac{4\pi\alpha(k^2)}{k^2} T_k^{\mu\nu} \gamma^{\mu}_{\alpha\alpha'} \gamma^{\nu}_{\beta\beta'}, \qquad (13)$$

which must also appear in the corresponding quark Dyson-Schwinger equation whose solution defines the renormalized dressed quark propagator:

$$S^{-1}_{\alpha\beta}(p) = Z_2(i\not{p} + m)_{\alpha\beta} + \int_q \mathcal{K}_{\alpha\alpha'\beta'\beta}(k) S_{\alpha'\beta'}(q). \quad (14)$$

The bare quark mass m enters as an input, and the gluon momentum is $k = p - q$. The inherent color structure of the kernel leads to prefactors $2/3$ and $4/3$ for the integrals in Eqs. (2) and (14), respectively.

Equations (13) and (14) define the rainbow-ladder (RL) truncation which has been extensively used in Dyson-Schwinger equation studies of mesons and baryons in the quark-diquark model, e.g., [26,27] and references therein. The nonperturbative dressing of the gluon propagator and the quark-gluon vertex are absorbed into an effective coupling $\alpha(k^2)$ for which we adopt the ansatz [28,29]

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 e^{-\eta^2(\frac{k^2}{\Lambda^2})} + \alpha_{\mathrm{UV}}(k^2). \qquad (15)$$

The second term reproduces the logarithmic decrease of QCD's perturbative running coupling and vanishes at $k^2 = 0$. The first term is parametrized by an infrared scale Λ and a dimensionless parameter η. It yields the nonperturbative enhancement at small and intermediate gluon momenta necessary to generate dynamical chiral symmetry breaking and hence a constituent-quark mass scale. ($\{\Lambda, \eta\}$ and the infrared parameters used in [29] are related by $\mathcal{C} = (\Lambda/\Lambda_t)^3$ and $\omega = \eta^{-1}\Lambda/\Lambda_t$, with $\Lambda_t = 1$ GeV.)

Beyond the present truncation, corrections arise from pseudoscalar meson-cloud contributions which provide a substantial attractive contribution to the "quark-core" of dynamically generated hadron observables in the chiral regime and vanish with increasing current-quark mass, but also from nonresonant contributions due to the infrared structure of the quark-gluon vertex. To anticipate corrections we exploit the freedom in adjusting the input scale Λ. We adopt two different choices established in the literature in the context of π and ρ properties [29]:

Setup A is determined by a fixed scale $\Lambda = 0.72$ GeV, chosen in [28] to reproduce the experimental pion decay constant and the phenomenological quark condensate. Corresponding results are, therefore, aimed in principle at a comparison to experimental data for meson and baryon properties (see [23,26] and references therein). Setup B defines a current-mass dependent scale which is deliberately inflated close to the chiral limit, where $\Lambda \approx 1$ GeV [29]. It is meant to describe a hadronic quark core which must subsequently be dressed by pion-cloud effects and other corrections. As a result, π, ρ, N, and Δ observables are consistently overestimated, but (with the exception of the Δ baryon) compatible with quark-core estimates from quark models and chiral perturbation theory (for a detailed discussion, see [22,23,29]). Irrespective of the choice of Λ, hadronic ground-state properties have turned out to be insensitive to the value of η in a certain range [26,28]. Consequently, with Eqs. (13) and (15) and Λ, the input of Eq. (2) is completely specified with all parameters already fixed to meson properties.

Since the dressed-gluon exchange kernel is flavor independent and we consider only equal quark masses, the equations for the Dirac amplitudes $\Psi_{\mathcal{M}_\mathcal{A}}$ and $\Psi_{\mathcal{M}_\mathcal{S}}$ in Eq. (12) decouple because of the orthogonality of the two flavor tensors $\mathsf{T}_{\mathcal{M}_\mathcal{A}}$ and $\mathsf{T}_{\mathcal{M}_\mathcal{S}}$. Hence one obtains two degenerate solutions of Eq. (2), where by virtue of the iterative solution method the symmetry of the start function determines the symmetry of the resulting amplitude. The massive computational demand in solving the equation primarily comes from the five Lorentz-invariant momentum combinations of Eq. (5) upon which the amplitudes depend. In analogy to the separability assumption of the nucleon amplitude in the quark-diquark model we omit the dependence on the angular variable $z_0 = \hat{p}_T \cdot \hat{q}_T$ but solve for all 64 dressing functions $f_k(p^2, q^2, 0, z_1, z_2)$.

The resulting nucleon masses at the physical pion mass in both setups A and B are presented in Table I. The difference of $\sim 2\%$ between the $\mathcal{M}_\mathcal{A}$ and $\mathcal{M}_\mathcal{S}$ solutions is presumably an artifact associated with the omission of the angle z_0. For either solution typically only a small number of covariants are relevant which are predominantly s wave with a small p-wave admixture. The angular dependence in the variable z_2 is small compared to z_1 in analogy to the quark-diquark model, where the dependence on the angle between the relative and total momentum of the two quarks in a diquark amplitude is weak.

The evolution of M_N and the ρ-meson mass from the BSE vs m_π^2 is plotted in Fig. 2 and compared to lattice results. The findings are qualitatively similar to those for

TABLE I. Nucleon masses (in GeV) obtained from the Faddeev equation in setups A and B and compared to the quark-diquark result. The η dependence is indicated for setup B in parentheses.

	Q-DQ [23]	Faddeev ($\mathcal{M}_\mathcal{A}$)	Faddeev ($\mathcal{M}_\mathcal{S}$)
Setup A	0.94	0.99	0.97
Setup B	1.26(2)	1.33(2)	1.31(2)

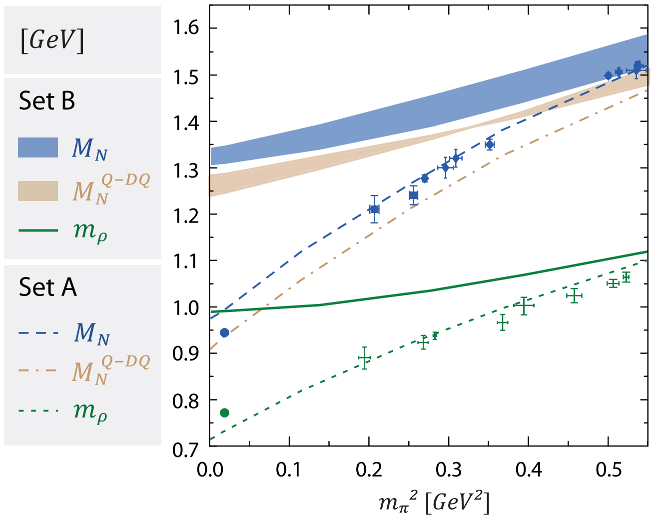

FIG. 2 (color online). Evolution with m_π^2 of m_ρ and M_N compared to lattice data; see [23] for references. The quark-diquark model result for M_N is plotted for comparison. Dashed and dash-dotted lines correspond to setup A; the solid line for m_ρ and the bands for M_N (mixed-antisymmetric solution) are the results of setup B, where the variation with η is explicitly taken into account. Dots denote the experimental values.

m_ρ: setup A, where the coupling strength is adjusted to the experimental value of f_π, agrees with the lattice data, which is reasonable in light of a recent study of corrections beyond RL truncation for the ρ meson [30]. Setup B provides a description of a quark core which overestimates the experimental values while it approaches the lattice results at larger quark masses.

A comparison to the quark-diquark model result of Refs. [22,23], where the same quark propagator and effective coupling were used, exhibits a discrepancy of only $\sim$5%. This surprising and reassuring result indicates that a description of the nucleon as a superposition of scalar and axial-vector diquark correlations that interact with the remaining quark provides a close approximation to the corresponding three-quark amplitude as obtained from the covariant Faddeev equation, Eq. (2), which, however, neglects irreducible three-quark interactions.

We have provided the first fully Poincaré-covariant three-quark amplitude for the nucleon as the result of a dynamical equation. The present study contains the first numerical results for the nucleon mass in this approach. Because of the considerable computational efforts involved, more results and an in-depth investigation with regard to the complete set of invariant variables will follow in subsequent publications. Future extensions of the present work will include an analogous investigation of the Δ-baryon, more sophisticated interaction kernels, e.g., in view of pionic corrections and the inclusion of three-quark irreducible components, and ultimately a comprehensive study of baryon resonances.

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