Comment on "Minimal Energy Cost for Thermodynamic Information Processing: Measurement and Information Erasure"

In a recent Letter [1], Sagawa and Ueda argue that Landauer's principle for information erasure [2] is only valid for symmetric memories. Landauer's principle is intimately connected to the second law and its violation in asymmetric memories would thus have dramatic consequences [3]. In the present Comment, we explain why the analysis of the one-bit memory depicted in Fig. 1 of [1] is not correct.

Following Ref. [1] we consider a one-particle ideal gas in a partitioned box as the model for a two-state memory. The particle has probability p_0 to be in the left (zero) state and $p_1 = 1 - p_0$ to be in the right (one) state. The initial Gibbs-Shannon entropy of the system is $S_i = -k\sum_n p_n \ln p_n$ and the final entropy S_f after the memory has been reset to state zero (or one) is zero. An elementary derivation of the erasure principle makes use of the Clausius inequality $T\Delta S \geq Q$: During a full erasure cycle, $\Delta U = W_{\text{era}} + Q = 0$ and the amount of heat $-Q$ given to the environment is equal to the work W_{era} done on the system. It then follows that the erasure work always satisfies, $W_{\text{era}} \geq \Delta F = -T\Delta S = -kT\sum_n p_n \ln p_n$. For the particular case of a symmetric memory, $p_0 = p_1 = 1/2$, we recover the standard "$kT\ln 2$" version of Landauer's principle. The above expression is completely general; it only requires that initial and final states of the system are in equilibrium, so that the corresponding entropies (and free energies) are defined. Sagawa and Ueda discuss quasistatic erasure in a box with symmetric probabilities, $p_0 = p_1 = 1/2$, but asymmetric volume partitioning, with volume ratio t: $1 - t$ $(0 < t < 1)$ [1]. They calculate the work required to reset the memory to state zero and find $W_{\text{era}} = kT\ln 2 - (kT/2)\ln[t/(1-t)]$. The latter lies below the Landauer bound when $t > 1/2$, leading the authors to the conclusion that Landauer's principle only holds for fully symmetric memories, in contradiction to the simple thermodynamic results derived above.

In order to clarify the issue, let us first point out that the initial state considered in [1] is not an equilibrium state—its entropy and free energy are therefore not defined. Equilibrium requires that temperatures and pressures in both partitions are respectively equal. Applying the ideal gas law, $PV = NkT$, we find that volume and probability ratios are identical, $V_1/V_0 = N_1/N_0 = p_1/p_0$ [the probabilities (p_0, p_1) of finding the particle in either partition being here given by the respective particle number fractions (N_0, N_1)]. At equilibrium, asymmetric volume partitioning thus implies asymmetric probabilities. Comparing the nonequilibrium result of Ref. [1] with the equilibrium result of Landauer is therefore inconsistent [4]. We next derive the minimal work required to reset a memory with asymmetric volume and probability ratio. We begin by noting that the state labelling is arbitrary; resetting the memory to either zero or one should be physically equivalent. This is obvious for symmetric memories. The work done by quasistatically compressing the gas from the right or the left, corresponding respectively to a reset to zero or one, is in both cases $kT\ln 2$. In the second case, the particle can be brought back to the right-hand side (state zero) by simply turning the box over, without additional work [5]. The situation seems different for asymmetric memories. Compression from the right (to zero) requires $W_R = -kT\ln t$, which is larger than the Landauer bound $W_{\text{Landauer}} = -kT[t\ln t + (1-t)\ln(1-t)]$ for $t < 1/2$, while from the left (to one) $W_L = -kT\ln(1-t)$, which is smaller than W_{Landauer}. However, the information content is unaffected by the substitution zero $\leftrightarrow$ one. The entropy, both before and after erasure, is indeed invariant under $t \leftrightarrow 1-t$; so should the work W_{era}. For $t < 1/2$, it is advantageous to first reset to one (left compression) and then turn the box over (to state zero with no extra cost), whereas for $t > 1/2$, it is advantageous to directly reset to zero (right compression). Since the relevant parameter is the ratio t $1-t$ (or its inverse) and not the values of t and $1-t$, the erasure work is minimized by averaging over right and left compressions; the minimal value being precisely the Landauer limit W_{Landauer}, which is clearly invariant under $t \leftrightarrow 1-t$, in contrast to the result presented in Ref. [1]. The erasure principle thus holds for both symmetric and asymmetric memories.

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Raoul Dillenschneider and Eric Lutz
Department of Physics
University of Augsburg
D-86135 Augsburg, Germany

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[1] T. Sagawa and M. Ueda, Phys. Rev. Lett. **102**, 250602 (2009).
[2] R. Landauer, IBM J. Res. Dev. **5**, 183 (1961).
[3] H. S. Leff and A. F. Rex, *Maxwell's Demon 2* (IOP Publishing, Bristol, 2003).
[4] The work extracted from the memory by letting it equilibrate could be used to partially erasure its information.
[5] R. P. Feynman, *Feynman Lectures on Computation* (Addison-Wesley, Reading, 1996), Chap. 5.

