Stimulated Brillouin Scatter in a Magnetized Ionospheric Plasma

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High power electromagnetic waves transmitted from the HAARP facility in Alaska can excite low-frequency electrostatic waves by magnetized stimulated Brillouin scatter. Either an ion-acoustic wave with a frequency less than the ion cyclotron frequency ($f_{\rm CI}$) or an electrostatic ion cyclotron (EIC) wave just above $f_{\rm CI}$ can be produced. The coupled equations describing the magnetized stimulated Brillouin scatter instability show that the production of both ion-acoustic and EIC waves is strongly influenced by the wave propagation relative to the background magnetic field. Experimental observations of stimulated electromagnetic emissions using the HAARP transmitter have confirmed that only ion-acoustic waves are excited for propagation along the magnetic zenith and that EIC waves can only be detected with oblique propagation angles. The ion composition can be obtained from the measured EIC frequency.

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The composition of the ionosphere is difficult to determine using ground-based sensors. HF excitation and ground reception of stimulated electromagnetic emission (SEE) down-shifted by an electrostatic ion cyclotron (EIC) wave frequency permits direct measurement of the ion mass. With the EIC mode excitation described here, high power radio wave facilities can be used as ground-based ion mass spectrometers to determine the composition of the E and F layers. This is especially valuable for identification of metallic ions of meteor origin in the E layer.

Stimulated Brillouin scatter by high power radio waves in the ionosphere involves three waves [1,2]. First an electromagnetic wave propagates from a source toward a plasma layer of increasing density. The pump wave can be either an ordinary or an extraordinary mode at frequency ${\omega_P}^{(O)}$ or ${\omega_P}^{(X)}$. As the wave approaches a cutoff point where the vertical component of refractive index vanishes, the electric field grows to a value exceeding a threshold for parametric decay into a scattered high-frequency electromagnetic wave and a low-frequency electrostatic wave. The low-frequency wave can be an ion-acoustic (IA) wave with frequency ω_{L-} or, in a magnetized plasma, an EIC wave with frequency ω_{L+}. The $\pm$ is used as a subscript because the EIC and IA waves are, respectively, above and below the local value of ion cyclotron frequency ($\Omega_i = eB/m_i = 2\pi f_{\rm ci}$) in the Earth's magnetic field $\mathbf{B}$ [2].

The received IA SEE spectra are distinguished by their frequencies. The IA emissions originating near the reflection altitude have a down-shifted peak (called SBS-1 or NP) frequency of about 8 to 15 Hz. The IA emissions from the upper-hybrid resonance region have a down-shifted peak (called BS-2 or 2NP) frequency of about 30 Hz. Both SBS modes can have weaker up-shifted (anti-Stokes) peaks with the same frequency offsets [1,2]. The IA frequency offset at the upper-hybrid region can be analyzed to yield the ion sound speed or electron temperature of the heated plasma [2].

The electric field vectors for the incident and scattered electromagnetic (EM) waves are $\mathbf{E}_P$ and $\mathbf{E}_S$, respectively. The IA and EIC waves are characterized by ion velocity fluctuation amplitudes $\tilde{\mathbf{v}}_i$ at frequency $\omega_{L\pm}$. The matching conditions for decay of an EM pump wave at frequency ω_P and wave number $\mathbf{k}_P$ into a scattered EM wave at frequency ω_S and wave number $\mathbf{k}_S$ and a low-frequency wave at frequency $\omega_{L\pm}$ and wave number $\mathbf{k}_{L\pm}$ are given by $\omega_P^{(O,X)} = \omega_P^{(O,X)} + \omega_{L\pm}$ and $\mathbf{k}_P^{(O,X)} = \mathbf{k}_P^{(O,X)} + \mathbf{k}_{L\pm}$ where the superscript O and X denote ordinary and extraordinary mode EM waves, respectively. The EM modes retain their identity after scatter by the magnetized stimulated Brillouin scatter (MSBS) process.

To better understand the generation of the MSBS, the theory for the scattered wave generation is derived assuming typical ionospheric conditions. The electric fields for the initial pump wave are given by the wave equation derived from Maxwell's equations [3]

$$-\nabla^2\mathbf{E}_P + \nabla(\nabla\cdot\mathbf{E}_P) - k_P^2[\mathbf{I} + \mathbf{X}_{eP}]\cdot\mathbf{E}_P = 0, \quad (1)$$

where the subscript P denotes the pump wave along the propagation path, $k_P = \omega_P/c$ is the free space wave number, and the EM wave susceptibility $\mathbf{X}_e$ at the wave frequency ω_P is proportional to the electron density and includes the effects of the electron gyro motion around $\mathbf{B}$ and collisions of the electrons with ions and neutrals [3].

Note that (1) provides the electric field vector components for both the ordinary and extraordinary EM modes.

The scattered EM wave at ω_S is driven by the pump wave electric fields $\mathbf{E}_P$ and the EIC/IA fluctuations in the ion density at $\omega_{L\pm}$. Both O-mode and X-mode electric fields may be excited during the SBS process with the following expression,

$$-\nabla^2\mathbf{E}_S + \nabla(\nabla\cdot\mathbf{E}_S) - k_S^2[\mathbf{I} + \mathbf{X}_{eS}]\cdot\mathbf{E}_S = k_S^2\tilde{\mathbf{X}}_{eL}\cdot\mathbf{E}_P, \tag{2}$$

where fluctuations in the electron susceptibility $\tilde{\mathbf{X}}_{eL}$ come from low-frequency oscillations in the electron density. Quasineutrality and the ion continuity equation are used to relate the ion velocity fluctuations to the wave-induced perturbations in electron density. The mixing between the pump wave and the low-frequency ion wave is described by the right side of (2).

The two electromagnetic waves mix to yield low-frequency electrostatic (IA and EIC) waves. The nonlinear ponderomotive force provides this coupling between the pump and scattered electromagnetic waves. The ponderomotive force in an electromagnetic field has been derived using the single particle approach for a magnetized plasma [4].

The low-frequency wave equation is based on the equations of continuity and momentum for the plasma [2]. For small density and velocity disturbances, linearized forms of these equations are combined giving

$$\nabla(\nabla\cdot\tilde{\mathbf{v}}_i) + \frac{\omega_{L\pm}^2}{c_{\mathrm{IA}}^2}\left(U_i\tilde{\mathbf{v}}_i - i\frac{\boldsymbol{\Omega}_i\times\tilde{\mathbf{v}}_i}{\omega_{L\pm}^2}\right) = \frac{i\omega_{L\pm}q_e^2}{4m_e m_i\omega_P^2 c_{\mathrm{IA}}^2}\left[\frac{\nabla(\boldsymbol{\Omega}_e\cdot\mathbf{E}_T)^2 - \omega_P^2\nabla(\mathbf{E}_T\cdot\mathbf{E}_T)}{\Omega_e^2 - \omega_P^2}\right]_{\omega_{L\pm}}, \tag{3}$$

where $\boldsymbol{\Omega}_i = e\mathbf{B}/m_i$, $U_i = 1 - i\nu_i/\omega_{L\pm}$ includes the damping term for the low-frequency waves, and $c_{\mathrm{IA}} = \sqrt{(\gamma_e T_e + \gamma_i T_i)/m_i}$ is the ion sound speed. The right side of (3) contains the ponderomotive force contribution with the total electric field ($\mathbf{E}_T = \mathbf{E}_P + \mathbf{E}_P$) squared yielding cross terms that drive low-frequency fluctuations of the form $e^{i\omega_{L\pm}t}$. The left side of (3) describes both the ion-acoustic ($-$) and electrostatic ion cyclotron ($+$) waves [2]. For wave propagation along the magnetic field, $\boldsymbol{\Omega}_i\times\tilde{\mathbf{v}}_i = 0$ and only the ion-acoustic wave mode remains. Electrostatic ion cyclotron waves require a finite perpendicular component to the propagation vector [5].

The coupled set of low- and high-frequency waves equations (1)–(3) provides a complete description of the SBS instability in a magnetized plasma. The growth rate for the MSBS is obtained by Fourier transform analysis of these equations assuming spatial variations with the form $e^{-i\mathbf{k}_{L\pm}\cdot\mathbf{x}}$, $e^{-i\mathbf{k}_S\cdot\mathbf{x}}$, and $e^{-i\mathbf{k}_P\cdot\mathbf{x}}$ for the low-frequency (EIC/IA), scattered EM and pump EM waves, respectively. All of the complex exponential can be dropped because of the wave matching conditions. For Brillouin scatter, $\mathbf{k}_P \cong -\mathbf{k}_S$ and $\mathbf{k}_L \cong 2\mathbf{k}_P$ [2,4,6].

The growth rates for the MSBS process are found by linearization of the coupled nonlinear equations (2) and (3) to form a single dispersion equation for the ion-acoustic and ion cyclotron waves driven by the pump given in (1). The general procedure for derivation of this dispersion equation is given by a number of authors [6]. Assuming that $|k_L c_{\mathrm{IA}}| \ll \Omega_i$, a simple expression is found for the ratio of the EIC and IA growth rates

$$\frac{\gamma_{LO+}}{\gamma_{LO-}} = \frac{\gamma_{\mathrm{EIC}}}{\gamma_{\mathrm{IA}}} = \sqrt{\frac{\omega_{\mathrm{IA}}}{\Omega_i}}\mathrm{Tan}^3\theta, \tag{4}$$

where the low-frequency wave has real and imaginary components with $\omega_{L\pm} = \omega_{L0\pm} - i\gamma_{L0\pm}$ and q is the angle between the wave vector and the magnetic field B. For small k_L, the real low-frequency components are given as $\omega_{L-} = \omega_{\mathrm{IA}} = c_{\mathrm{IA}}k_L\mathrm{Cos}\theta$ and $\omega_{L\pm} = \omega_{\mathrm{EIC}} = \sqrt{\Omega_i^2 + c_{\mathrm{IA}}^2 k_L^2}$ [7]. The full expressions for these frequencies are derived by Bernhardt *et al.* [2]. Threshold fields for each mode are also dependent on the propagation angle in the plasma.

The theory predicts a beam angle dependence for the excitation of the ion-acoustic and electrostatic ion cyclotron modes of MSBS. To find the EIC mode inSEE, it is necessary to move the EM pump wave vector far from magnetic zenith. A search for the IA and EIC frequency offsets was conducted by tilting the High Frequency Active Auroral Research Program (HAARP) EM O-mode beam to large angles away from the vertical and the magnetic zenith. Previous tilting of the beam at HAARP [1,2] was not far enough from $\mathbf{B}$ to excite the EIC mode.

For propagation at small angles to the magnetic field, Eq. (4) is much less than unity and the ion-acoustic waves grow much faster than the electrostatic ion cyclotron waves. For propagation nearly perpendicular to B, Eq. (4) is much greater than unity and the EIC mode is dominant. The transition between IA and EIC wave excitation occurs at a critical angle θ_{Crit} found by setting Eq. (4) to unity using $k_L \cong 2k_P$. The propagation angle θ that changes with distance into the plasma starts out with an initial value of θ_0 on the ground. The MSBS modes excited depend on the value of θ at the point of maximum electric field in the standing wave of the EM pump.

The HAARP transmitter is operated at O-mode with 3.6 MW power at 4.2 MHz. The HF signal data were acquired with the Australian developed GBox-5 receiver that digitizes a 250 kHz band around the pump frequency for digital signal processing to produce the SEE spectra [2]. The MSBS experiments 13 March 2009 used a full power beam pointed at different directions θ_0 between 0 and 22 degrees with respect to the magnetic zenith (202° azimuth and 14° zenith angle). For θ_0 greater than 15 degrees, Figs. 1(b) and 1(c) show strongly excited EIC modes at 48.6 Hz corresponding to the ion-gyro frequency for the

O^+ ion. Using the International Geomagnetic Reference Field model of the magnetic field over HAARP, gyro frequency at 225 km altitude was estimated to be 48.6 Hz

The strength of the EIC SBS line is not constant. A 110 s spectrogram illustrates that the EIC SBS line near f_{CI} fluctuates by 40 dB with constant power intervals of a few seconds as does its much weaker second harmonic line at 2 f_{CI} (Fig. 2). This is attributed to a mode near threshold for excitation. Reduction of the pump amplitude by the SBS conversion process may bring its amplitude below the mode threshold and turn off the EIC SBS instability. The IA SBS lines are much more constant indicating that, for the 21.4 degree EM pump beam angle, the ion-acoustic mode is not near its threshold. The EIC spectral lines are much narrower than the IA spectral lines. This is consistent with the wave matching conditions given by Bernhardt *et al.* [2] where the resonant EIC mode frequency is nearly constant with altitude and the IA resonant frequency has much more variation with altitude. Over the same range of interaction altitudes, the EIC mode will have a much smaller spectral range than the IA mode.

Both the down-shifted (Stokes) and up-shifted (anti-Stokes) emissions are easily seen in the spectra of Fig. 2. The anti-Stokes emissions for the EIC mode are rare but, as indicated by the white circles, can be found when the EIC Stokes mode is strong and constant. The EIC anti-Stokes lines are the result of scatter by the reflected pump wave from the EIC waves to produce an upgoing, up-shifted EM wave that reflects in the ionosphere and is received on the ground [2].

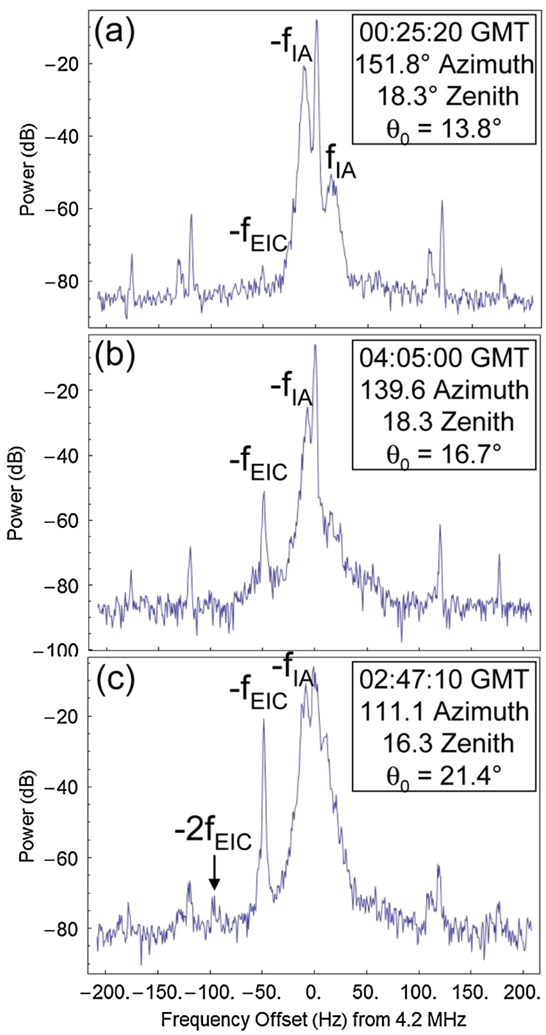

FIG. 1 (color online). Low-frequency SEE spectra showing ion-acoustic and electrostatic ion cyclotron spectral lines. When the beam is offset from the magnetic field by (a) 13.8 degrees, a strong IA and an extremely weak EIC line are produced. Increasing the offset angle to (b) 16.7 degrees increases the strength of the EIC line by 24 dB. With (c) 21.4 degree injection angle, the EM pump wave yields an additional 29 dB increase in the EIC emission.

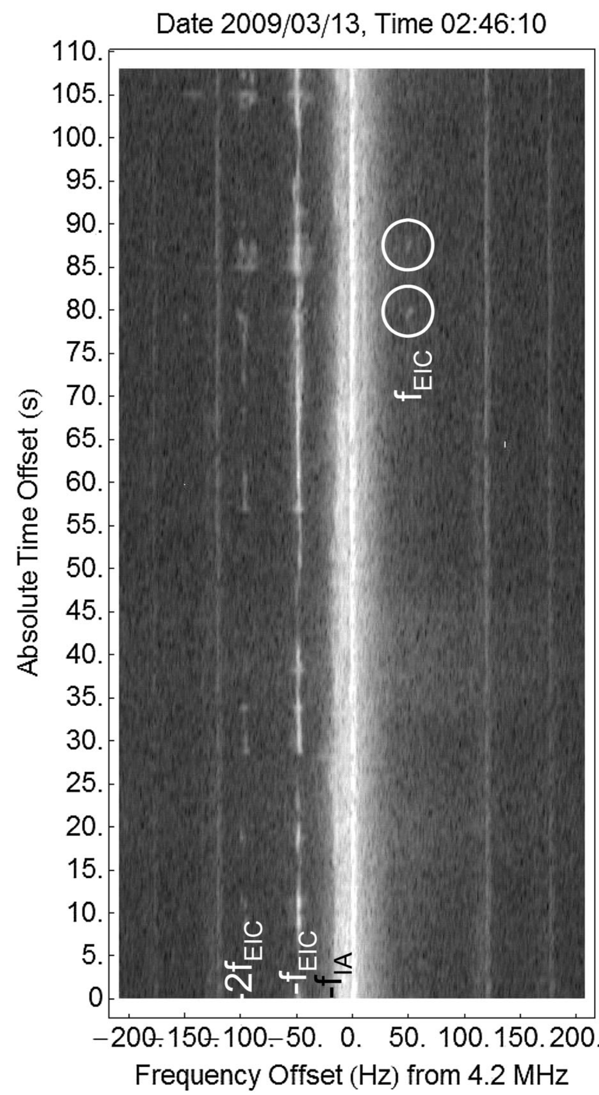

FIG. 2. Spectrogram of the stimulated Brillouin scatter lines for a pump wave beam offset 21.4 degrees from the magnetic field direction.

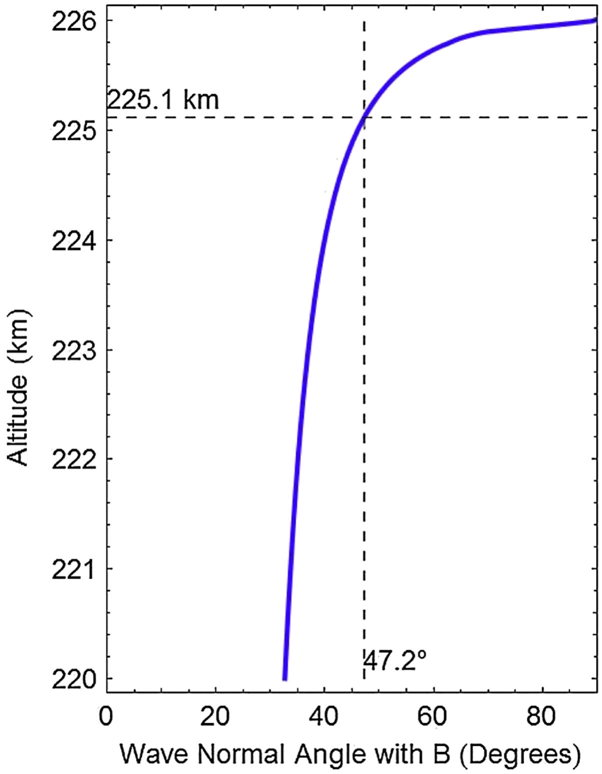

FIG. 3 (color online). Computed wave normal angle in the plasma for an oblique, O-mode plane wave launched from the ground at 4.2 MHz. The plasma layer profile for the computation is derived from the ionogram taken at the time of the HAARP experiment shown in Fig. 2.

The angle (θ) of the wave normal relative to B is computed by solving Eq. (1) to produce a standing wave in a horizontally stratified ionosphere. The same full-wave solution was used to determine the pump wave profile that drives the SBS modes for vertical incidence [2]. With $\theta_0 = 21.4$ degrees at a geographic azimuth of 111.1° on the ground, the wave normal tilt increases with altitude having a maximum value of 90 degrees near the reflection altitude. Figure 3 illustrates the computed variation of θ as a function of altitude in the measured background ionosphere. The computed EM pump amplitude peaks at 225 km altitude where the k vector makes an angle $\theta = 47.2$ degrees with B. Substitution of this angle along with the measured values of 8.8 Hz for the ion-acoustic frequency and 48.6 Hz for the ion cyclotron frequency into Eq. (4) gives $\gamma_{\rm EIC}/\gamma_{\rm IA} = 0.5$ so the ion-acoustic growth rate is twice the EIC growth rate. Tilting of the HF beam farther from the magnetic zenith should yield even stronger EIC waves.

Practical, remote sensing applications for HAARP can be based on the measurements of the EIC SBS emissions. Besides validating the theory that predicts a wave angle dependence for the MSBS instabilities, the measurements demonstrate a sensitive method for determining ion species. The ion mass is measured from the ion-gyro frequency offset by the SBS EIC mode using $m_i = eB/\Omega_I$ where models such as the International Geomagnetic Reference Field can provide a precise value of magnetic field strength B. The composition of the lower ionosphere can be changed by meteorite ablation and this technique could detect the presence of metallic ions. Future experiments will search for EIC modes in strong sporadic E layers excited by power HF waves. Additional theory is required to obtain the relative abundance of the ion species and to account for enhanced collision frequencies at lower altitudes.

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