Two-Particle Separation Energy Trends in the Superdeformed Well

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(Received 18 November 2009; published 20 April 2010)

A measurement of the energy and spin of superdeformed states in ^{190}Hg, obtained through the observation of transitions directly linking superdeformed and normal states, expands the number of isotopes in which binding energies at superdeformation are known. Comparison with neighboring nuclei shows that two-proton separation energies are higher in the superdeformed state than in the normal state, despite the lower Coulomb barrier and lower total binding energy. This unexpected result provides a critical test for nuclear models.

DOI: 10.1103/PhysRevLett.104.162501 PACS numbers: 23.20.−g, 21.10.Dr, 23.20.Lv, 27.80.+w

The location of nuclear closed shells, as evidenced through discontinuities in binding energy and one- and two-particle separation energy systematics, remains one of the simplest tests of global nuclear models. How shell gaps evolve, whether with increasing mass, increasing neutron:proton ratio or increasing deformation, is still uncertain, and it has recently been suggested that one must go beyond a static mean-field picture to include the effects of dynamic fluctuations in the nuclear shape even in the ground-state (see, e.g., [1] and references therein). The identification of key properties which may distinguish between competing approaches is thus vital.

The present work reports on the establishment of three such properties in the Hg/Pb region of superdeformed (SD) nuclei. Superdeformation in this mass region is associated with a distinct excited minimum, well-separated from the spherical or weakly oblate normal (ND) states, in which an effectively independent level structure is formed. The nonrotating SD state can thus be thought of as a second ground state or quasivacuum, and the SD nucleus as a distinct system from its ND counterpart.

In principle, such quasivacuum states offer an excellent opportunity to test our understanding of nuclear structure through measurements of quantities such as binding energies or two-particle separation energies in the deformed system. However, although 20 nuclei are known to support SD shapes in the Hg/Pb region, it has proved frustratingly difficult to make use of these relatively simple level structures to evaluate different nuclear models. This is largely because fundamental quantities—excitation energy, angular momentum and parity—have proved difficult to determine.

The key to a precise measurement of these quantities is the unambiguous determination of decay sequences connecting SD and ND states. Unfortunately, the decay is distributed among many pathways, and the spectrum of γ rays connecting the SD and ND states is dominated by unresolved transitions. Only occasionally is sufficient strength concentrated in a transition directly linking SD and ND states that it can be resolved. Fortunately, the unresolved or quasicontinuum (QC) component of the spectrum itself contains information about the energy and spin of the SD states. The high energy part of the QC component is composed of transitions between SD states and low-lying, discrete ND states, with the QC end point determined by the excitation energy of the decaying state. Isolation of this part of the spectrum can therefore yield the average energy and spin removed in the decay, and hence a somewhat less precise measurement of the excitation energy and spin of the SD states (as has been carefully demonstrated in ^{194}Hg [2]).

"Single-step" transitions linking SD and ND states have been identified in three Pb isotopes (^{192}Pb [3], ^{194}Pb [4,5], and ^{196}Pb [6]). A quasicontinuum analysis has established a lower limit on the SD energy of ^{195}Pb [7].

Excitation energy measurements via the observation of discrete linking transitions have proved somewhat more difficult in Hg nuclei, and it is only in ^{194}Hg that direct links between SD and ND states have been unambiguously established [8,9]. However, quasicontinuum analyses have furnished measurements in ^{192}Hg [2] and ^{191}Hg [10], with candidate single-step links giving a lower limit on the energy in the latter.

The present work reports on candidate transitions linking states in the SD quasivacuum band in ^{190}Hg with known states in the ground-state minimum. The resulting excitation energy and spin are supported by an analysis of the QC spectrum. The measured SD binding energy, combined with data on neighboring nuclei, sets the stage for systematic study of binding and two-particle separation energies in SD nuclei in this region.

The data were obtained using the EUROBALL IV detector array [11]. Excited states in ^{190}Hg were populated following the ^{160}Gd(^{34}S, $4n$) reaction. The beam of 156 MeV ^{34}S was incident on a target consisting of a stack of two thin (500 μg/cm^2), self-supporting foils of ^{160}Gd. A total of 1.76×10^9 events (when 3 or more γ rays were detected) were obtained. Further details of the experiment and data analysis procedures are given elsewhere [12].

Figure 1(a) shows the spectrum obtained by triple-gating on the SD quasivacuum band [13,14]; Fig. 1(b) shows the same spectrum in the 2–3 MeV region. A peak is clearly visible at 2717(2) keV. There is some evidence for other peaks at lower energies, including two peaks with intensities $\sim 2\sigma$ above the background at 2207(2) keV and 2400(2) keV. We have confirmed that the same peaks are present in lower statistics data obtained in an earlier study of ^{190}Hg carried out using the Eurogam II array [14], strongly supporting their assignment as in true coincidence with the SD band.

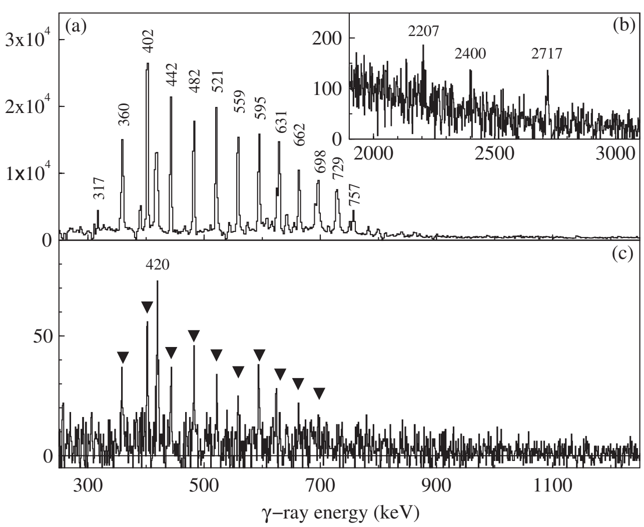

FIG. 1. (a) and (b) Spectrum obtained by triple-gating on the SD quasivacuum band in ^{190}Hg. (c) Spectrum in coincidence with at least two SD transitions (marked with closed triangles) and the 2717 keV transition.

Figure 1(c) shows the spectrum in coincidence with at least two SD transitions and the 2717 keV γ ray. The 420 keV E2 transition which feeds the 23 ns 12^+ isomer [15] in the ND well is clearly visible. Its intensity is consistent with that of the 360 and 402 keV in-band SD transitions (with the effects of gating taken into account) and is significantly greater than the limit on the intensity of the 317 keV in-band transition, which is not visible in the spectrum. The 2717 keV transition is therefore assigned as deexciting the second level in the SD band and feeding the ND 14^+ state at 3041 keV [15].

Although the SD band and ND transitions are visible in spectra gated on the weaker 2207 or 2400 keV γ-rays in combination with transitions from the SD band, low statistics prevent their unambiguous location within the level scheme on the basis of the coincidence data alone. However, energy-summing considerations suggest that the 2400 keV transition deexcites the lowest observed SD state and feeds the 14^+ state at 3041 keV. Similarly, it is likely that the 2207 keV transition deexcites the same state as the 2717 keV transition and feeds the known ND 13^- state at 3549 keV [15]. (The coincidence data confirm that this level is fed in the band's decay.)

The proposed decay paths are shown in Fig. 2. Assuming the linking transitions carry no more than $2\hbar$, the spin of the lowest observed SD state could be $12\hbar$ or $13\hbar$. Since the band may be thought of as a quasivacuum band in an even-even SD nucleus, it is unlikely that the in-band levels will have odd spin; we therefore prefer the spin assignment indicated in the figure, which is consistent with the ND levels fed in the decay [14].

The excitation energy and spin assignment is supported by analysis of the QC component of the decay spectrum. The procedures followed in the present work are based on those developed and validated by Lauritsen *et al.* [2,16] and are the same in detail as used for the neighboring isotope ^{191}Hg [10]. The intensities of the three lowest-energy transitions in the SD band in ^{190}Hg indicate the

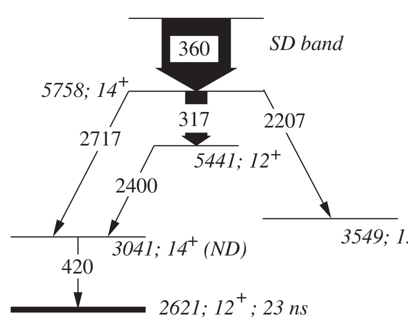

FIG. 2. Partial decay scheme.

average exit point is 427 keV and $2.6\hbar$ above the lowest observed SD level. The energy and spin of this point obtained from this analysis are 6.0 ± 0.5 MeV and $14.4 \pm 0.7\hbar$, leading to an excitation energy of 5.6 ± 0.5 MeV and spin of $11.8 \pm 0.7\hbar$ for the lowest level, supporting the assignment based on the candidate linking transitions. A variable-moment-of-inertia extrapolation to $I = 0$ based on this assignment yields an excitation energy of 4.53(2) MeV for the SD quasivacuum state.

Figures 3(a) and 3(b) compare the known energies of the SD quasivacuum states in even-even nuclei in this region [2–6,8,9]. The results of three theoretical approaches are also shown: static mean-field (Hartree-Fock-Bogolyubov) calculations employing state-of-the-art parameterizations of the competing Skyrme (SLy4) and Gogny (D1S) interactions [17,18], and generator coordinate method (GCM) calculations [18] (also using the Gogny D1S interaction). The main difference between the HFB and GCM approaches is that the latter takes into account long-range correlations and thus allows for the effects of quadrupole vibrational modes. Although the static mean-field calculations employing the Skyrme (Gogny) interaction reproduce the Hg (Pb) energies well, it is only the calculations including dynamic shape fluctuations that simultaneously reproduce the excitation energies in both isotope chains to within 0.5 MeV.

The quasivacuum state excitation energies can be combined with ground-state masses [19] to obtain SD binding energies. These are compared with the ground-state binding energies in Fig. 3(c). The maximum binding energy per nucleon appears to be reached at lower neutron number in the SD system compared to the ND system, with the maximum occurring at $N \approx 112$ for the SD Hg isotopes compared to $N \approx 114$ in the ND case.

The data shown in Figs. 3(a) and 3(b) depend on the properties of both the deformed and normal systems. These can be disentangled by separate examination of properties within the SD systems, and differences between the SD and ND systems. Two-particle separation energies, which measure the derivative of the binding energy trends, provide particularly sensitive probes with which to do this [20,21].

Table I compares the energies required to remove pairs of nucleons from the SD well, $S_{2p,\mathrm{SD}}$ and $S_{2n,\mathrm{SD}}$, to the results of the three theoretical approaches. There is good agreement between the static mean-field calculations and the experimental values of $S_{2p,\mathrm{SD}}$, with the Skyrme interaction performing somewhat better. The Skyrme calculations also reproduce the experimental values of $S_{2n,\mathrm{SD}}$ quite well in the Hg isotopes. However, they result in an unphysical increase from $^{194}\mathrm{Pb}_{112}$ to $^{196}\mathrm{Pb}_{114}$ (previously noted in [21]), indicating an underlying flaw in this approach. In contrast, use of the Gogny interaction results in a large drop in $S_{2n,\mathrm{SD}}$ between $N = 112$ and $N = 114$ for both isotopes (consistent with predictions of a SD shell gap at $N = 112$). This drop is much larger than that observed experimentally. Thus neither interaction produces good

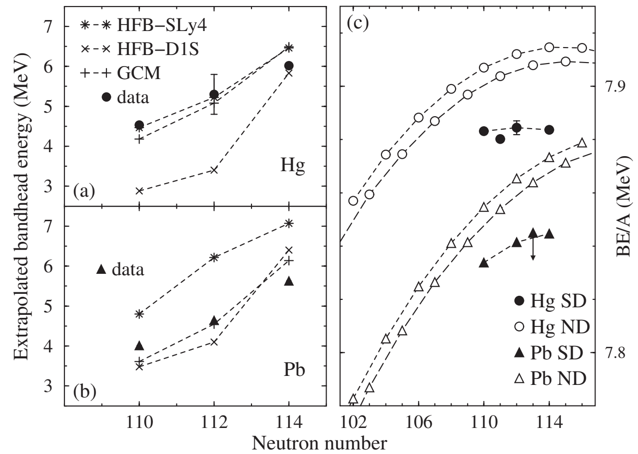

FIG. 3. (a) and (b) Known energies of SD quasivacuum states in even-even Hg and Pb nuclei compared with the results of static (SLy4 Skyrme [17] and D1S Gogny [18]) and dynamic (GCM [18]) mean-field calculations. (c) Binding energies per nucleon in the SD and ND systems.

agreement across the SD systems when used in static mean-field calculations.

The addition of dynamic shape fluctuations via the GCM approach smooths out the apparent shell gap at $N = 112$, in a manner similar to the quenching of the $N/Z = 50$ ND shell gap seen in GCM calculations using the Skyrme interaction [1]; however the resulting predictions for $S_{2n,\mathrm{SD}}$ ($S_{2p,\mathrm{SD}}$) are lower (higher) than the experimentally observed values. Thus despite their success in reproducing the relative energies of the true and quasivacuum states, these calculations underestimate the slope of the SD bind-

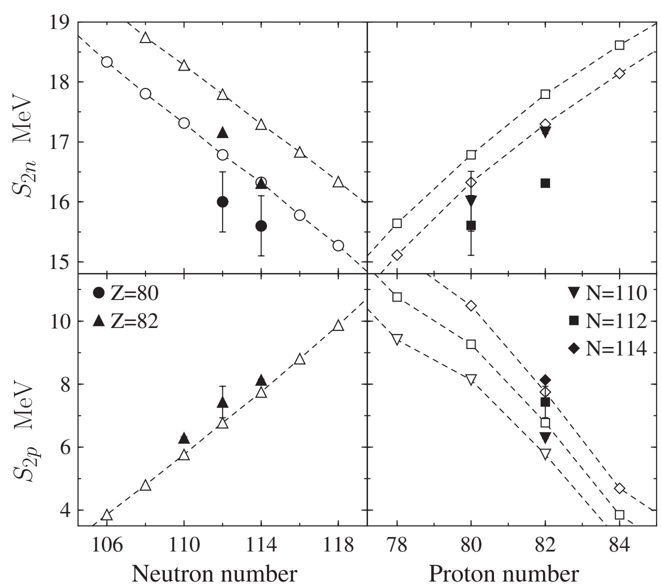

FIG. 4. Two-neutron (S_{2n}) and two-proton (S_{2p}) separation energies as a function of neutron number (left-hand panels) and proton number (right-hand panels). Open (closed) symbols indicate ground-state (SD) separation energies.

TABLE I. Two-proton and two-neutron separation energies in the SD well. All values are in MeV.

		HFB-SLy4	HFB-D1S	GCM	Experiment
$S_{2p,\mathrm{SD}}$	^{192}Pb	6.480	6.61	7.02	6.29(3)
	^{194}Pb	6.801	7.51	8.02	7.4(5)
	^{196}Pb	8.267	8.91	8.95	8.13(2)
$S_{2n,\mathrm{SD}}$	^{192}Hg	16.060	16.05	15.47	16.0(5)
	^{194}Hg	15.120	13.57	14.47	15.6(5)
	^{194}Pb	16.381	16.95	16.47	17.16(2)
	^{196}Pb	16.587	14.97	15.41	16.31(2)

ing energy trends, and overestimate the difference between the energies of the SD systems in the Pb and Hg isotopes.

The change in the energy required to remove a pair of nucleons as the deformation increases provides a test of the models' ability to simultaneously handle both true and quasivacuum systems. Naively, one might expect a reduction in both S_{2n} and S_{2p} at superdeformation, since both the binding energies per nucleon and the Coulomb barrier are lower. Figure 4 compares S_{2n} and S_{2p} in the normal [19] and SD wells. The values of $S_{2n,\mathrm{SD}}$ are consistently lower than $S_{2n,\mathrm{ND}}$. In contrast, the values of $S_{2p,\mathrm{SD}}$ are consistently larger than $S_{2p,\mathrm{ND}}$. Although this result is implicit in the data shown in Fig. 3, Fig. 4 clarifies these important differences.

This somewhat counterintuitive result provides a particularly stringent test for the models, which are compared with the data in Table II. Only the GCM calculations reproduce both the decrease in S_{2n} and the increase in S_{2p} between ND and SD wells, suggesting that mixing due to the presence of quadrupole vibrational modes may significantly suppress the proton Fermi surface in the SD system. However, these calculations overestimate the difference in S_{2n} between the true and quasivacuum states, and so do not consistently reproduce the data.

In conclusion, the growing body of data on excitation energies allows for the use of SD states as fine probes of different approaches to modeling the nuclear potential. The data show three critical features: (i) Lower excitation energies relative to the true ground states in Pb isotopes compared to Hg isotopes; (ii) only a small reduction in the energy required to remove a pair of neutrons from the SD system at $N = 114$; and (iii) a reduction in the energy required to remove a pair of neutrons and an increase in the energy required to remove a pair of protons compared to the ND systems. Of the calculations available, only those including collective quadrupole modes reproduce these features. Although the failure to reproduce all aspects of the data shows there is still room for improvement, the results suggest that dynamic shape fluctuations may play a decisive role in the superdeformed quasivacuum, and that nuclear models which do not include such long-range correlations cannot be relied on to predict the evolution of the magic numbers.

TABLE II. Differences (in MeV) between two-particle separation energies in the normal and deformed systems.

	HFB-SLy4	HFB-D1S	GCM	Experiment
$S_{2n,\mathrm{ND}} - S_{2n,\mathrm{SD}}$				
^{192}Hg	0.755	0.52	0.90	0.8(5)
^{194}Hg	1.242	2.43	1.41	0.6(5)
^{194}Pb	1.412	0.62	0.94	0.63(2)
^{196}Pb	0.856	2.30	1.58	0.99(2)
$S_{2p,\mathrm{ND}} - S_{2p,\mathrm{SD}}$				
^{192}Pb	0.335	0.60	−0.57	−0.53(3)
^{194}Pb	0.992	0.70	−0.53	−0.6(5)
^{196}Pb	0.607	0.57	−0.93	−0.60(2)

The authors are grateful to Paul-Henri Heenen for making the results of his Skyrme force calculations available. This work has been supported by the German BMBF through Grant no. 06BN907 and the EU under contracts ERBFMGECT98-0145 and ERBFMRXCT97-0123, the OTKA through Grant no. K72566 and the U.S. Department of Energy, Office of Nuclear Physics, under Contract No. DE-AC02-06CH11357.

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