Instabilities of Non-Abelian Vortices in Dense QCD

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We construct a low-energy effective theory describing non-Abelian vortices in the color superconducting quark matter under stress. We demonstrate that all the vortices are radically unstable against decay into the only one type of vortices due to the potential term induced by the explicit flavor symmetry breaking by the strange quark mass. A simple analytical estimate for the lifetime of unstable vortices is provided under the controlled weak-coupling calculations. We briefly discuss the (non)existence of magnetic monopoles at high density.

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Introduction.—Topological or quantized vortices commonly arise in a wide area of physics from condensed matter physics and cosmology to particle physics [1]. In the context of nuclear physics, the dynamical breaking of $U(1)_B$ baryon number due to the neutron superfluidity in nuclear matter gives rise to topologically stable vortices characterized by the first homotopy group $\pi_1[U(1)_B] = \mathbb{Z}$. They are phenomenologically important since the sudden increase of the rotation of neutron stars, the so-called glitches, may be attributed to the unpinning of vortices which releases an angular momentum transfer from the nuclear "mantle" to the outer crust [2]. Topological superfluid vortices also emerge in the color superconducting quark matter [3] presumably existing in the "core" of neutron stars: the $U(1)_B$ symmetry is broken by the condensation of diquark pairs in the color-flavor locked (CFL) phase [4] which is indeed shown to be the most stable ground state at asymptotic high density in quantum chromodynamics (QCD). Recently, however, it has been found that minimal topological vortices in quark matter are not $U(1)_B$ vortices [5] but non-Abelian vortices [6] referred to as the semisuperfluid vortices [7], which have only winding number $1/3$ inside $U(1)_B$. Actually it is energetically favorable for a single $U(1)_B$ vortex to split into three (red, green, and blue) non-Abelian vortices. At first glance, one may expect that all the resultant three non-Abelian vortices are stable.

In this Letter, we show that these remaining non-Abelian vortices are still unstable against decay into the only one type of stable vortices when the effect of nonzero strange quark mass m_s is taken into account. In order to elucidate the (in)stabilities of non-Abelian vortices in a model-independent manner, we use the Ginzburg-Landau (GL) approach near the transition temperature T_c, and construct the low-energy effective theory of vortices with the potential term induced by the explicit breaking of flavor symmetry. Owing to the asymptotic freedom of QCD, all the calculations throughout this Letter are under theoretical control at high density regime where the QCD coupling constant is weak. We remark that the existence of non-Abelian vortices by itself does not rely on the domain of applicability of the GL Lagrangian, but only on the dynamical symmetry breaking induced by the diquark condensation. On the other hand, the symmetry argument is not enough to ensure their stabilities which depend on the details of the dynamics. In the following, we neglect the effect of $U(1)_{\rm EM}$ electromagnetism since the mixing between broken $SU(3)_C$ color and $U(1)_{\rm EM}$ is sufficiently small at high density. The generalization to include the effect is straightforward.

Ginzburg-Landau Lagrangian.—We consider the diquark pairing in the most attractive CFL and spin-parity 0^+ channel [4]: $(\Phi_L)^i_a \sim \epsilon_{abc}\epsilon_{ijk}\langle (q_L)^j_b C (q_L)^k_c\rangle$ and $(\Phi_R)^i_a \sim \epsilon_{abc}\epsilon_{ijk}\langle (q_R)^j_b C (q_R)^k_c\rangle$, where i, j, k (a, b, c) are flavor (color) indices and C is the charge conjugation operator. Here we take $\Phi_L = -\Phi_R = \Phi$ so that the ground state is the positive parity state.

The time-dependent Ginzburg-Landau (TDGL) Lagrangian up to the second order in time and space derivatives [8] at large quark chemical potential $\mu \gg m_s \gg m_{u,d} \simeq 0$ near T_c is given by [9,10]:

$$\begin{aligned}\mathcal{L}_{\rm GL} &= {\rm Tr}(K_0 \mathcal{D}_0\Phi^\dagger \mathcal{D}_0\Phi - K_3 \mathcal{D}_i\Phi^\dagger\mathcal{D}_i\Phi) \\ &\quad + {\rm Tr}(K_D\Phi^\dagger\mathcal{D}_0\Phi + {\rm H.c.}) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V_{\rm GL}, \\ V_{\rm GL} &= {\rm Tr}\left[\Phi^\dagger\left\{\left(\alpha + \frac{2\epsilon}{3}\right)\mathbf{1}_3 + \epsilon X_3\right\}\Phi\right] \\ &\quad + \beta_1[{\rm Tr}(\Phi^\dagger\Phi)]^2 + \beta_2{\rm Tr}[(\Phi^\dagger\Phi)^2], \end{aligned} \tag{1}$$

where $\mathcal{D}_\mu\Phi = \partial_\mu\Phi - ig_sA_\mu\Phi$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s[A_\mu, A_\nu]$, and $X_3 = \frac{1}{2}{\rm diag}(0, 1, -1)$. K_D is a dissipative term reflecting the decay of Cooper pairs into fermionic excitations. The ϵ terms originate from a Fermi surface splitting due to the nonzero strange quark mass together with the constraints of electric and color charge neutrality and weak interaction equilibration [10].

The GL parameters $K_{0,3}$, α, $\beta_{1,2}$, and ϵ are obtained from the weak-coupling calculations [9,10]:

$$\alpha = 4N(\mu)\log\frac{T}{T_c}, \quad \beta_{1,2} = \frac{7\zeta(3)}{8(\pi T_c)^2}N(\mu) \equiv \beta,$$
$$K_3 = \frac{1}{3}K_0 = \frac{7\zeta(3)}{12(\pi T_c)^2}N(\mu), \quad \epsilon = N(\mu)\frac{m_s^2}{\mu^2}\log\frac{\mu}{T_c}, \tag{2}$$

where $N(\mu) = \mu^2/(2\pi^2)$ is the density of state at the Fermi surface and $T_c = 2^{1/3}e^{\gamma}\Delta/\pi$ is the critical temperature of the CFL phase in the absence of m_s. K_0 and K_D have not been calculated in the literature, but can be derived following the same procedure of Ref. [11].

The ground state of the GL potential is given by $\Phi = [(-\frac{\alpha}{8\beta} - \frac{\epsilon}{12\beta})\mathbf{1}_3 - \frac{\epsilon}{2\beta}X_3]^{1/2} \equiv \mathrm{diag}(\Delta_1, \Delta_2, \Delta_3)$ where the gap parameters Δ_1, Δ_2, and Δ_3 denote down-strange, strange-up, and up-down Cooper pairs, respectively. Due to the gap ordering, $\Delta_3 > \Delta_1 > \Delta_2$, the symmetry breaking pattern is [10]

$$\mathrm{SU}(3)_C \times \mathrm{SU}(3)_{L,R} \times \mathrm{U}(1)_B \xrightarrow{\Phi} \mathrm{SU}(3)_{C+F} \xrightarrow{m_s} \mathrm{U}(1)_V^2. \tag{3}$$

For clarity and completeness, we will first neglect the ϵX_3 term and later treat it as a perturbation. Without the ϵX_3 term, the order parameter is given by $\Phi = \bar{\Delta}\mathbf{1}_3 \equiv \sqrt{-\frac{\alpha}{8\beta} - \frac{\epsilon}{12\beta}}\mathbf{1}_3$.

Mass spectra around this ground state are

$$m_G^2 = 2g_s^2\bar{\Delta}^2K_3, \quad m_1^2 = -\frac{2}{K_3}\left(\alpha + \frac{2\epsilon}{3}\right), \quad m_8^2 = \frac{4\beta\bar{\Delta}^2}{K_3}, \tag{4}$$

where m_G is the mass of the gluons, m_1 and m_8 are the masses of quarks in the $3 \otimes \bar{3} = 1 \oplus 8$ representation under the unbroken $\mathrm{SU}(3)_{C+F}$ symmetry, respectively. From Eqs. (2) and (4), we have $m_G \sim g_s\mu$ and $m_1 \simeq 2m_8 \sim \bar{\Delta}$; then, the relation $g_s\mu \gg \bar{\Delta}$ at high density indicates that the CFL phase is a type-I superconductor [12]. Note that non-Abelian vortices can appear even in this type-I system, since their interactions are repulsive at large distances due to the exchange of the Nambu-Goldstone (NG) boson associated with the $\mathrm{U}(1)_B$ symmetry breaking [13]. This is in contrast to the case of the metallic (Abelian) superconductor where vortices can exist only in a type-II system. Non-Abelian vortices are rather superfluid vortices; they are created under a rapid rotation.

Profiles of non-Abelian vortices.—Corresponding to the three types of vortices, the order parameter Φ asymptotically behaves as

$$\Phi \xrightarrow{r\to 0} \begin{cases} \mathrm{diag}(0, *, *) \\ \mathrm{diag}(*, 0, *) \\ \mathrm{diag}(*, *, 0) \end{cases}, \qquad \Phi \xrightarrow{r\to\infty} \begin{cases} \mathrm{diag}(e^{i\theta}\Delta_1, \Delta_2, \Delta_3) \\ \mathrm{diag}(\Delta_1, e^{i\theta}\Delta_2, \Delta_3) \\ \mathrm{diag}(\Delta_1, \Delta_2, e^{i\theta}\Delta_3) \end{cases}$$

where (r, θ) is the polar coordinate and "$*$" stand for some nonzero constants. All the above three asymptotic forms at infinity can be brought into a unique form $\Phi \xrightarrow{r\to\infty} e^{i\theta/3}\,\mathrm{diag}(\Delta_1, \Delta_2, \Delta_3)$ by regular $\mathrm{SU}(3)_C$ gauge transformations [13]. The overall phase $e^{i\theta/3}$ manifestly shows that the non-Abelian vortex winds $2\pi/3$ inside $\mathrm{U}(1)_B$. Their tensions logarithmically diverge as $T \simeq \frac{2\pi\bar{\Delta}^2}{3}\log\frac{L}{r_0} + \mathcal{O}(1)$ where L is a long-distance cutoff and r_0 is a short-distance cutoff.

Let us take a diagonal ansatz for a single vortex

$$\Phi = \bar{\Delta}e^{i\theta[(1/\sqrt{3})T_0 - \sqrt{2/3}(\nu_3T_3 + \nu_8T_8)]} \times \left[\frac{F(r)}{\sqrt{3}}T_0 - \sqrt{\frac{2}{3}}G(r)(\nu_3T_3 + \nu_8T_8)\right], \tag{5}$$

$$A_i = \frac{1}{g_s}\frac{\epsilon_{ij}x^j}{r^2}[1 - h(r)]\sqrt{\frac{2}{3}}(\nu_3T_3 + \nu_8T_8), \tag{6}$$

with $T_0 = \frac{1}{\sqrt{3}}\mathrm{diag}(1, 1, 1)$, $T_3 = \frac{1}{\sqrt{2}}\mathrm{diag}(0, 1, -1)$, and $T_8 = \frac{1}{\sqrt{6}}\mathrm{diag}(-2, 1, 1)$. We impose $(F, G, h) \to (3, 0, 0)$ as $r \to \infty$ to satisfy $\Phi \to \bar{\Delta}\mathbf{1}_3$. The single-valuedness condition for Φ requires $(\nu_3, \nu_8) = (0, 1), (\pm\frac{\sqrt{3}}{2}, -\frac{1}{2})$.

In the presence of each vortex, the remaining $\mathrm{SU}(3)_{C+F}$ symmetry is further broken down to $[\mathrm{U}(1) \times \mathrm{SU}(2)]_{C+F}$. Hence, the vortex solution is labeled by the NG modes (or the orientational modes) living on the coset space $\mathrm{SU}(3)_{C+F}/[\mathrm{U}(1) \times \mathrm{SU}(2)]_{C+F} \simeq \mathbb{C}P^2$, which we parametrize by introducing $\phi = (\phi_1, \phi_2, \phi_3)^T$ ($\phi^\dagger\phi = 1$, $\phi \sim e^{i\alpha}\phi$) defined as $U[\sqrt{\frac{2}{3}}(\nu_3T_3 + \nu_8T_8)]U^\dagger \equiv \phi\phi^\dagger - \frac{1}{3}\mathbf{1}_3$. The most general solution can be obtained by acting $U \in \mathrm{SU}(3)_{C+F}$ as $\Phi \to U\Phi U^\dagger$ and $A_i \to UA_iU^\dagger$:

$$\Phi = \bar{\Delta}e^{(i\theta)/3}\left[\frac{F(r)}{\sqrt{3}}T_0 + G(r)\left(\phi\phi^\dagger - \frac{1}{3}\mathbf{1}_3\right)\right], \tag{7}$$

$$A_i = \frac{1}{g_s}\frac{\epsilon_{ij}x^j}{r^2}h(r)\left(\phi\phi^\dagger - \frac{1}{3}\mathbf{1}_3\right). \tag{8}$$

For concreteness, let us choose $(\nu_3, \nu_8) = (0, 1)$ as a reference solution. Equations of motion for the profile functions read [14]:

$$f'' + \frac{f'}{r} - \frac{(2h+1)^2}{9r^2}f - \frac{m_1^2}{6}f(f^2 + 2g^2 - 3) - \frac{m_8^2}{3}f(f^2 - g^2) = 0, \tag{9a}$$

$$g'' + \frac{g'}{r} - \frac{(h-1)^2}{9r^2}g - \frac{m_1^2}{6}g(f^2 + 2g^2 - 3) + \frac{m_8^2}{6}g(f^2 - g^2) = 0, \tag{9b}$$

$$h'' - \frac{h'}{r} - \frac{m_G^2}{3}[g^2(h-1) + f^2(2h+1)] = 0, \tag{9c}$$

with $f \equiv \frac{1}{3}(F + 2G)$ and $g \equiv \frac{1}{3}(F - G)$. These equations are solved with the boundary conditions, $(f, g, h) \to (1, 1, 0)$ as $r \to \infty$ and $(f, g', h) \to (0, 0, 1)$ as $r \to 0$.

Low-energy effective theory.—The NG modes $\phi \in \mathbb{C}P^2$ propagate along the non-Abelian vortex string. The philosophy of constructing the low-energy effective Lagrangian is similar to that of the chiral perturbation theory (ChPT) describing the low-energy dynamics of QCD. Remembering that the ChPT is constrained by the $[\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R]/\mathrm{SU}(N_f)$ symmetry, the form of the Lagrangian in our case is determined solely by the $\mathrm{SU}(3)/[\mathrm{U}(1) \times \mathrm{SU}(2)]$ symmetry, and is described by the $\mathbb{C}P^2$ nonlinear sigma model [15]:

$$\mathcal{L}_{\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha[\partial^\alpha \phi^\dagger \partial_\alpha \phi + (\phi^\dagger \partial^\alpha \phi)(\phi^\dagger \partial_\alpha \phi)], \tag{10}$$

where ϕ is promoted to a dynamical field as $\phi \to \phi(x_0, x_3)$ depending on the vortex world-sheet coordinates x_0 and x_3. K_α are the stiffness parameters in Eq. (1) and we have only one unknown constant C. Note that the K_D term in Eq. (1) gives no contribution to Eq. (10) since it is traceless in the vortex background solutions [15].

In order to determine the constant C, we have to go back to the original GL Lagrangian (1) and we have to know the ϕ dependences of Φ and A_μ. It is easy for Φ and $A_{i=1,2}$ because we have already solved background vortex solutions as $\Phi(x_{1,2}; \phi(x_{0,3}))$ and $A_{1,2}(x_{1,2}; \phi(x_{0,3}))$. The missing part is $A_{0,3}(\phi(x_{0,3}))$ which vanishes in the background solutions. Therefore we make an ansatz in an appropriate gauge following Ref. [16]: $A_\alpha(x_{1,2}; \phi(x_{0,3})) = \frac{i\rho(r)}{g_s} \times [\phi\phi^\dagger, \partial_\alpha(\phi\phi^\dagger)]$ $(\alpha = 0, 3)$ where $\rho(r)$ is an unknown function. Then we finally arrive at

$$C = \frac{4\pi}{g_s^2} \int dr \, \frac{r}{2}\left[m_G^2\left((1-\rho)(f-g)^2 + \frac{\rho^2}{2}(f^2+g^2)\right) + \frac{(1-\rho)^2 h^2}{r^2} + \rho'^2 \right], \tag{11}$$

where ρ should be determined so that the integral (11) is minimized. Using the Euler-Lagrange equation for ρ, $\rho'' + \frac{\rho'}{r} + (1-\rho)\frac{h^2}{r^2} - \frac{m_G^2}{2}[(f^2+g^2)\rho - (f-g)^2] = 0$, one finds that C is indeed finite and ϕ is normalizable [14].

Unstable non-Abelian vortices.—We now turn on the ϵX_3 term and consider the regime $\epsilon \ll \alpha$, which allows for an analytical treatment. Since this term explicitly breaks $\mathrm{SU}(3)_{C+F}$ symmetry, the NG modes in Eq. (10) are lifted via an effective potential over the $\mathbb{C}P^2$ space. Let us consider a single vortex whose field configuration satisfies Eq. (9). Variations of its tension can be thought of as the potential

$$V_{\mathbb{C}P^2} = \epsilon \int d^2x \, \mathrm{Tr}[\Phi^\dagger X_3 \Phi] = D(|\phi_3|^2 - |\phi_2|^2), \tag{12}$$

where we have used $|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 = 1$ and have defined

$$D = \pi\epsilon\bar{\Delta}^2 \int_0^\infty dr \, r(g^2 - f^2). \tag{13}$$

Note that D is positive and finite because $g - f$ is always positive and gets exponentially small as going away from the vortex [14]; thus, the effective potential is well-defined.

The effective potential in $\mathbb{C}P^2$ space is shown in Fig. 1. Since the potential has one minimum at $(\phi_1, \phi_2, \phi_3) = (0, 1, 0)$, any vortices away from $(0, 1, 0)$ are unstable against decay into the $(0, 1, 0)$ vortex. This matches the fact that the pairing gap Δ_2 is smaller than Δ_1 and Δ_3 so that the vortex whose string tension is proportional to Δ_2 is easier to be created than others; the details of the dynamics even suggest that the $(1, 0, 0)$ and $(0, 0, 1)$ vortices are no longer local minima.

Let us estimate the lifetime of unstable vortices. As an example, we consider the decay from the $(1, 0, 0)$ vortex at the left-bottom corner of Fig. 1 to the $(0, 1, 0)$ vortex at the right-bottom corner. The discussion here holds for the $(0, 0, 1)$ vortex. In what follows, we set $\phi_3 = 0$, implying that we will consider a $\mathbb{C}P^1$ submanifold (corresponding to the bottom edge of Fig. 1) inside $\mathbb{C}P^2$. It is useful to introduce an inhomogeneous coordinate $u(t) \in \mathbb{C}P^1$ by $(\phi_1, \phi_2) = (1/\sqrt{1+|u|^2}, u/\sqrt{1+|u|^2})$. Then the low-energy effective Lagrangian can be rewritten as

$$\mathcal{L}_{\mathbb{C}P^1} = CK_0 \frac{|\dot{u}|^2}{(1+|u|^2)^2} + D\frac{|u|^2}{1+|u|^2}. \tag{14}$$

A typical time scale of this equation is $\tau = \sqrt{CK_0/D}$.

In principle, we can numerically calculate τ for each μ. Here we provide a simple analytical estimate instead. Since the profile function f (g, h, and ρ) increases (decrease) with a typical scale $r \sim \bar{\Delta}^{-1}$ for $m_G \gg m_{1,8}$ [14], we find $C \sim (\mu/\bar{\Delta})^2$ from Eq. (11). Furthermore, D is estimated from Eq. (13) as $D \sim \epsilon \sim m_s^2 \log(\mu/\bar{\Delta})$. Therefore the lifetime of unstable vortices is given by

$$\tau \sim m_s^{-1}\eta(\mu/\bar{\Delta}), \qquad \eta(x) \equiv x^2(\log x)^{-1/2}. \tag{15}$$

In the limit $m_s \to 0$, $\tau \to \infty$ as anticipated.

(Non)existence of magnetic monopoles.—Let us discuss the (non)existence of magnetic monopoles in QCD at high density. One may expect that the symmetry breaking pattern (3) would support the magnetic monopoles characterized by $\pi_2[\mathrm{SU}(3)/\mathrm{U}(1)^2] = \mathbb{Z}^2$. If so, monopoles must be

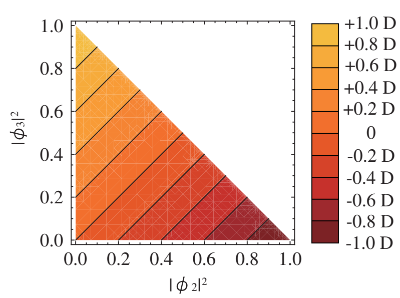

FIG. 1 (color online). Contour plot of the effective potential for the $\mathbf{C}P^2$ NG modes in the $|\phi_2|^2$-$|\phi_3|^2$ plane. The color represents the height of the potential.

confined due to the color Meissner effect of the color superconductivity because it is in the Higgs phase. In fact, such a confined monopole exists in the $\mathcal{N} = 2$ supersymmetric QCD in the Higgs phase with the same symmetry breaking pattern (3) [17], where magnetic fluxes are squeezed into vortex strings confining the monopole from both sides. This composite object has been understood as a kink in the low-energy effective world-sheet theory on the vortex string with a suitable potential term admitting more than or equal to two minima. If the low-energy theory (10) in our case had a potential similar to supersymmetric QCD, this would realize the dual of the confinement scenario advocated in the QCD vacuum where monopoles are condensed and quarks are confined [18]. However, this is not the case. The potential (12) has only one minimum and allows no kink solutions but implies the instabilities of non-Abelian vortices instead, as we have seen.

One should note that this conclusion may not be valid if one includes the nonperturbative quantum effects which account for the mass gap of NG modes as indicated by the Coleman-Mermin-Wagner theorem in two dimensions. Actually, such effects may lead to multiple local minima in the potential, and thus, the monopole-antimonopole meson attached to the vortex [16,19]. In the original four-dimensional GL theory at sufficiently high density, instanton effects are highly suppressed due to the screening of instantons together with the asymptotic freedom of QCD [20], and another mechanism responsible for the quantum effects should be present. We will defer this issue to a future work.

Discussion.—It is interesting to investigate possible astrophysical implications of our results. When the core of a neutron star cools down below the critical temperature of the CFL phase, a network of non-Abelian vortices will be formed by the Kibble mechanism. Remarkably, the extrapolation of our formula (15) to the intermediate density regime relevant to the core of neutron stars ($\mu \sim 500$ MeV) with $\Delta \sim 10$ MeV and $m_s \simeq 150$ MeV suggests that all the vortices decay radically with the lifetime of order $\tau \sim 10^{-21}$ second. Although this result should be taken with some care due to the uncertainty of numerical factor in Eq. (15), it is reasonable to expect that only one type of non-Abelian vortices, which correspond to the point (0, 1, 0) in the $\mathbb{C}P^2$ space, survive as a response to the rotation of neutron stars in reality. The other decaying non-Abelian vortices will emit NG bosons, quarks, gluons, or photons during thermal evolution of neutron stars. In relation to the glitch phenomena, it would be also important to understand how the Abelian $U(1)_B$ vortices in hadronic matter are connected to the stable non-Abelian vortices in color superconducting quark matter in the interior of neutron stars. This may be relevant to the question of continuity of hadronic matter and quark matter [21,22].

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