Gravity-Induced Vacuum Dominance

William C. C. Lima* and Daniel A. T. Vanzella†

Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, CEP 15980-900, São Carlos, SP, Brazil

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It has been widely believed that, except in very extreme situations, the influence of gravity on quantum fields should amount to just small, subdominant contributions. This view seemed to be endorsed by the seminal results obtained over the last decades in the context of renormalization of quantum fields in curved spacetimes. Here, however, we argue that this belief is false by showing that there exist well-behaved spacetime evolutions where the vacuum energy density of free quantum fields is forced, by the very same background spacetime, to become dominant over any classical energy-density component. By estimating the time scale for the vacuum energy density to become dominant, and therefore for back-reaction on the background spacetime to become important, we argue that this (infrared) vacuum dominance may bear unexpected astrophysical and cosmological implications.

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In the absence of a full quantum gravity theory, the influence of gravity on quantum fields can be properly analyzed only in the semiclassical approximation, in which matter (and other interaction) fields are quantized on classical background spacetimes. This semiclassical approach, known as quantum field theory in curved spacetimes (QFTCS) [1–6], gives meaningful results as long as it deals with situations far away from the Planck scale. The Hawking effect [7,8], according to which black holes should emit a thermal bath of particles, provides an example of the strength of the QFTCS formalism. However, in spite of its conceptual importance, it has been widely believed that except in very extreme situations (near singularities, Cauchy horizons, tiny black holes), the influence of gravity on quantum phenomena should amount only to small, subdominant contributions. Here we argue that this commonly held belief is false. For the sake of simplicity, we focus on the vacuum energy density of a free quantum scalar field and show that on some well-behaved spacetimes it can become dominant over any classical energy-density component, even though it is bound to remain finite everywhere. We also show, by performing a simple estimate, that the natural time scale for this semiclassical gravity effect to become important, if it is triggered, is of tiny fractions of a second in some astrophysical contexts, while in cosmological contexts it would be of a few billion years.

Let us begin by considering a real, free scalar field Φ with mass m satisfying the usual Klein-Gordon equation with the additional coupling to the scalar curvature R:

$$(-\Box + m^2 + \xi R)\Phi = 0, \tag{1}$$

where ξ is a real constant. (We adopt natural units in which $\hbar = c = 1$, unless stated otherwise.) The associated quantum field $\hat{\Phi}$ is formally written as $\hat{\Phi} = \int d\mu(\alpha)[\hat{a}_\alpha u_\alpha^{(+)} + \hat{a}_\alpha^\dagger u_\alpha^{(-)}]$, where $u_\alpha^{(+)}$ and $u_\alpha^{(-)} \equiv (u_\alpha^{(+)})^*$ are positive- and negative-norm solutions of Eq. (1), respectively, which together form a complete set of normal modes, satisfying $(u_\alpha^{(+)}, u_\beta^{(+)})_{\rm KG} = -(u_\alpha^{(-)}, u_\beta^{(-)})_{\rm KG} = \delta(\alpha, \beta)$ and $(u_\alpha^{(+)}, u_\beta^{(-)})_{\rm KG} = 0$, with $\delta(\alpha, \beta)$ being the Dirac's "delta function" associated with the measure $\mu(\alpha)$ on the set of "quantum numbers" α. Recall that the Klein-Gordon inner product defined on the space $\mathcal{S}$ of complex solutions of Eq. (1) is given by $(u, v)_{\rm KG} := i\int_\Sigma d\Sigma n^a[u^*\nabla_a v - v\nabla_a u^*]$, where $d\Sigma$ is the proper-volume element on the Cauchy surface Σ and n^a is the future-pointing unit vector field orthogonal to Σ. The operators $\hat{a}_\alpha$ and $\hat{a}_\alpha^\dagger$ are taken to satisfy the canonical commutation relations (CCR) $[\hat{a}_\alpha, \hat{a}_\beta^\dagger] = \delta(\alpha, \beta)$, $[\hat{a}_\alpha, \hat{a}_\beta] = 0$, from where the mode-annihilation and -creation interpretation follows, as well as the Fock-space construction based on the "vacuum" state $|0\rangle$ defined through $\hat{a}_\alpha|0\rangle = 0$ for all α. Obviously, the choice of the solutions to constitute the positive-norm modes $u_\alpha^{(+)}$ is far from unique, and different choices can lead to different (i.e., unitarily inequivalent) Fock spaces of states where the CCR is implemented. In the absence of a timelike symmetry, with respect to which a preferred notion of positive-frequency solutions can be defined, there is no natural way of picking one space out of the infinite possibilities. As a consequence, no natural notion of particles exists in a general curved spacetime. This, however, poses no impediment to the formalism of QFTCS, as is well known.

The effect we shall discuss here does not rely on this "indeterminacy" of the particle concept. Therefore, in order to avoid unnecessary complications we shall assume a globally hyperbolic spacetime which is conformally static in both the asymptotic past and future. To be even more conservative, we focus attention on a spacetime which is conformally flat in the asymptotic past:

$$ds^2 \sim \begin{cases} f_{\rm in}^2(-dt^2 + d\vec{x}^2) & \text{, asymp. past} \\ f_{\rm out}^2(-dt^2 + h_{ij}dx^i dx^j) & \text{, asymp. future} \end{cases}, \tag{2}$$

where $f_J = f_J(t, \vec{x}) > 0$, $J \in \{\text{in}, \text{out}\}$, are smooth functions and $h_{ij} = h_{ij}(\vec{x})$, $i, j = 1, 2, 3$, are the components of an arbitrary spatial metric. [We use the same labels t and $\vec{x} = (x^1, x^2, x^3)$ for coordinates in the asymptotic past and future only for simplicity; they are obviously defined on nonintersecting regions of the spacetime.] In each of these asymptotic regions the field Φ can be written as $\Phi = \tilde{\Phi}/f_J$, where $\tilde{\Phi}$ satisfies

$$-\frac{\partial^2}{\partial t^2}\tilde{\Phi} = -\Delta_J\tilde{\Phi} + V_J\tilde{\Phi}, \tag{3}$$

where Δ_{in} is the usual (flat) Laplace operator, Δ_{out} is the Laplace operator associated with the spatial metric h_{ij}, and the effective potential V_J is given by

$$\begin{aligned} V_J &= \frac{(\Delta_J f_J - \ddot{f}_J)}{f_J} + f_J^2(m^2 + \xi R) \\ &= (1 - 6\xi)\frac{(\Delta_J f_J - \ddot{f}_J)}{f_J} + f_J^2 m^2 + \xi K_J, \end{aligned} \tag{4}$$

with $K_{\text{in}} = 0$, $K_{\text{out}} = K_{\text{out}}(\vec{x})$ the scalar curvature associated with the spatial metric h_{ij}, and the dots denoting differentiation with respect to the variable t.

Although Eq. (3) is already in a form upon which our main line of reasoning could be constructed, let us simplify our analysis further by assuming that $V_{\text{in}} = 0$ and V_{out} does not depend on t, $V_{\text{out}} = V_{\text{out}}(\vec{x})$. This is certainly not the case in general for spacetimes whose metric satisfies Eq. (2), but there are very interesting situations which do satisfy this condition: (i) the massless ($m = 0$) field with arbitrary coupling ξ in spacetimes which are asymptotically flat in the past and asymptotically static in the future [$f_{\text{in}} = 1$ and $f_{\text{out}} = f_{\text{out}}(\vec{x})$], as those describing the formation of a static star from matter initially scattered throughout space, and (ii) the massless, conformally coupled field ($m = 0$ and $\xi = 1/6$). With this assumption for the potential, two different sets of positive-norm modes, $u_{\vec{k}}^{(+)}$ and $v_\alpha^{(+)}$, can be naturally defined by the requirement that they are the solutions of Eq. (1) which satisfy the asymptotic conditions:

$$u_{\vec{k}}^{(+)} \overset{\text{past}}{\sim} (16\pi^3\omega_{\vec{k}})^{-1/2} f_{\text{in}}^{-1} e^{-i(\omega_{\vec{k}}t - \vec{k}\cdot\vec{x})} \tag{5}$$

and

$$v_\alpha^{(+)} \overset{\text{future}}{\sim} (2\varpi_\alpha)^{-1/2} f_{\text{out}}^{-1} e^{-i\varpi_\alpha t} F_\alpha(\vec{x}), \tag{6}$$

where $\vec{k} \in \mathbb{R}^3$, $\omega_{\vec{k}} := \| \vec{k} \|$, $\varpi_\alpha > 0$, and $F_\alpha(\vec{x})$ are solutions of

$$[-\Delta_{\text{out}} + V_{\text{out}}(\vec{x})]F_\alpha(\vec{x}) = \varpi_\alpha^2 F_\alpha(\vec{x}) \tag{7}$$

satisfying the normalization

$$\int_{\Sigma_{\text{out}}} d^3x\sqrt{h}F_\alpha(\vec{x})^* F_\beta(\vec{x}) = \delta(\alpha, \beta) \tag{8}$$

on a Cauchy surface Σ_{out} in the asymptotic future. (Each F_α can be chosen to be real with no loss of generality.)

The fact that in general the modes $v_\alpha^{(+)}$ cannot be expanded in terms of $u_{\vec{k}}^{(+)}$ alone ($u_{\vec{k}}^{(-)}$ might be needed) is responsible for the almost-forty-year-old effect of particle creation due to the (change in the) gravitational background: the vacuum state $|0\rangle_{\text{in}}$ associated with the modes $u_{\vec{k}}^{(+)}$, which represents absence of particles in the asymptotic past, represents a particle-filled state according to the natural notion of particles in the asymptotic future (associated with $v_\alpha^{(+)}$). This stands at the root of the Hawking effect and of particle creation in expanding universes [1,2]. Here, however, we want to call attention to a different effect, independent of particle creation, which seems to have passed unnoticed in the general context: there are reasonable situations where the modes $v_\alpha^{(+)}$, given in Eq. (6), together with $v_\alpha^{(-)}$ fail to form a complete set of normal modes. This happens whenever the operator $[-\Delta_{\text{out}} + V_{\text{out}}(\vec{x})]$ in Eq. (7) happens to possess normalizable [i.e., satisfying Eq. (8)] eigenfunctions with negative eigenvalues, $\varpi_\alpha^2 = -\Omega_\alpha^2 < 0$. In this case, additional positive-norm modes $w_\alpha^{(+)}$ with the asymptotic behavior

$$w_\alpha^{(+)} \overset{\text{future}}{\sim} \frac{(e^{\Omega_\alpha t - i\pi/12} + e^{-\Omega_\alpha t + i\pi/12})F_\alpha(\vec{x})}{\sqrt{2\Omega_\alpha} f_{\text{out}}(t, \vec{x})} \tag{9}$$

and their complex conjugates $w_\alpha^{(-)}$ are necessary in order to expand an arbitrary solution of Eq. (1). As a direct consequence, at least some of the in-modes $u_{\vec{k}}^{(\pm)}$ (typically those with low $\omega_{\vec{k}}$) eventually undergo an exponential growth (assuming that f_{out} remains polynomially bounded). This asymptotic divergence is reflected on the unbounded increase of the vacuum fluctuations,

$$\langle\Phi^2\rangle \overset{\text{future}}{\sim} \frac{\kappa e^{2\bar{\Omega}t}}{2\bar{\Omega}}\left(\frac{\bar{F}(\vec{x})}{f_{\text{out}}(t, \vec{x})}\right)^2 [1 + \mathcal{O}(e^{-\epsilon t})], \tag{10}$$

where $\bar{F}(\vec{x})$ is the eigenfunction of Eq. (7) associated with the lowest negative eigenvalue allowed, $\varpi_\alpha^2 = -\bar{\Omega}^2$, ϵ is some positive constant, and κ is a dimensionless constant (typically of order unity) whose exact value depends globally on the spacetime structure (since it crucially depends on the projection of each $u_{\vec{k}}^{(\pm)}$ on the mode $w_\alpha^{(\pm)}$ whose $\varpi_\alpha^2 = -\bar{\Omega}^2$; κ also depends on the initial state, here assumed to be the vacuum $|0\rangle_{\text{in}}$).

As one would expect, these wild quantum fluctuations give an important contribution to the vacuum energy stored in the field. In fact, the expectation value of its energy-momentum tensor, $\langle T_{\mu\nu}\rangle$, in the asymptotic future is found to be dominated by this exponential growth:

$$\langle T_{00}\rangle \overset{\text{future}}{\sim} \langle\Phi^2\rangle\bigg\{\frac{(1-4\xi)}{2}\bigg(\bar{\Omega}^2+\frac{(D\bar{F})^2}{\bar{F}^2}+m^2f^2+\xi K\bigg)+(1-6\xi)\bigg(\frac{2\xi\ddot{f}}{f}-\frac{2\xi D^2f}{f}+\frac{\dot{f}^2}{2f^2}-\frac{\bar{\Omega}\,\dot{f}}{f}+\frac{(Df)^2}{2f^2}-\frac{D_ifD^i\bar{F}}{f\bar{F}}\bigg)+\mathcal{O}(e^{-\epsilon t})\bigg\}, \tag{11}$$

$$\langle T_{0i}\rangle \overset{\text{future}}{\sim} \langle\Phi^2\rangle\bigg\{(1-4\xi)\frac{\bar{\Omega}D_i\bar{F}}{\bar{F}}+(1-6\xi)\times\bigg(\frac{\dot{f}D_if}{f^2}-\frac{\dot{f}D_i\bar{F}}{f\bar{F}}-\frac{\bar{\Omega}D_if}{f}\bigg)+\mathcal{O}(e^{-\epsilon t})\bigg\}, \tag{12}$$

$$\langle T_{ij}\rangle \overset{\text{future}}{\sim} \langle\Phi^2\rangle\bigg\{(1-2\xi)\frac{D_i\bar{F}D_j\bar{F}}{\bar{F}^2}-2\xi\frac{D_iD_j\bar{F}}{\bar{F}}+\xi\tilde{R}_{ij}+\frac{(1-4\xi)h_{ij}}{2}\bigg(\bar{\Omega}^2-\frac{(D\bar{F})^2}{\bar{F}^2}-m^2f^2-\xi K\bigg)+(1-6\xi)\bigg[\frac{D_ifD_jf}{f^2}-\frac{D_ifD_j\bar{F}}{f\bar{F}}-\frac{D_jfD_i\bar{F}}{f\bar{F}}+h_{ij}\bigg(\frac{2\xi D^2f}{f}-\frac{2\xi\ddot{f}}{f}+\frac{\dot{f}^2}{2f^2}-\frac{\bar{\Omega}\,\dot{f}}{f}-\frac{(Df)^2}{2f^2}+\frac{D_kfD^k\bar{F}}{f\bar{F}}\bigg)\bigg]+\mathcal{O}(e^{-\epsilon t})\bigg\}, \tag{13}$$

where D_i is the derivative operator compatible with the metric h_{ij} (so that $\Delta_{\rm out}=D^2$), $\tilde{R}_{ij}$ is the associated Ricci tensor (so that $K_{\rm out}=h^{ij}\tilde{R}_{ij}$), and we have omitted the subscript out in $f_{\rm out}$ and $K_{\rm out}$ for simplicity. The Eqs. (11)–(13), together with Eq. (10), imply that on time scales determined by $\bar{\Omega}^{-1}$, the vacuum fluctuations of the field should overcome any other classical source of energy, therefore taking control over the evolution of the background geometry through the semiclassical Einstein equations (in which $\langle T_{\mu\nu}\rangle$ is included as a source term for the Einstein tensor). We are then confronted with a startling situation where the quantum fluctuations of a field, whose energy is usually negligible in comparison with classical energy components, are forced by the background spacetime to play a dominant role.

We are still left with the task of showing that there exist indeed well-behaved background spacetimes in which the operator $[-\Delta_{\rm out}+V_{\rm out}(\vec{x})]$ possesses negative eigenvalues $\varpi_\alpha^2<0$, condition on which depends all the discussion presented above. Experience from usual quantum mechanics tells us that this typically occurs when $V_{\rm out}$ gets sufficiently negative over a sufficiently large region. It is easy to see from Eq. (4) that, except for very special geometries (as the flat one), one can generally find appropriate values of $\xi\in\mathbb{R}$ which make $V_{\rm out}$ as negative as would be necessary in order to guarantee the existence of negative eigenvalues. Therefore, the question is not if negative eigenvalues are possible, but how natural are the scenarios in which they appear. For massless fields with coupling ξ of order unity, $V_{\rm out}$ is of order R [see Eq. (4)], which in turn is of order $8\pi G\rho_c$ (assuming the validity of the classical Einstein equations), where ρ_c is the energy density of the classical matter governing the spacetime evolution and G is Newton's constant. Note also that we can manipulate the sign of $V_{\rm out}$ by choosing ξ properly (but still with values of order 1). Combining all these observations suggests that background geometries associated with matter distributions whose density variations are of order $\delta\rho_c$ over regions of typical linear size L, satisfying $8\pi G\delta\rho_cL^2\sim 1$ or larger, are promising candidates where a massless field with appropriate coupling ξ (with $|\xi|\sim 1$) would exhibit the vacuum-dominance effect presented above. Recovering units appropriate in different contexts, we have

$$\frac{8\pi G\delta\rho_cL^2}{c^2}\approx\bigg(\frac{\delta\rho_c}{10^{15}\ {\rm g/cm^3}}\bigg)\bigg(\frac{L}{7\ {\rm km}}\bigg)^2\approx\bigg(\frac{\delta\rho_c}{\rho_{m0}}\bigg)\bigg(\frac{L}{4.7\times10^3\ {\rm Mpc}}\bigg)^2\sim 1, \tag{14}$$

where $\rho_{m0}\approx 2.5\times10^{-30}\ {\rm g/cm^3}$ is the matter density (baryonic and dark) averaged over the observable Universe, whose linear size is comparable to the Hubble length 4.1×10^3 Mpc [9].

This crude estimate serves only to suggest the scenarios in which the vacuum-dominance effect might play some role: compact objects [10] and cosmology. Only a thorough analysis can properly reveal the relevance of the mechanism in each of these contexts. Notwithstanding, although the main goal of this Letter is to lay the general basis of the mechanism, next we summarize the results of a detailed analysis performed in the simplest (nontrivial) instance where the vacuum dominance is found to be triggered: the background geometry of a uniform-density, spherically symmetric compact object [11]. In such an idealized case, the Tolman-Oppenheimer-Volkoff equation (which relates pressure and density inside the object) can be analytically solved (see, e.g., Ref. [12]), from where the background geometry [$f_{\rm out}$ and h_{ij} in Eq. (2)] can be calculated and substituted into the expression for $V_{\rm out}$, Eq. (4). Then, it is simply a matter of verifying (numerically) the existence of bound eigenfunctions for the operator $(-\Delta_{\rm out}+V_{\rm out})$ appearing in Eq. (7). After performing this procedure for several values of the compact-object mass M and radius r_o, it is found that there always exist classically stable compact-object configurations (i.e., with $M/r_o<4/9$ in geometric units) which awake the vacuum energy of massless fields with any value of $\xi>1/6$ or $\xi<\xi_0$ (with $\xi_0\approx -2$). Preliminary results [11] show that more realistic compact objects (like some neutron stars) can also trigger the effect for massless fields with appropriate couplings. This leads to an interesting (and rare) possible interconnection between observational astrophysics and semiclassical gravity, where the observation of stable neutron-star configurations may rule out the existence of certain fields in nature.

Back to the general context, the time scale $\bar{\Omega}^{-1}$ (typically or order $|V_{\rm out}|^{-1/2}$), which determines how sharp would be the transition from classical to vacuum dominance, can be estimated as being given by L when condition (14) is verified. Therefore, for compact objects we have $\bar{\Omega}^{-1} \sim 10^{-4}$ s, while in the cosmological context $\bar{\Omega}^{-1} \sim 10^{10}$ years (this latter time scale might be considerably smaller since matter is not evenly distributed over the whole observable Universe).

We conclude with some final remarks. First, it is worth mentioning that in spite of the unbounded growth in Eqs. (11)–(13), $\langle T_{\mu\nu}\rangle$ is covariantly conserved: $\nabla_\mu \langle T^\mu_\nu\rangle = 0$. In the static case [$f_{\rm out} = f_{\rm out}(\vec{x})$], for instance, this implies that the total vacuum energy is kept constant, although it continuously flows from spatial regions where its density is negative (and ever decreasing) to spatial regions where it is positive (and ever increasing). (This is an example of a spontaneous timelike symmetry breaking.) Also, in the massless conformally coupled case ($m = 0$ and $\xi = 1/6$), the exponentially increasing terms give no contribution to the anomalous value of the trace $\langle T^\mu_\mu\rangle$. Finally, notice that the exponential behavior appearing in Eqs. (10)–(13) leads only to asymptotic divergences; strictly speaking, all the quantities remain finite everywhere. This is in agreement, as it should be, with the seminal results obtained over the last decades on the topic of renormalization in QFTCS, which in summary show that a state (satisfying a positivity condition) which is renormalizable and free from infrared divergences at a particular time (i.e., with the only singular behavior of its two-point function being of a Hadamard form, for points in the same normal neighborhood of a given Cauchy surface), will remain so throughout the spacetime; no divergences can appear due to a well-behaved evolution of the background spacetime [13–15]. This seminal result, whose importance cannot be stressed enough, seems to have discouraged further investigation on the topic of "infrared behavior of fields in curved spacetime" in the general context, as if it offered no more surprises. (For a thorough investigation in the case of de Sitter spacetime, see Ref. [16].) The vacuum-dominance effect presented here illustrates that this "mathematical good behavior" may still harbor interesting and wild physical phenomena. In fact, it is quite natural to expect that the infrared sector of a field theory should be very sensitive to the nontriviality of the background geometry, giving rise to legitimate QFTCS effects. We have made use of some idealizations (e.g., free scalar field, conformally static asymptotic metrics) only to put in evidence the main idea behind the vacuum-dominance mechanism, avoiding unnecessary complications. The fact that this mechanism already manifests itself in such a simple and classically well-behaved situation leads us to speculate that it might be of relevance in other, more complicated (and possibly realistic) scenarios (for instance, during the collapse of stars which classically would lead to the formation of black holes, or in the course of structure formation during cosmological expansion). Some of these legitimate QFTCS effects may still be waiting to be uncovered.

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*william@ursa.ifsc.usp.br
†vanzella@ifsc.usp.br

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