Ivanov and Kienle Reply: Nowadays it is well established experimentally that neutrinos ν_α with lepton flavors $\alpha = e, \mu$, and τ are superpositions $|\nu_\alpha\rangle = \sum_j U^*_{\alpha j}|\nu_j\rangle$ of massive neutrino mass eigenstates $|\nu_j\rangle$ with masses m_j, where $U^*_{\alpha j}$ are elements of the 3×3 unitary mixing matrix U, defined by mixing angles θ_{ij} [1]. The wave functions $|\nu_\alpha\rangle$ and $|\nu_j\rangle$ each represent orthogonal bases and are used for the description of neutrino oscillations $\nu_\alpha \longleftrightarrow \nu_\beta$ with frequencies $\omega_{ij} = \Delta m^2_{ij}/2E$, where E is the neutrino energy and $\Delta m^2_{ij} = m^2_i - m^2_j$ [1]. In K-shell electron capture (EC) decays of the H-like heavy ions $p \to d + \nu_e$, where p and d are the parent and daughter ions in their ground states [2,3], one deals with an emission of electron neutrinos $|\nu_e\rangle = \sum_j U^*_{ej}|\nu_j\rangle$. Thus, the EC-decay rates of the H-like heavy ions are defined by the decay channels $p \to d_j + \nu_j$, where the final states are described by the orthogonal wave functions $\langle \nu_i d_i | d_j \nu_j\rangle = 0$ for $i \neq j$. The states of the daughter ions d_j differ in 3-momenta $\vec{q}_j$ and energies $E_d(\vec{q}_j)$. The massive neutrinos ν_j are produced with 3-momenta $\vec{k}_j = -\vec{q}_j$ and energies $E_j(\vec{k}_j)$, caused by conservation of energy and momentum in the decay channels $p \to d_j + \nu_j$. Since massive neutrinos ν_j are not detected they appear in the asymptotic states with 3-momenta $\vec{k}_j$, energies $E_j(\vec{k}_j)$, and energy differences $\omega_{ij} = \Delta m^2_{ij}/2M_p$, where M_p is the mass of the parent ion p [3]. In the GSI experiments [2,3] the EC-decay channels $p \to d_j + \nu_j$ are measured by detecting the daughter ions d_j. If the daughter ions were detected in the asymptotic states with 3-momenta $\vec{q}_j$ and energies $E_d(\vec{q}_j)$, the probability per unit time of the EC decay $p \to d + \nu_e$ would be equal to

$$P(p \to d\nu_e)(t) = \sum_j |U_{ej}|^2 P(p \to d_j\nu_j)(t) = \sum_j |U_{ej}|^2 \frac{d}{dt}|A(p \to d_j\nu_j)(t)|^2, \quad (1)$$

where $A(p \to d_j\nu_j)(t)$ is the amplitude of the decay channel $p \to d_j + \nu_j$. However, this is not the case in the GSI experiments, where the time differential detection of the daughter ions with a time resolution τ_d leads to indistinguishability of daughter ions in the decay channels $p \to d_j + \nu_j$. As a result the daughter ions d_j are measured in the asymptotic state d with a 3-momentum $\vec{q}$ and an energy $E_d(\vec{q})$ such that $\vec{q} \simeq \vec{q}_j$ and $E_d(\vec{q}) \simeq E_d(\vec{q}_j)$ [3]. This does not violate the orthogonality of the wave functions in the final state $\langle \nu_i d | d \nu_j\rangle = 0$ for $i \neq j$. The energy and momentum uncertainties δE_d and $|\delta\vec{q}_d|$, respectively, induced by the time differential detection of the daughter ions, provide the overlap of the wave functions of the daughter ions if $\delta E_d \gg |\omega_{ij}|$ and $|\delta\vec{q}_d| \gg |\vec{q}_i - \vec{q}_j| = |\vec{k}_i - \vec{k}_j|$, where ω_{ij} present also the differences of the recoil energies of the daughter ions. The time differential detection of the daughter ions d_j in the asymptotic state with the 3-momentum $\vec{q}$ and an energy $E_d(\vec{q})$ results in a smearing of momenta and energies in the decay channels $p \to d_j + \nu_j$ around $\vec{q} + \vec{k}_j \simeq 0$ and $E_d(\vec{q}) + E_j(\vec{k}_j) \simeq M_p$. This is the origin of the nonvanishing interference terms in the probability per unit time of the EC decay $p \to d + \nu_e$ [3]

$$\begin{aligned} P(p \to d\nu_e)(t) = {} & \sum_j |U_{ej}|^2 P(p \to d\nu_j)(t) \\ & + 2\sum_{i>j} \frac{d}{dt}\,\mathrm{Re}[U^*_{ei}U_{ej}A^*(p \to d\nu_i)(t) \\ & \times A(p \to d\nu_j)(t)], \end{aligned} \quad (2)$$

where in comparison with Eq. (1) the second sum is caused by the interference terms. Since uncertainties δE_d and $|\delta\vec{q}_d|$ are rather small [3] and $|\vec{k}_j| = |\vec{q}_j| \simeq |\vec{q}| \simeq Q_{EC}$ and $Q_{EC} \gg m_j$, where Q_{EC} is the Q value of the EC decay $p \to d + \nu_e$, one can set the neutrino masses zero everywhere except for energy differences ω_{ij} and mixing angles θ_{ij} in the interference terms [3], and neutrino 3-momenta equal $\vec{k}_j = \vec{k}$. As a result the EC-decay rate is given by [3]

$$\begin{aligned} \lambda_{\mathrm{EC}}(t) &= \frac{1}{2M_p}\int P(p \to d\nu_e)(t)\frac{d^3q}{(2\pi)^3 2E_d}\frac{d^3k}{(2\pi)^3 2E_{\nu_e}} \\ &= \lambda_{EC}\Big(1 + 2\sum_{i>j}\mathrm{Re}[U^*_{ei}U_{ej}]\cos(\omega_{ij}t)\Big), \end{aligned} \quad (3)$$

Thus, the asymptotic orthogonality of the final state wave functions does not influence the observation of the time modulation of the EC decays, observed in the GSI experiments with a time resolution $\tau_d \ll T_{ij}$ much shorter than the modulation periods $T_{ij} = 2\pi/\omega_{ij}$ [3]. This is unlike the assertion by Flambaum [4]. For $\tau_d \gg T_{ij}$ the time modulation vanishes [see Eq. (1)], since the daughter ions d_j become distinguishable with 3-momenta $\vec{q}_j$ and energies $E_d(\vec{q}_j)$. For the theoretical description of the GSI data [2], accounting for the procedure for the detection of the daughter ions, one can use time-dependent perturbation theory and wave packets for the wave functions of the daughter ions, related to the density matrix description of unisolated quantum systems. For technical details we refer to [3].

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