Multiple-Time Scaling and Universal Behavior of the Earthquake Interevent Time Distribution

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The interevent time distribution characterizes the temporal occurrence in seismic catalogs. Universal scaling properties of this distribution have been evidenced for entire catalogs and seismic sequences. Recently, these universal features have been questioned and some criticisms have been raised. We investigate the existence of universal scaling properties by analyzing a Californian catalog and by means of numerical simulations of an epidemic-type model. We show that the interevent time distribution exhibits a universal behavior over the entire temporal range if four characteristic times are taken into account. The above analysis allows us to identify the scaling form leading to universal behavior and explains the observed deviations. Furthermore, it provides a tool to identify the dependence on the mainshock magnitude of the c parameter that fixes the onset of the power law decay in the Omori law.

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Seismic occurrence is a phenomenon of great complexity involving different processes acting on different time and space scales. In the last decade a unifying picture of seismic occurrence has been proposed via the investigation of $D(\Delta t)$, the distribution of interevent times Δt between successive earthquakes [1–3]. These studies have shown that, rescaling interevent times by the average occurrence rate R, $D(\Delta t)$ follows the scaling relation

$$D(\Delta t) = Rf(R\Delta t) \tag{1}$$

where the functional form of $f(x)$ is quite independent of the geographic zone and the magnitude threshold. The above relation suggests that R is a nonuniversal quantity and is the only typical inverse time scale affecting $D(\Delta t)$. This result, obtained for periods of stationary rate, has been generalized to nonstationary periods [4] and Omori sequences [5,6]. The scaling relation (1) has been also observed for volcanic earthquakes [7]. On the other hand, recent studies have questioned the universality of the interevent time distribution [8–14]. In particular, deviations from universality at small Δt have been related to the interplay between correlated earthquakes, following a gamma distribution, and uncorrelated events, following a pure exponential decay [14]. This behavior is well described by numerical simulations of the epidemic-type aftershock sequence (ETAS) model [15]. Indeed, analytical studies [9] and a previous numerical analysis of the ETAS model [10], have shown that the functional form of $D(\Delta t)$ depends on the ratio between correlated and independent earthquakes $\mathcal{K}$. The problem has been also attacked within the theoretical framework of probability generating functions [11–13]. Saichev and Sornette have obtained an exact nonlinear integral equation for the ETAS model and solved it analytically at linear order [11,12]. This solution shows that the function $f(x)$ in Eq. (1) depends on $\mathcal{K}$ and on some other parameters of the model. This behavior is confirmed if nonlinear contributions are taken into account [13].

Multiplicity of characteristic times is often observed in the dynamics of complex systems, where different temporal scales are associated to the relaxation of different spatial regions or structures. For instance, their existence is a well established property in glassy materials, polymers or gelling systems, where they originate from the relaxation of complex structures at different mesoscopic scales, or else from the emergence of competing interactions [16]. Moreover, the coexistence of different physical mechanisms acting at different spatiotemporal scales may also give rise to complex temporal scaling [17]. Therefore, the identification of the number of relevant time scales controlling universal behaviors is a very debated subject in complex systems.

In this Letter, we do not assume the existence of a unique time scale $1/R$, as in Eq. (1). We show that four typical time scales are relevant for the interevent time distribution scaling: the inverse rate of independent events μ, the average inverse rate of correlated events, the time parameter c defined in the Omori law, and the catalog duration T. These different time scales lead to deviations from the simple scaling (1). Nevertheless, we show that the interevent time distribution can be expressed in a universal scaling form in terms of these four characteristic times. The scaling form allows us to better enlighten the mechanism leading to universality for $D(\Delta t)$ and the deviations from it. The above analysis also clarifies the dependence of c on the mainshock magnitude for intermediate mainshock sizes.

We assume that seismic occurrence can be modeled by a time-dependent Poisson process with instantaneous rate $\lambda(t)$. In this case, the interevent time distribution for the temporal interval $[0, T]$ is [5]

$$D(\Delta t) = \frac{1}{N}\left[\int_0^{T-\Delta t} ds\lambda(s)\lambda(s+\Delta t)e^{-\int_s^{s+\Delta t} du\lambda(u)} + \lambda(\Delta t)e^{-\int_0^{\Delta t} du\lambda(u)}\right] \quad (2)$$

where $N = \int_0^T ds\lambda(s)$ is the number of events. The widely accepted scenario is that seismic occurrence can be considered as the superposition of Poissonian events occurring at constant rate μ and independent aftershock sequences, which gives

$$\lambda(t) = \mu + \sum_{i:t_i<t} a_i(p-1)\left(\frac{t-t_i}{c_i}+1\right)^{-p} \quad (3)$$

where $p > 1$ is the exponent of the Omori law. The quantity a_i is proportional to the rate of aftershocks correlated to the ith mainshock since, from Eq. (3), the total number of events triggered by the ith mainshock is $a_i c_i$. The productivity law [18] indicates that a_i is exponentially related to the mainshock magnitude $a_i = A10^{\alpha m_i}$. The ETAS model assumes $c_i = c$ whereas recent studies on experimental catalogs have obtained $c_i = c10^{\alpha m_i}$ which leads to the so-called generalized Omori law [5,19,20]. The dependence of c_i on mainshock magnitudes has been attributed to a dynamical scaling relation involving time, space and energy [20–23] or to catalog incompleteness [24]. Inserting Eq. (3) in Eq. (2), we obtain a scaling form for $D(\Delta t)$ expressing time in unit of $1/\mu$,

$$D(\Delta t) = \mu G(\mu\Delta t, \mu/\bar{a}, \mu\bar{c}, \mu T) \quad (4)$$

where $\bar{a}$ ($\bar{c}$) is the value of a_i (c_i) averaged over all mainshocks. We show that Eq. (1) represents a particular case of the more general scaling form (4). Indeed, by definition, R in the time interval $[0, T]$ is the inverse of the average Δt, and from Eq. (4) $R = \mu/H(\mu/\bar{a}, \mu\bar{c}, \mu T)$, with $H(y, z, w) = \int dx G(x, y, z, w)x$. Therefore, expressing μ in terms of R and setting $\mathcal{K} = \bar{a}\,\bar{c}$, we obtain

$$D(\Delta t) = RG_1(R\Delta t, \mathcal{K}, \mu\bar{c}, \mu T). \quad (5)$$

For the ETAS model $\mathcal{K} = \bar{a}c$ is the branching ratio, i.e., the number of direct aftershocks per earthquake.

The complex form of Eq. (3) does not allow the full derivation of an analytical expression for $D(\Delta t)$, unless one uses specific assumptions. Saichev and Sornette [12], for instance, have exactly calculated $D(\Delta t)$ for $T \to \infty$ in the hypothesis that each earthquake triggers, on average, the same number of aftershocks, i.e., $a_i = A$ and $c_i = c$. This solution exactly follows the scaling form Eq. (5) with $G_1(x, y, z, \infty) = \exp\{-x(1-y) - (\omega^{2-p} - 1)zy/[(1-y)(2-p)]\}((1 - y + y\omega^{1-p})^2 + (p-1)\omega^{-p}y(1-y)/z)$ with $\omega = 1 + (1-y)x/z$. For a given choice of the parameters $\mathcal{K}$, μc and p the above expression leads to a $D(\Delta t)$ in good agreement with the experimental distribution. Interestingly, the above expression coincides with the linear order of the ETAS model expansion, in the limit $\Delta t \gg c$. Higher order terms lead to small differences with the above solution [13].

A useful example to understand the role of the different time scales in $D(\Delta t)$ can be obtained if we limit the calculation to events in a single aftershock sequence. A scaling form consistent with Eq. (5) has been already obtained in Ref. [5] assuming $\mu = 0$. Here, we restrict to large T and $\Delta t \ll T$ assuming that $\lambda(t)$ is about constant in Δt. This choice does not represent a loss of generality for sufficiently small Δt. Under these assumptions, Eq. (2) becomes $D(\Delta t) = \frac{1}{N}\int_0^T ds\lambda^2(s)e^{-\lambda(s)\Delta t}$ which can be analytically integrated. Three typical regimes can be identified: (i) At large times $\mu\Delta t > 1$ (i.e., $x > 1$), with $\Delta t/T \ll 1$, $D(\Delta t)$ decays as $\exp(-\mu\Delta t)$ as already observed [9]. (ii) At intermediate times $c < \Delta t < 1/\mu$ ($y < x < 1$), we observe a power law decay $\Delta t^{-\epsilon}$ with $\epsilon = 2 - 1/p$, as predicted by Utsu [25]. (iii) At small times $\Delta t \ll c(x/y \ll 1)$, $D(\Delta t)$ becomes Δt independent. The three regimes can be identified in Fig. 1 where, for a fixed value of μ and different c, we plot $\Delta t D(\Delta t)$ vs $R\Delta t$. This is equivalent to the representation adopted by [14] and allows us to better enlighten deviations from the scaling relation (1). We observe that all curves present a peak at $R\Delta t \simeq 1$ and then exponentially decay for $R\Delta t > 1$. Conversely, at small times ($\Delta t < c$), all the curves increase linearly since $D(\Delta t)$ is constant. The intermediate regime can be observed only for the two smallest values of c, since only in these cases $\mu c \ll 1$ and the intermediate regime has a finite extension. In this regime an about flat behavior is observed since for $p = 1.05$, $\epsilon \simeq 1$. Notice that one of the theoretical curves provides a good qualitative fit of the distribution obtained from experimental data [26].

In Fig. 2 we present the results of numerical simulations of the ETAS model obtained following the method of Ref. [27]. We perform extensive simulations in order to

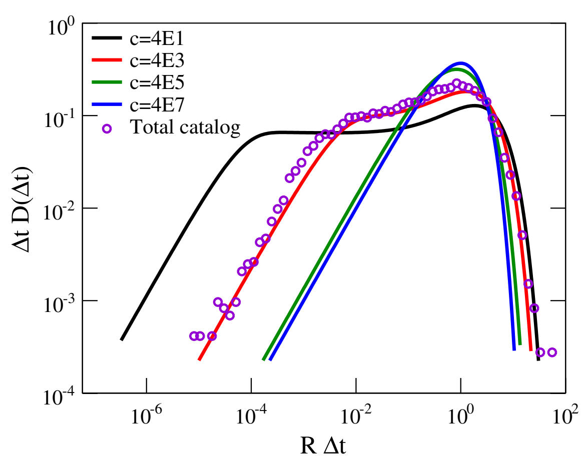

FIG. 1 (color online). $\Delta t D(\Delta t)$ vs $R\Delta t$ for the single Omori sequence with $\mu = 3 \times 10^{-7}$ events per second and varying c from 40 to 4×10^7 sec (from left to right). Other model parameters are $T = 1 \times 10^7$ sec, $A = 1$ and $p = 1.05$. Open circles represent $\Delta t D(\Delta t)$ for the California catalog [26].

recover the limit $T \to \infty$ and neglect the dependence on μT. Previous studies [10] have proposed that $D(\Delta t)$ only depends on the branching ratio and used this result to measure the ratio between triggered and independent events in experimental catalogs. Sornette *et al.* [13] have shown that the functional form of $D(\Delta t)$ also depends on p, whose experimental values fluctuate around one. Our numerical simulations with a random $p \in]1, 1.6]$ indicate that $D(\Delta t)$ depends on the average p value. In the following we present results for simulations with $p = 1.2$. A very similar pattern is obtained for other p values. Figure 2(a) shows that for fixed values of μ and c the curves exhibit different behaviors for different $\mathcal{K}$, in agreement with previous results. We then focus on the role of the parameter c. In Fig. 2(b), we plot $\Delta t D(\Delta t)$ at constant $\mathcal{K}$ and c and for different values of μ, as in Ref. [14]. We confirm the existence of deviations from scaling (1) at small Δt which can be attributed to c. In order to show that the dependence on c enters in the scaling form as $\mu\bar{c}$, in Fig. 2(c), we fix the branching ratio $\mathcal{K}$ and vary μ as in Fig. 2(b), but now c is allowed to vary keeping constant μc. In this case all the distributions collapse on the same master curve revealing a "universal" behavior also at small Δt. This result indicates that deviations from Eq. (1) rely on the presence of the variable $\mu\bar{c}$ in Eq. (4). The dependence on this variable is relevant for $\Delta t < c$ and becomes negligible at larger Δt. This accounts for the appearance of deviations from universality only at small Δt.

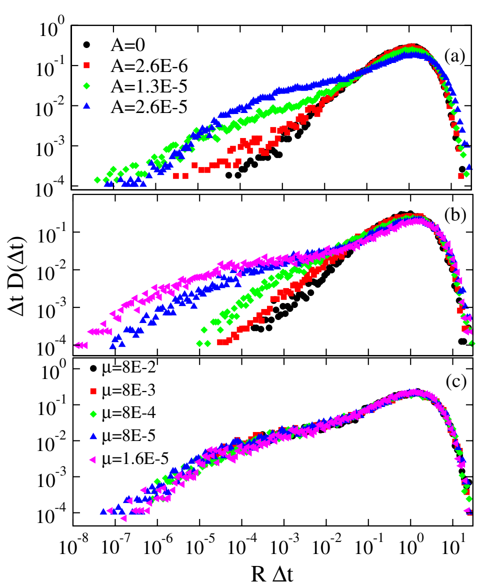

FIG. 2 (color online). $\Delta t D(\Delta t)$ vs $R\Delta t$ for the simulated catalogs with $T = 10^7$ and $p = 1.2$. (a) $\Delta t D(\Delta t)$ for $c = 3000$, $\mu = 8 \times 10^{-6}$ events per unit time and varying $\mathcal{K}$ using different A values reported in the legend. (b) $\Delta t D(\Delta t)$ for $c = 3000$, $\mathcal{K} = 0.5$ and varying μ according to the legend in panel (c). (c) We fix $c\mu = 0.24$ obtaining data collapse with $\mathcal{K} = 0.5$. In all figures the time unit is the iteration step of the model.

We now explore the role of the characteristic time c on the scaling properties of $\Delta t D(\Delta t)$ for the experimental catalog [26]. We first consider the whole catalog. In this case μT is very large and does not affect the scaling form (5). To isolate the dependence on $\mu\bar{c}$, we plot $\Delta t D(\Delta t)$ including in the analysis only earthquakes above a lower magnitude threshold $m_{\rm th}$. $\mu/\bar{a}$, indeed, should not depend on $m_{\rm th}$ [12] and $\Delta t D(\Delta t)$ is expected to depend only on $R\Delta t$ and $\mu\bar{c}$. Figure 3(a) shows deviations from universality. Since these deviations are confined at small Δt they are not easy to detect in the usual plot $R^{-1}D(\Delta t)$ vs $R\Delta t$ [2]. Deviations must be attributed to $\mu\bar{c}$ and allow us to identify $\bar{c}$ from the crossover points separating the linear growth from the plateau [identified by arrows in Fig. 3(a)]. Using the known values of R we obtain that $\bar{c}$ is quite independent of $m_{\rm th}$. This is consistent with c_i independent of mainshock magnitudes but also with $c_i \propto 10^{\alpha(m_i - m_{\rm th})}$ and $\alpha < b$ [28]. The dependence of c_i on m_i and its influence on the $D(\Delta t)$ can be obtained by restricting the analysis to temporal periods soon after mainshocks. In these intervals, μ can be neglected simplifying the scaling relation (5). We further observe that for a single Omori sequence $R = N/T = a_i c_i [1 - (T/c_i)^{(1-p)}]/[T(p-1)]$ and therefore R/a_i is a function only of the ratio T/c_i. In this case the scaling simplifies to

$$D(\Delta t) = R G_2\left(R\Delta t, \frac{T}{c_i}\right). \tag{6}$$

We start by considering the main-aftershock sequences for the three largest shocks recorded in the catalog: Landers, Northridge, and Hector Mine. We consider as aftershocks

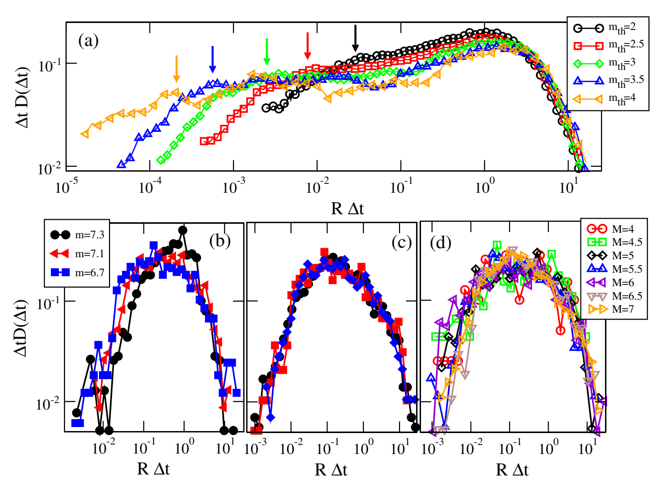

FIG. 3 (color online). (a) $\Delta t D(\Delta t)$ vs $R\Delta t$ for different magnitude thresholds $m_{\rm th}$; (b) for the Landers, Northridge, and Hector Mine sequences (mainshock magnitudes $m = 7.3$, 6.7, 7.1, respectively) with fixed duration $T = 10$ days; (c) for the Landers, Northridge, and Hector Mine sequences with T chosen following the criterion of Ref. [6]; (d) for all sequences with a mainshock magnitude m greater than 4 grouped in classes of m, $m \in [M, M + 0.5[$.

all events with $m \geq 2.5$ occurring in a temporal window [0, T] after the main event and within the aftershock zone, i.e., a radius $L = 0.01 \times 10^{0.5m_i}$ km from the mainshock. Different definitions of the aftershock zone [29] lead to very similar results. We first fix $T = 10$ days for all sequences. In Fig. 3(b) the curves do not collapse but show a progressive shift as the mainshock magnitude increases. This effect can be attributed to dependence of c_i on m_i. Therefore, at fixed T the variable T/c_i assumes different values for each sequence violating the collapse Eq. (1). As an alternative approach we use the criterion proposed in Ref. [6] to identify the end of a sequence: namely, a sequence ends when the rate $\lambda(t)$ reaches the average Poisson rate $\mu \simeq 2$ events/day. Figure 3(c) clearly indicates a very good data collapse for the different sequences in good agreement with Eq. (1). This can be understood from the scaling relation (6) where the variable T/c_i assumes the same value for all sequences. Indeed, according to the Omori law, for $t \gg c$ the occurrence rate can be expressed as $\lambda(t) \sim 10^{\alpha m_i} t^{-p}$ and the condition $\lambda(T) = 2$ provides $T \sim 10^{\alpha m_i/p}$. For the largest mainshocks, $c_i \sim 10^{b m_i/p}$ [5,6] and therefore, the condition $\alpha \simeq b/p$ implies that the ratio T/c_i is almost constant for the three sequences.

Next, we extend the above analysis to all sequences with mainshock magnitude $m > 4$. To improve the statistics, we group mainshocks in classes of magnitude $m \in [M, M + 0.5[$. Mainshocks are identified with the criterion suggested in Ref. [6] and the duration is again fixed by the condition $\mu(T) = 2$. Other methods [29] for aftershock identification provide similar results. Figure 3(d) shows data collapse for all M values. Since the criterion $\mu(T) = 2$ roughly implies $T \sim 10^{\alpha M}$, the collapse of Fig. 3(d), suggests that the dependence of c_i on the mainshock magnitude as $c_i \sim 10^{b m_i/p}$ is valid also for intermediate mainshock magnitudes.

In conclusion, we address recent criticisms to the universal behavior of the interevent distribution. We follow the approach of Ref. [14] and show that $D(\Delta t)$ does exhibit universal features on the whole temporal range if four characteristic time scales are taken into account. In particular, deviations at small Δt can be attributed to c scaling differently from μ. Whereas, by keeping constant T/c for different sequences, the $D(\Delta t)$ collapse onto a unique master curve.

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