

## Orbital-Fluctuation-Mediated Superconductivity in Iron Pnictides: Analysis of the Five-Orbital Hubbard-Holstein Model

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In iron pnictides, we find that the moderate electron-phonon interaction due to the Fe-ion oscillation can induce the critical  $d$ -orbital fluctuations, without being prohibited by the Coulomb interaction. These fluctuations give rise to the strong pairing interaction for the  $s$ -wave superconducting (SC) state without sign reversal ( $s_{++}$ -wave state), which is consistent with experimentally observed robustness of superconductivity against impurities. When the magnetic fluctuations due to Coulomb interaction are also strong, the SC state shows a smooth crossover from the  $s$ -wave state with sign reversal ( $s_{\pm}$ -wave state) to the  $s_{++}$ -wave state as impurity concentration increases.

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The mechanism of high- $T_c$  superconductivity in iron pnictides has been an important open problem. By considering the Coulomb interaction at Fe ions, an antiferromagnetic (AFM) fluctuation mediated fully-gapped sign-reversing  $s$ -wave state ( $s_{\pm}$ -wave state) is expected theoretically [1,2]. Regardless of the beauty of the mechanism, there are several serious discrepancies for the  $s_{\pm}$ -wave state. For example, although the  $s_{\pm}$ -wave state is expected to be very fragile against impurities due to the interband scattering [3], the superconducting (SC) state is remarkably robust against impurities [4] and  $\alpha$ -particle irradiation [5]. Moreover, clear “resonancelike” peak structure observed by neutron scattering measurements [6] is reproduced by considering the strong correlation effect via quasiparticle damping, without the necessity of sign reversal in the SC gap [7]. These facts indicate that a conventional  $s$ -wave state without sign reversal ( $s_{++}$ -wave state) is also a possible candidate for iron pnictides.

Then, a natural question is whether the electron-phonon ( $e$ -ph) interaction is important or not. Although first principle study predicts a small  $e$ -ph coupling constant  $\lambda \sim 0.21$  [8], several experiments indicate the significance of the  $e$ -ph interaction. For example, the structural transition temperature  $T_S$  is higher than the Néel temperature in underdoped compounds, although the structural distortion is small. Also, prominent softening of the shear modulus is observed towards  $T_S$  or  $T_c$  in Ba122 [9]. Raman spectroscopy [10] also indicates larger  $e$ -ph interaction.

Interestingly, there are several “high- $T_c$ ” compounds with nodal SC gap structure, like BaFe<sub>2</sub>(As<sub>1-x</sub>P<sub>x</sub>)<sub>2</sub> [11] and some 122 systems [12]. Although the nodal  $s_{\pm}$ -wave state can appear in the spin-fluctuation scenario due to the competition between the dominant  $\mathbf{Q} = (\pi, 0)$  and subdominant fluctuations [1,13], the  $T_c$  is predicted to be very low. Thus, it is a crucial challenge to explain the rich variety of the gap structure in high- $T_c$  compounds.

In this Letter, we introduce the five-orbital Hubbard-Holstein (HH) model for iron pnictides, considering the

$e$ -ph interaction by Fe-ion vibrations. We reveal that a relatively small  $e$ -ph interaction ( $\lambda \lesssim 0.3$ ) induces the large orbital fluctuations, which can realize the high- $T_c$   $s_{++}$ -wave SC state. Moreover, the orbital fluctuations are *accelerated* by Coulomb interaction. In the presence of impurities, the  $s_{++}$ -wave state dominates the  $s_{\pm}$ -wave state for a wide range of parameters.

First, we derive the  $e$ -ph interaction term, considering only Einstein-type Fe-ion oscillations for simplicity. Here, we describe the  $d$  orbitals in the  $XYZ$  coordinate [1], which is rotated by  $\pi/4$  from the  $xyz$  coordinate given by the Fe-site square lattice: We write the  $Z^2$ ,  $XZ$ ,  $YZ$ ,  $X^2 - Y^2$ , and  $XY$  orbitals as 1, 2, 3, 4, and 5, respectively [1]. We calculate the  $e$ -ph matrix elements due to the Coulomb potential, by following Ref. [14]. The potential for a  $d$  electron at  $\mathbf{r}$  (with the origin at the center of the Fe ion) due to the surrounding As<sup>3-</sup>-ion tetrahedron is  $U^{\pm}(\mathbf{r}; \mathbf{u}) = 3e^2 \sum_{s=1}^4 |\mathbf{r} + \mathbf{u} - \mathbf{R}_s^{\pm}|^{-1}$ , where  $\mathbf{u}$  is the displacement vector of the Fe ion, and  $\mathbf{R}_s^{\pm}$  is the location of the surrounding As ions;  $\sqrt{3}\mathbf{R}_s^+ / R_{\text{Fe-As}} = (\pm\sqrt{2}, 0, 1)$  and  $(0, \pm\sqrt{2}, -1)$  for Fe<sup>(1)</sup>, and  $\sqrt{3}\mathbf{R}_s^- / R_{\text{Fe-As}} = (\pm\sqrt{2}, 0, -1)$  and  $(0, \pm\sqrt{2}, 1)$  for Fe<sup>(2)</sup> in the unit cell with two Fe-sites. Note that  $u_{X,Y}$  and  $u_Z$  belong to  $E_g$  and  $B_{1g}$  phonons [10]. The  $\mathbf{u}$  linear term of  $U^{\pm}$ , which gives the  $e$ -ph interaction, is obtained as  $V^{\pm}(\mathbf{r}; \mathbf{u}) = \pm A [2XZu_X - 2YZu_Y + (X^2 - Y^2)u_Z] + O(\mathbf{r}^4)$ , where  $A = 30e^2 / \sqrt{3}R_{\text{Fe-As}}^4$ . Then, its nonzero matrix elements are given as

$$\begin{aligned} \langle 2|V|4 \rangle &= \pm 2a^2 A u_X / 7, & \langle 3|V|4 \rangle &= \pm 2a^2 A u_Y / 7, \\ \langle 2|V|2 \rangle &= \pm 2a^2 A u_Z / 7, & \langle 3|V|3 \rangle &= \mp 2a^2 A u_Z / 7, \end{aligned} \quad (1)$$

where  $a$  is the radius of the  $d$  orbital. Here, we consider  $\langle i|V|j \rangle$  only for orbitals  $i, j = 2-4$  that compose the Fermi surfaces (FSs) in Fig. 1(a) [1]. The obtained  $e$ -ph interaction does not couple to the charge density since  $\langle i|V|j \rangle$  is traceless. Thus, the Thomas-Fermi screening for the coefficient  $A$  is absent. The local phonon Green function is  $D(\omega_l) = 2\bar{u}_0^2 \omega_D / (\omega_l^2 + \omega_D^2)$ , which is given by the

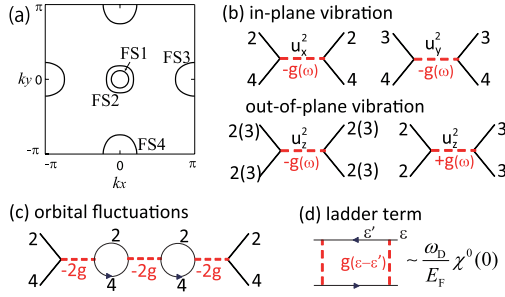


FIG. 1 (color online). (a) FSs in the unfolded Brillouin zone. (b) Phonon-mediated electron-electron interaction. (c) A bubble-type diagram that induces the critical orbital fluctuations between (2,4) orbitals. (d) A ladder-type diagram that is ignorable when  $\omega_D \ll E_F$ .

Fourier transformation of  $\langle T_\tau u_\mu(\tau) u_\mu(0) \rangle$  ( $\mu = X, Y, Z$ ).  $\bar{u}_0 = \sqrt{\hbar/2M_{\text{Fe}}\omega_D}$  is the position uncertainty of Fe ions,  $\omega_D$  is the phonon frequency, and  $\omega_l = 2\pi lT$  is the boson Matsubara frequency. Then, for both Fe<sup>(1)</sup> and Fe<sup>(2)</sup>, the phonon-mediated interaction is given by

$$\begin{aligned} V_{24,42} &= V_{34,43} = -(2Aa^2/7)^2 D(\omega_l) \equiv -g(\omega_l), \\ V_{22,22} &= V_{33,33} = -V_{22,33} = -g(\omega_l), \end{aligned} \quad (2)$$

as shown in Fig. 1(b). Note that  $V_{ll',mm'}$  is symmetric with respect to  $l \leftrightarrow l'$ ,  $m \leftrightarrow m'$ , and  $(ll') \leftrightarrow (mm')$ . We obtain  $g(0) \approx 0.4$  eV if we put  $R_{\text{Fe-As}} \approx 2.4$  Å,  $a \approx 0.77$  Å (Shannon crystal radius of Fe<sup>2+</sup>), and  $\omega_D \approx 0.018$  eV. We have neglected the  $e$ -ph coupling due to  $d$ - $p$  hybridization [14] considering the modest  $d$ - $p$  hybridization in iron pnictides [15]. Thus, we obtain the multiorbital HH model for iron pnictides by combining Eq. (2) with the on-site Coulomb interaction; the intra- (inter-) orbital Coulomb  $U$  ( $U'$ ), Hund coupling  $J$ , and pair hopping  $J'$ .

Now, we study the rich electronic properties realized in the multiorbital HH model [16]. The irreducible susceptibility in the five-orbital model is given by  $\chi_{ll',mm'}^0(q) = -(T/N) \sum_k G_{lm}^0(k+q) G_{m'l'}^0(k)$ , where  $\hat{G}^0(k) = [i\epsilon_n + \mu - \hat{H}_k^0]^{-1}$  is the  $d$ -electron Green function in the orbital basis:  $q = (\mathbf{q}, \omega_l)$ ,  $k = (\mathbf{k}, \epsilon_n)$ , and  $\epsilon_n = (2n+1)\pi T$  is the fermion Matsubara frequency.  $\mu$  is the chemical potential, and  $\hat{H}_k^0$  is the kinetic term given in Ref. [1]. Then, the susceptibilities for spin and charge sectors in the random phase approximation (RPA) are given as [17]

$$\hat{\chi}^{s(c)}(q) = \hat{\chi}^0(q) [1 - \hat{\Gamma}^{s(c)} \hat{\chi}^0(q)]^{-1}. \quad (3)$$

For the spin channel,  $\Gamma_{l_1 l_2, l_3 l_4}^s = U, U', J$ , and  $J'$  for  $l_1 = l_2 = l_3 = l_4$ ,  $l_1 = l_3 \neq l_2 = l_4$ ,  $l_1 = l_2 \neq l_3 = l_4$ , and  $l_1 = l_4 \neq l_2 = l_3$ , respectively [1]. For the charge channel,  $\hat{\Gamma}^c = -\hat{C} - 2\hat{V}(\omega_l)$ , where  $\hat{V}(\omega_l)$  is given in Eq. (2), and  $C_{l_1 l_2, l_3 l_4} = U, -U' + 2J, 2U' - J$ , and  $J'$  for  $l_1 = l_2 = l_3 = l_4$ ,  $l_1 = l_3 \neq l_2 = l_4$ ,  $l_1 = l_2 \neq l_3 = l_4$ , and  $l_1 = l_4 \neq l_2 = l_3$ , respectively [1]. Figure 1(c) shows one of the bubble diagrams for the (2,4)-channel due to the ‘‘nega-

tive exchange coupling  $V_{24,42}$ ’’ that leads to a critical enhancement of  $\hat{\chi}^c(q)$  [18]. We neglect the ladder diagrams given by  $\hat{V}(\omega_l)$  in Fig. 1(d) since  $\omega_D \ll W_{\text{band}}$  [8,10]. We put  $\omega_D = 0.02$  eV,  $U'/U = 0.69$ ,  $J/U = 0.16$ , and  $J = J'$ , and fix the electron number  $n = 6.1$  (10% electron doping); the density of states per spin is  $N(0) = 0.66$  eV<sup>-1</sup>. Numerical results are not sensitive to these parameters. We use 128<sup>2</sup>  $\mathbf{k}$  meshes, and 512 Matsubara frequencies. Hereafter, the unit of energy is eV.

Figure 2(a) shows the obtained  $U$ - $g(0)$  phase diagram.  $\alpha_{s(c)}$  is the spin (charge) Stoner factor, given by the maximum eigenvalue of  $\hat{\Gamma}^{s(c)} \hat{\chi}^0(\mathbf{q}, 0)$ . Then, the enhancement factor for  $\chi^{s(c)}$  is  $(1 - \alpha_{s(c)})^{-1}$ , and  $\alpha_{s(c)} = 1$  gives the spin (orbital) order boundary. Because of the nesting of the FSs, the AFM fluctuation with  $\mathbf{Q} \approx (\pi, 0)$  develops as  $U$  increases, and  $s_{\pm}$ -wave state is realized for  $\alpha_s \leq 1$  [1]. In contrast, we find that the orbital fluctuations develop as  $g(0)$  increases. For  $U = 1$ , the critical value  $g_{\text{cr}}(0)$  for  $\alpha_c = 1$  is 0.4, and the critical  $e$ -ph coupling constant is  $\lambda_{\text{cr}} \equiv g_{\text{cr}}(0)N(0) = 0.26$  [19]. Since the obtained  $\lambda_{\text{cr}}$  is close to  $\lambda$  given by the first principle study [8], strong orbital fluctuations are expected to occur in iron pnictides. At fixed  $U$ ,  $\lambda_{\text{cr}}$  decreases as  $J/U$  approaches zero.

Figures 2(b) and 2(c) show the obtained  $\chi_{ll',mm'}^c(\mathbf{q}, 0)$  for  $(ll', mm') = (24, 42)$  and  $(22, 22)$ , respectively, for  $U = 1.14$  and  $\alpha_c = 0.97$  ( $g(0) = 0.40$ ): Both of them are the most divergent channels for electron-doped cases. The enhancement of (24, 42)-channel is induced by the multiple scattering by  $V_{24,42}$ . The largest broad peak around  $\mathbf{q} = (0, 0)$  originates from the forward scattering in the electron-pocket (FS3 or 4) composed of 2–4 orbitals. (FS1,2 are composed of only 2 and 3 orbitals.) These ferro-orbital fluctuations would induce the softening of shear modulus [9], and also reinforce the ferro-orbital-ordered state below  $T_S$  [20] that had been explained by

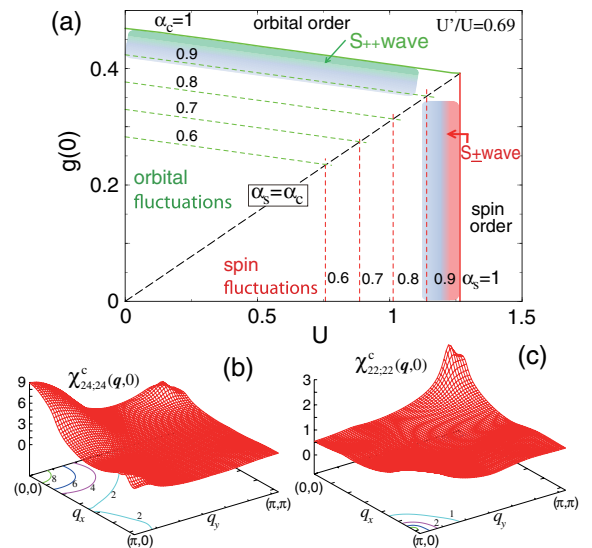


FIG. 2 (color online). (a) Obtained  $U$ - $g(0)$  phase diagram. (b) Obtained  $\chi_{24,42}^c(\mathbf{q}, 0)$  and  $\chi_{22,22}^c(\mathbf{q}, 0)$  for  $\alpha_c = 0.97$ .

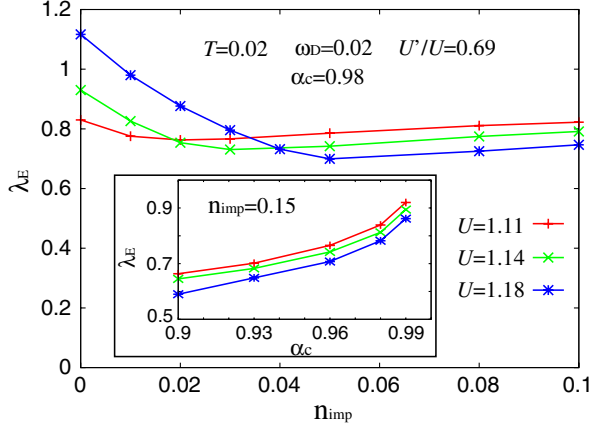


FIG. 3 (color online).  $n_{\text{imp}}$  dependence of  $\lambda_E$  at  $\alpha_c = 0.98$ . If we put  $g(0) = 0$  ( $s_{\pm}$  state),  $\lambda_E$  at  $n_{\text{imp}} = 0$  decreases by  $0.1 \sim 0.15$ , since the ferro-orbital fluctuations enhance both the  $s_{++}$  and  $s_{\pm}$  wave states. Inset:  $\alpha_c$  dependence of  $\lambda_E$ .

different theoretical approaches [21]: The divergence of  $\chi_{24,42}^c$  ( $\chi_{34,43}^c$ ) pushes the 2,4 (3,4) orbitals away from the Fermi level, and the Fermi surfaces in the ordered state will be formed only by 3 (2) orbital, consistently with ref. [20]. The lower peak around  $\mathbf{Q} = (\pi, 0)$  comes from the nesting between hole- and electron-pockets. Also, the enhancement of (22, 22)-channel for  $\mathbf{Q} = (\pi, 0)$  is induced by the nesting via multiple scattering by  $V_{22,22}$  and  $V_{22,33}$ . In contrast, the charge susceptibility  $\sum_{l,m} \chi_{ll,mm}^c(\mathbf{q}, 0)$  is finite even if  $\alpha_c \rightarrow 1$  since  $\chi_{22,33}^c \approx -\chi_{22,22}^c$ .

Now, we will show that large orbital fluctuations, which are not considered in the first principle study of  $T_c$  [8], can induce the  $s_{++}$ -wave state when  $g(0) > 0$ . We analyze the following linearized Eliashberg equation using the RPA [1], by taking both the spin and orbital fluctuations into account on the same footing:

$$\lambda_E \Delta_{l'l'}(k) = \frac{T}{N} \sum_{k', m_i} W_{l m_1, m_4 l'}(k - k') G_{m_1 m_2}(k') \Delta_{m_2 m_3}(k') \times G_{m_4 m_3}(-k'), \quad (4)$$

where  $\hat{W}(q) = -\frac{3}{2} \hat{\Gamma}^s \hat{\chi}^s(q) \hat{\Gamma}^s + \frac{1}{2} \hat{\Gamma}^c \hat{\chi}^c(q) \hat{\Gamma}^c - \frac{1}{2} (\hat{\Gamma}^s - \hat{\Gamma}^c)$  for singlet states. The eigenvalue  $\lambda_E$  increases as  $T \rightarrow 0$ , and it reaches unity at  $T = T_c$ . In addition, we take the impurity effect into consideration since many iron pnictides show relatively large residual resistivity. Here, we assume the Fe-site substitution, where the impurity potential  $I$  is diagonal in the  $d$ -orbital basis [3]. Then, the  $T$  matrix in the normal state is given by  $\hat{T}(\epsilon_n) = [I^{-1} - N^{-1} \sum_k \hat{G}(\mathbf{k}, \epsilon_n)]^{-1}$  in the orbital basis [3]. Then, the normal self-energy is  $\hat{\Sigma}^n(\epsilon_n) = n_{\text{imp}} \hat{T}(\epsilon_n)$ , where  $n_{\text{imp}}$  is the impurity concentration. Also, the linearized anomalous self-energy is given by

$$\Sigma_{l'l'}^a(\epsilon_n) = \frac{n_{\text{imp}}}{N} \sum_{k, m_i} T_{l m_1}(\epsilon_n) G_{m_1 m_2}(\mathbf{k}, \epsilon_n) \Delta_{m_2 m_3}(\mathbf{k}, \epsilon_n) \times G_{m_4 m_3}(-\mathbf{k}, -\epsilon_n) T_{l' m_4}(-\epsilon_n). \quad (5)$$

Then, the Eliashberg equation for  $n_{\text{imp}} \neq 0$  is given by using the full Green function  $\hat{G}(k) = [i\epsilon_n + \mu - \hat{H}_k^0 - \hat{\Sigma}_{l'l'}^a(\epsilon_n)]^{-1}$  in Eqs. (4) and (5), and adding  $\Sigma_{l'l'}^a(\epsilon_n)$  to the right hand side of Eq. (4). Hereafter, we solve the equation at relatively high temperature  $T = 0.02$  since the number of  $\mathbf{k}$  meshes ( $128^2$ ) is not enough for  $T < 0.02$ .

Figure 3 shows the  $n_{\text{imp}}$  dependence of  $\lambda_E$  at  $\alpha_c = 0.98$ , for  $U = 1.11, 1.14$  and  $1.18$ . Considering large  $\lambda_E \geq 0.8$  at  $T = 0.02$ , relatively high- $T_c$  ( $\leq 0.02$ ) is expected. For the smallest  $U$  ( $U = 1.11$ ;  $\alpha_s = 0.85$ ), we find that nearly isotropic  $s_{++}$ -wave state is realized; the obtained  $\lambda_E$  is almost independent of  $n_{\text{imp}}$ , indicating the absence of the impurity effect on the  $s_{++}$ -wave state, as discussed in Refs. [3,22]. For the largest  $U$  ( $U = 1.18$ ;  $\alpha_s = 0.91$ ), the  $s_{\pm}$ -wave state is realized at  $n_{\text{imp}} = 0$ ;  $\lambda_E$  decreases slowly as  $n_{\text{imp}}$  increases from zero, whereas it saturates for  $n_{\text{imp}} \geq 0.05$ , indicating the smooth crossover from  $s_{\pm}$ - to  $s_{++}$ -wave states due to the interband impurity scattering. For  $U = 1.14$  ( $\alpha_s = 0.88$ ), the SC gap at  $n_{\text{imp}} = 0$  is a hybrid of  $s_{++}$  and  $s_{\pm}$ ; only  $\Delta_{\text{FS2}}$  is different in sign.

The inset of Fig. 3 shows  $\lambda_E$  for the  $s_{++}$ -wave state in the presence of impurities: Since  $\lambda_E(\alpha_c = 0.98) - \lambda_E(\alpha_c = 0.9)$  is only  $\sim 0.15$  for each value of  $U$ , we expect that relatively large  $T_c$  for  $s_{++}$ -wave state is realized even if orbital fluctuations are moderate. We stress that the obtained  $\lambda_E$  is almost constant for  $\omega_D = 0.02-0.1$ , suggesting the absence of isotope effect in the  $s_{++}$ -wave state due to the strong retardation effect [14]. By the same reason,  $\lambda_E$  for the  $s_{++}$ -wave state is seldom changed if we put  $U = 3$  in the Hartree-Fock term  $\frac{1}{2}(\hat{\Gamma}^s - \hat{\Gamma}^c)$  in  $W(q)$ , indicating that the Morel-Anderson pseudopotential almost saturates.

Here, we discuss the case  $U = 1.18$  in detail: Fig. 4 shows the SC gap on the FSs in the band representation for (a)  $n_{\text{imp}} = 0$ , (b) 0.03, and (c) 0.08. They satisfy the condition  $N^{-1} \sum_{k, lm} |\Delta_{lm}(\mathbf{k})|^2 = 1$ . The horizontal axis is the azimuth angle for the  $\mathbf{k}$  point with the origin at  $\Gamma$  (M) point for FS1,2 (FS4);  $\theta = 0$  corresponds to the  $k_x$  direction. In case (a), the  $s_{\pm}$  state with strong imbalance,  $|\Delta_{\text{FS1}}|, |\Delta_{\text{FS2}}| \ll \Delta_{\text{FS4}}$ , is realized, and  $\Delta_{\text{FS4}}$  takes the largest value at  $\theta = \pi/2$ , where the FS is mainly composed of orbital 4. In case (c), the impurity-induced isotropic  $s_{++}$  state [23] with  $\Delta_{\text{FS1}} \sim \Delta_{\text{FS2}} \sim \Delta_{\text{FS4}}$  is realized, consistently with many ARPES measurements [24]. In case (b),  $\Delta_k$  on FS1 is almost gapless. However, considering the  $k_z$  dependence of the FSs, a (horizontal-type) nodal structure is expected to appear on FS1,2. In real compounds with  $T_c \sim 50$  K, the  $s_{\pm} \rightarrow s_{++}$  crossover should be induced by small residual resistivity  $\rho_{\text{imp}} \sim 20 \mu\Omega\text{cm}$  ( $n_{\text{imp}} \sim 0.01$  for  $I = 1$ ), as estimated in Ref. [3].

We comment that at  $n_{\text{imp}} = 0$ ,  $s_{\pm}$ -wave state is realized in the RPA even if  $\alpha_s \leq \alpha_c$ , due to factor 3 in front of  $\frac{1}{2} \hat{\Gamma}^s \hat{\chi}^s(q) \hat{\Gamma}^s$  in  $W(q)$ . For the same reason, however, reduction in  $\alpha_s$  (or increment of  $U_{\text{cr}}$  for  $\alpha_s = 1$ ) due to the ‘‘self-energy correction by  $U$ ’’ is larger, which will be unfavor-

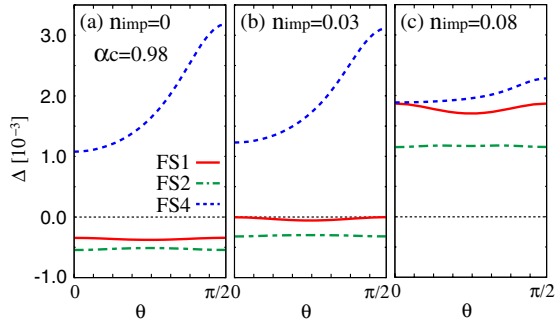


FIG. 4 (color online). SC gap functions for  $U = 1.18$  as functions of  $\theta$  at (a)  $n_{\text{imp}} = 0$ , (b) 0.03, and (c) 0.08, respectively.

able for the  $s_{\pm}$ -wave state. Therefore, self-consistent calculation for the self-energy is required to discuss the value of  $\alpha_{c,s}$  and the true pairing state.

Here, we discuss where in the  $\alpha_s$ - $\alpha_c$  phase diagram in Fig. 2(a) real compounds are located. Considering the weak  $T$  dependence of  $1/T_1T$  in electron-doped SC compounds [25], we expect that they belong to the area  $\alpha_c \gg \alpha_s$ . Then, the  $s_{++}$ -wave SC state will be realized without (or very low density) impurities, like the case of  $U = 1.11$  or 1.14 in Fig. 3. On the other hand, impurity-induced  $s_{\pm} \rightarrow s_{++}$  crossover may be realized in  $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$  (undoped) or  $(\text{Ba}_{1-x}\text{K}_x)\text{Fe}_2\text{As}_2$  (hole-doped) SC compounds, where AFM fluctuations are rather strong.

Finally, we discuss the non-Fermi-liquid-like transport phenomena in iron pnictides. For example, the resistivity is nearly linear in  $T$ , and the Hall coefficient  $R_H$  increases at lower temperatures [4,26]. Although the forward scattering induced by ferro-orbital fluctuations might be irrelevant, antiferro-orbital and AFM fluctuations with  $\mathbf{Q} = (\pi, 0)$  are expected to cause the anomalous transport, due to the current vertex correction [27].

In summary, we have proposed a mechanism of the  $s_{++}$ -wave SC state induced by orbital fluctuations, due to the phonon-mediated electron-electron interaction. Three orbitals ( $XZ$ ,  $YZ$ , and  $X^2 - Y^2$ ) are necessary to lead the ferro-orbital fluctuations. The SC gap structure drastically changes depending on parameters  $\alpha_s$ ,  $\alpha_c$ , and  $n_{\text{imp}}$ , consistent with the observed rich variety of the gap structure that is a salient feature of iron pnictides. The orbital-fluctuation-mediated  $s_{++}$ -wave state is also obtained for hole-doped cases, although the antiferro-orbital fluctuations become stronger than the ferro-orbital ones.

The  $s$ -wave superconductivity induced by orbital fluctuations had been discussed in Ref. [17] for  $U' > U$ ; this condition can be realized by including the  $A_{1g}$  phonon [28]. In the present model, however, the  $A_{1g}$  phonon is negligible since  $g_{\text{cr}}(0)$  given by the  $A_{1g}$  phonon is much greater than  $g_{\text{cr}}(0) \sim 0.4$  in Fig. 2(a): The ferro-orbital fluctuations in Fig. 2(b) originate from the *negative* exchange interaction caused by the  $E_g$  phonon, as shown in Fig. 1(c).

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*Note added in proof.*—After the acceptance of this work, we found that  $g_{\text{cr}}(0) \sim 0.4$  in Fig. 2(a) reduced to half if all the  $e$ -ph matrix elements including the 1,5 orbitals are taken into account. Results similar to Fig. 3 are obtained by using  $g(0) \sim 0.2$ , whereas (vertical-type) nodes appear on FS3,4 during the  $s_{++} \rightarrow s_{\pm}$  crossover for  $U = 1.18$ .

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