Halo-Shape and Relic-Density Exclusions of Sommerfeld-Enhanced Dark Matter Explanations of Cosmic Ray Excesses

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(Received 15 December 2009; published 15 April 2010)

Dark matter with Sommerfeld-enhanced annihilation has been proposed to explain observed cosmic ray positron excesses in the 10 GeV to TeV energy range. We show that the required enhancement implies thermal relic densities that are too small to be all of dark matter. We also show that the dark matter is sufficiently self-interacting that observations of elliptical galactic dark matter halos exclude large Sommerfeld enhancement for light force carriers. Resonant Sommerfeld enhancement does not modify these conclusions, and the astrophysical boosts required to resolve these discrepancies are disfavored, especially when significant self-interactions suppress halo substructure.

DOI: 10.1103/PhysRevLett.104.151301 PACS numbers: 95.35.+d, 95.85.Ry, 98.70.Sa

Introduction.—Recently, PAMELA [1], ATIC [2], Fermi [3], and HESS [4] have observed the spectrum of cosmic ray positrons with energies between 10 GeV and a few TeV. Some of the data show excesses over background expectations [5]. The excesses have plausible astrophysical explanations [6,7]. At the same time, signals from many dark matter candidates are expected in this energy range, and this possibility has not escaped attention.

By far the most researched possibility is that the observed positrons are produced by dark matter annihilation. If dark matter X is a thermal relic, the relic density implies that its thermally averaged annihilation cross section times relative velocity at freeze-out is $\langle \sigma_{\rm an} v_{\rm rel} \rangle \approx \sigma_0^{\rm th} \equiv 3 \times 10^{-26}\ {\rm cm}^3/{\rm s}$. Unfortunately, if this is the annihilation cross section now, the resulting signal is too small by 2 to 3 orders of magnitude to explain the observed cosmic ray excesses.

A seemingly attractive solution is to postulate that dark matter interacts with a light force carrier ϕ with fine structure constant $\alpha_X \equiv \lambda^2/(4\pi)$ [8,9]. For $m_\phi = 0$, the annihilation is enhanced by the Sommerfeld factor [10]

$$S = \frac{\pi \alpha_X / v_{\rm rel}}{1 - e^{-\pi \alpha_X / v_{\rm rel}}}. \tag{1}$$

For massive ϕ, the enhancement is typically cut off at a value $\propto \alpha m_X / m_\phi$ [8,9,11,12], but for fine-tuned choices of α_X, m_X, and m_ϕ, there are also resonance regions where the enhancement may exceed this cutoff, as we discuss below. The velocity of dark matter particles is $\sim 1/3$ at freeze-out and $\sim 10^{-3}$ now. Sommerfeld enhancement therefore provides an elegant mechanism for boosting annihilations now. Constraints from dark matter annihilation in protohalos with $v_{\rm rel} \sim 10^{-8}$ exclude $m_\phi = 0$ [13]. However, taking $m_X \sim$ TeV and $m_\phi \sim$ MeV-GeV, and assuming $\langle \sigma_{\rm an} v_{\rm rel} \rangle \approx \sigma_0^{\rm th}$, one may still generate $S \sim 10^3$ to explain the positron excesses, while the cutoff allows one to satisfy the protohalo constraint.

Of course, for a viable solution, dark matter must not only annihilate with the correct rate, it must also be produced with the right density and form structure in accord with observations. Here, we find that the desired thermal relic density cannot be achieved in Sommerfeld-enhanced models designed to explain the positron excesses. In addition, we show that the new force carrier ϕ induces dark matter self-interactions that may contradict current observations. As is well-known, if ϕ were massless, the resulting long range Coulomb force would lead to large energy transfers that make halos spherical, and observations of triaxial halos constrain this possibility [14,15]. For $m_\phi \sim$ 100 MeV, the force's range is only $\sim$10 fm, but, as we show below, the implied cross section is still large enough to play a role in galactic dynamics.

Thermal relic density.—If XX annihilation is enhanced by ϕ exchange, there is an "irreducible" annihilation process $XX \to \phi\phi$ through t-channel X. For $m_\phi \ll m_X$, the thermally averaged annihilation cross section is the typical WIMP cross section

$$\langle \sigma_{\rm an} v_{\rm rel} \rangle \approx \pi \alpha_X^2 / m_X^2, \tag{2}$$

with $\mathcal{O}(1)$ corrections depending on the details of the initial and final states. Requiring that $\langle \sigma_{\rm an} v_{\rm rel} \rangle$ be small enough that X can be all of the dark matter implies

$$\alpha_X \leq \sqrt{\sigma_0^{\rm th}/\pi}\, m_X. \tag{3}$$

This bound is conservative. In fact, the Sommerfeld effect enhances the annihilation cross section even at freeze-out [8,16], and the bound may be significantly strengthened in the presence of other annihilation channels.

Self-interactions.—Self-interactions allow dark matter particles to transfer energy. The average rate for dark matter particles to change velocities by $\mathcal{O}(1)$ factors is

$$\Gamma_k = \int d^3 v_1 d^3 v_2 f(v_1) f(v_2) (n_X v_{\rm rel} \sigma_T)(v_{\rm rel}^2 / v_0^2), \tag{4}$$

where $f(v) = e^{-v^2/v_0^2}/(v_0\sqrt{\pi})^3$ is the dark matter's assumed (Maxwellian) velocity distribution, n_X is its number density, $v_{\rm rel} = |\vec{v}_1 - \vec{v}_2|$, and $\sigma_T = \int d\Omega_*(d\sigma/d\Omega_*)(1 - \cos\theta_*)$ is the energy transfer cross section, where θ_* is the scattering angle in the center-of-mass frame.

Dark matter particles coupled to a massive force carrier ϕ scatter through the Yukawa potential $V(r) = -\alpha_X e^{-m_\phi r}/r$. In the Born approximation, keeping only the dominant t-channel contribution present in all interactions, the transfer cross section is

$$\sigma_T = \frac{2\pi}{m_\phi^2}\beta^2\left[\ln(1+R^2) - \frac{R^2}{1+R^2}\right], \tag{5}$$

where $\beta \equiv 2\alpha_X m_\phi/(m_X v_{\rm rel}^2)$ is the ratio of the potential energy at $r \sim m_\phi^{-1}$ to the kinetic energy of the particle, and $R \equiv m_X v_{\rm rel}/m_\phi$ is the ratio of the interaction range to the dark matter particle's de Broglie wavelength. For typical values of interest here, $v_{\rm rel} \sim 10^{-3}$ and $m_X/m_\phi \gtrsim 10^3$, and so $R \gtrsim 1$. For $R \gg 1$, $\sigma_T \approx \frac{8\pi\alpha_X^2}{v_{\rm rel}^4 m_X^2}(\ln R^2 - 1)$. As in the Coulomb case, this is greatly enhanced for small $v_{\rm rel}$, but here, the finite interaction length of the Yukawa potential cuts off the logarithmic divergence.

Equation (5) receives significant corrections in the strong interaction regime, where $\beta \gg 1$. Our focus in this work will be on the $R \gg 1$ region of parameter space. In this region, quantum effects are subdominant, and so classical studies [17] of particles moving in Yukawa potentials are applicable. Although the authors of these studies were interested in slow and highly charged particles moving in plasmas with screened Coulomb potentials, they approximated these potentials by Yukawa potentials, and so their results are exactly applicable in the current context. The numerical results of these studies are accurately reproduced by [17]

$$\begin{aligned} \sigma_T &\simeq \frac{4\pi}{m_\phi^2}\beta^2\ln(1+\beta^{-1}), && \beta < 0.1, \\ \sigma_T &\simeq \frac{8\pi}{m_\phi^2}\frac{\beta^2}{1+1.5\beta^{1.65}}, && 0.1 < \beta < 1000. \end{aligned} \tag{6}$$

We use these analytical fits to obtain the results below.

Halo shapes.—Self-interactions that are strong enough to create $\mathcal{O}(1)$ changes in the energies of dark matter particles will isotropize the velocity dispersion and create spherical halos. These expectations are borne out by simulations of self-interacting dark matter in the hard sphere limit [18–20]. The shapes of dark matter halos of elliptical galaxies and clusters are decidedly elliptical, which constrains self-interactions [21]. The ellipticity of galactic halos provides the strongest constraints on these models [15]. To implement these constraints, we consider the well-studied, nearby (about 25 Mpc away) elliptical galaxy NGC 720. In Ref. [22], x-ray isophotes were used to extract the ellipticity of the underlying matter distribution. Comparing it to the ellipticity induced by the stellar mass profile, the dark matter halo of NGC 720 was found to be elliptical at about 5 kpc and larger radii.

To compute Γ_k, we use the measured total mass profile and the decomposition into stars plus dark matter for NGC 720 [23] and obtain the radial velocity dispersion $\bar{v}_r^2(r) = v_0^2(r)/2$ and the dark matter density. For the radius, we pick 5 kpc. Our constraints would be stronger if we could use the higher densities inside this radius, but the constraints on the ellipticity weaken for radii below 5 kpc [22]. For the dark matter density, we choose the average value within 5 kpc, which is roughly 4 GeV/cm^3. To compute the dispersion, we assume isotropy and that the total (stellar plus dark matter) mass profile scales approximately linearly with radius. For a Navarro-Frenk-White profile with best fit scale radius [23], $\bar{v}_r^2(r) \simeq (240\ \text{km/s})^2$. Varying within the quoted error range for the scale radius [23] only changes this dispersion by about 10%.

Results.—To derive constraints on the particle physics parameters from the observed halo shapes, we require

$$\Gamma_k^{-1} > 10^{10}\ \text{years}, \tag{7}$$

i.e., that the average time for self-interactions to create $\mathcal{O}(1)$ changes in dark matter particle velocities is greater than the galaxy's lifetime. Imposing Eqs. (3) and (7) from the relic density and the observation of ellipticity in the dark matter halo of NGC 720 yields the constraints shown in Fig. 1. The relic-density constraint is independent of m_ϕ, and the extremely stringent halo shape constraint for $m_\phi = 0$ [15] remains significant for $m_\phi \lesssim 30$ MeV. The crucial point is that when the interaction range is larger than the de Broglie wavelength, although the Coulomb logarithm enhancement is lost, the enhancement from low $v_{\rm rel}$ remains. Note that our assumption of a locally Gaussian velocity distribution is supported by recent simulation of

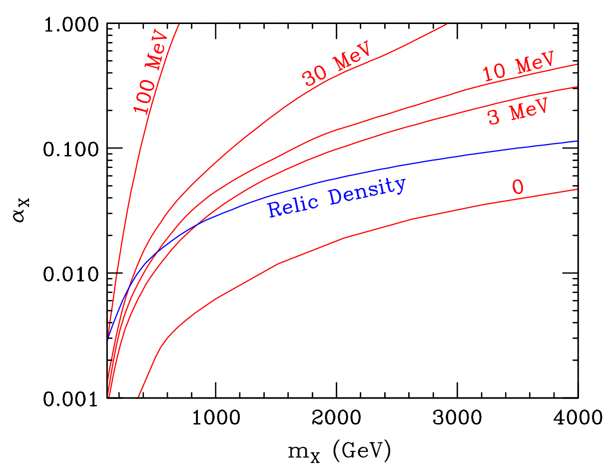

FIG. 1 (color online). Regions above the contours are excluded by the relic-density constraint and by halo ellipticity observations for the m_ϕ indicated. The classical approximation used to obtain the halo bounds becomes inaccurate for $m_\phi \gtrsim 100$ MeV.

Milky Way-sized dark matter halos [24]. Γ_k does not change by more than a factor of about 2 when we allow the distribution to become anisotropic or introduce a velocity cutoff at the escape speed. At the same time, we have checked that our halo-shape bounds are consistent with the predictions from simulations with hard sphere scattering [18].

In Fig. 2, we present the regions of the (m_X, S) plane required to explain PAMELA and Fermi as determined in Ref. [25]. These are for $m_\phi = 250$ MeV, which is large enough to allow contributions to positrons through $\phi \to \mu^+\mu^-$, but small enough to forbid contributions to antiprotons, where no excess is seen [9]. Upper bounds from relic density and halo shapes are also given. We see that the large Sommerfeld enhancements required to explain the positron excesses are significantly excluded by the relic-density constraint for all m_X. For $m_\phi \lesssim 30$ MeV, the halo-shape constraints also exclude the required Sommerfeld enhancements.

Discussion.—The results of Fig. 2 are not surprising. For the relic density, the WIMP miracle implies that for $m_X \sim 250$ GeV, the correct relic density is obtained for $\alpha \sim 10^{-2}$. Given $v_{\rm rel} \sim 10^{-3}$, this implies an upper bound of $S \sim 10$, and this bound scales as m_X. Of course, X need not be all the dark matter, but in this case, the Sommerfeld-enhanced flux scales as $n^2\langle\sigma_{\rm an}v\rangle S \sim \alpha_X^{-1}$, and so the signal is maximal for $S \sim 1$.

In deriving our results, we have ignored the cutoff of the Sommerfeld enhancement factor for massive ϕ. Including this cutoff will reduce the maximal possible S for low m_X, strengthening the disagreement between the allowed values of S and the experimentally favored regions. To reduce the disagreement, one might consider resonant Sommerfeld enhancement. As with resonances from additional postulated particles [26], these resonances require fine tuning and are bounded by astrophysical observations [27]. In addition, resonance enhancement occurs at $m_\phi/m_X \simeq 6\alpha_X/(\pi^2 n^2)$, $n = 1, 2, \ldots$ [28] and is significant only for low n. For $m_\phi \sim$ GeV and the relevant range of $\alpha_X \gtrsim 0.01$, this implies $m_X \lesssim 500$ GeV; the resonances are, therefore, ineffective in reaching the favored regions given in Fig. 2. Most importantly, as noted above, our bounds are conservative in that they do not include the Sommerfeld effect on freeze-out [16]. This effect suppresses the largest possible S, especially at resonances. Self-consistently including the effects of resonances on annihilation in both the early Universe and now, we find that the maximal possible enhancement factor is $S \sim 100$, even allowing for resonances [29].

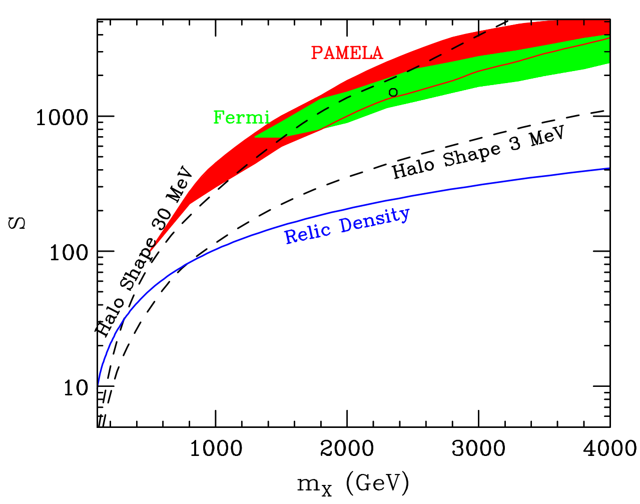

FIG. 2 (color online). Upper bounds on Sommerfeld enhancement factor S from relic density (solid line), along with PAMELA- and Fermi-favored regions and the best fit point $(m_X, S) = (2.35\ \text{TeV}, 1500)$ [25], all for $m_\phi = 250$ MeV. Halo-shape bounds are also shown for the values of m_ϕ indicated (dashed line).

As an alternative approach to evade the relic-density constraints, one may consider other production mechanisms or modify early Universe cosmology, but this sacrifices the WIMP miracle and also removes the motivation for considering Sommerfeld enhancement in the first place. Alternatively, one might appeal to boosts of $\sim$10 from cold and dense dark matter substructure in the local neighborhood. Such large values at a distance of only 10 kpc from the Milky Way center are, however, not motivated by simulations with collisionless dark matter [24]. The presence of the stellar disk would further reduce these expectations. The self-scatterings among the particles in the substructure would also serve to reduce the inner densities [18] and hence the expected boost. In addition, for $m_\phi \lesssim 30$ MeV, interactions with the dark matter particles of the Milky Way could evaporate substructure because $v_{\rm rel}$ is much larger than the internal velocity dispersion of the substructure [30].

The halo-shape bounds are obtained from inferred dark matter halo ellipticity, which depends on merger histories and the environment. For example, a major merger at a redshift of $z = 0.5$ for NGC 720 would effectively halve the age that Γ_k^{-1} should be compared to and weaken the bound on m_ϕ by roughly a factor of $\sqrt{2}$. However, the lack of large scale disturbances in the gas argues against such a recent major merger. These bounds may be made more robust by deeper data sets of NGC 720, which will further constrain point source contamination and rotation or large scale disturbances in the gas, as well as by measuring ellipticities and mass profiles in other galaxies and clusters [31].

A second prediction of strongly self-interacting dark matter is the formation of constant density cores, if gravothermal collapse does not occur. The time scale for the formation of these cores is of order Γ_k^{-1}, suggesting that NGC 720 should have $\mathcal{O}$(kpc) sized core. Future tests for the presence of cores in galaxy and cluster halos may provide comparable or stronger limits. Self-interactions should also dramatically alter the dark matter halos of smaller galaxies, such as the dwarf galaxies in the Local Group. The central dark matter densities measured in these dwarf satellites of the Milky Way are $\mathcal{O}(\mathrm{GeV/cm^3})$ [32] and fit neatly within the standard CDM predictions. For the parameter space disfavored by NGC 720 observations, and using simulation results [18], we estimate that core sizes would be of order the luminous extent of the dwarfs or larger. The tidal force of the Milky Way would significantly reduce the central densities of the dwarfs with such large cores and likely make it impossible to explain the large observed dark matter densities in all the dwarfs [33]. In parameter regions with more moderate Sommerfeld enhancements, these cores would be smaller and consistent with current data [34].

Conclusions.—Cosmic positron data have motivated dark matter candidates with Sommerfeld-enhanced annihilations. The required enhancement is large, requiring large couplings to light force carriers. Annihilation to these force carriers provides an upper limit on the thermal relic abundance of these dark matter candidates. With or without resonances, this constraint excludes the existence of enhancements that can explain the positron excesses. These models also predict self-interactions that may make galactic dark matter halos spherical. The ellipticity of the halo of NGC 720 also excludes the required Sommerfeld enhancements for $m_\phi \lesssim 30$ MeV. Interestingly, viable models with moderate Sommerfeld enhancements, although unable to explain the positron data, may predict constant density spherical cores in small galactic halos and other departures from the standard cold dark matter paradigm that are consistent with current data.

We thank Matthew Buckley, David Buote, Patrick Fox, Phil Humphreys, Masahiro Ibe, Alessandro Strumia, and Huitzu Tu for helpful conversations. The work of J. L. F and H. Y was supported in part by NSF Grants No. PHY–0653656 and No. PHY–0709742. The work of M. K was supported in part by NSF Grant No. PHY–0855462 and NASA Grant No. NNX09AD09G.

Note added.—As the first version of this work was being completed, we learned of related work in progress. This work [35] agrees with Eq. (6) in the classical regime.

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