Unified Framework for Correlations in Terms of Local Quantum Observables

A. Acín,[1,2] R. Augusiak,[1] D. Cavalcanti,[1] C. Hadley,[1] J. K. Korbicz,[3,4] M. Lewenstein,[1,2] Ll. Masanes,[1] and M. Piani[5]

[1]ICFO–Institut de Ciències Fotòniques, E–08860 Castelldefels, Barcelona, Spain
[2]ICREA–Institució Catalana de Recerca i Estudis Avançats, Lluis Companys 23, 08010 Barcelona, Spain
[3]Institute of Theoretical Physics and Astrophysics, University of Gdańsk, 80-952 Gdańsk, Poland
[4]National Quantum Information Center of Gdańsk, 81-824 Sopot, Poland
[5]Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, N2L 3G1 Waterloo, Ontario, Canada

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We provide a unified framework for nonsignalling quantum and classical multipartite correlations, allowing all to be written as the trace of some local (quantum) measurements multiplied by an operator. The properties of this operator define the corresponding set of correlations. We then show that if the theory is such that all local quantum measurements are possible, one obtains the correlations corresponding to the extension of Gleason's Theorem to multipartite systems. Such correlations coincide with the quantum ones for one and two parties, but we prove the existence of a gap for three or more parties.

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Introduction.—Physical principles impose limits on the correlations observed by distant parties. It is known, for instance, that the principle of no-signalling—that is, the fact that the correlations cannot lead to any sort of instantaneous communication—implies no-cloning [1] and no-broadcasting [2] theorems, and the possibility of secure key distribution [3]. Moving to the quantum domain, the main goal of quantum information theory is precisely to understand how quantum properties may be used for information processing. It is then important to understand how the quantum formalism constrains the correlations amongst distant parties. For instance, an asymptotically convergent hierarchy of necessary conditions for some correlations to have a quantum realization has been introduced in Ref. [4] (see also Ref. [5]). All these conditions provide nontrivial bounds to the set of quantum correlations.

The standard scenario when studying correlations consists of N distant parties, $A_1, \ldots, A_N$, who can perform m possible measurements, each with r possible outcomes, on their local systems. Denote by $x_1, \ldots, x_N$ the measurement applied by the parties and by $a_1, \ldots, a_N$ the obtained outcomes. The observed correlations are described by the joint probability distribution $P(a_1, \ldots, a_N|x_1, \ldots, x_N)$, giving the probability that the parties obtain the outcomes $a_1, \ldots, a_N$ when performing the measurements $x_1, \ldots, x_N$. In full generality, $P(a_1, \ldots, a_N|x_1, \ldots, x_N)$ is an arbitrary vector of $m^N \times r^N$ positive entries satisfying the normalization conditions $\sum_{a_1,\ldots,a_N} P(a_1, \ldots, a_N|x_1, \ldots, x_N) = 1$ for all $x_1, \ldots, x_N$. These objects however become nontrivial if one wants them to be compatible with a physical principle.

Indeed, imposing that the observed correlations should not contradict the no-signalling principle, requires that the marginal probability distribution observed by a group of parties, say the first k parties, be independent of the measurements performed by the remaining $N - k$ parties. Nonsignalling correlations, then, are such that

$$\sum_{a_{k+1},\ldots,a_N} P(a_1, \ldots, a_N|x_1, \ldots, x_N) = P(a_1, \ldots, a_k|x_1, \ldots, x_k), \quad (1)$$

for any splitting of the N parties into two groups.

Assume now that the correlations have a quantum origin, i.e., they can be established by performing local measurements on a multipartite quantum state. Precisely

$$P_Q = \{\mathrm{tr}(\rho M_{a_1}^{x_1} \otimes \cdots \otimes M_{a_N}^{x_N})\}, \quad (2)$$

where ρ is a positive operator of unit trace acting on a composite Hilbert space $\mathcal{H}_{A_1} \otimes \ldots \otimes \mathcal{H}_{A_N}$, while $M_{a_i}^{x_i}$ are positive operators in each local space i defining the m local measurements, i.e., $\sum_{a_i} M_{a_i}^{x_i} = \mathbb{1}_{A_i}, \quad \forall\ x_i$. It is well known that the set of nonsignalling correlations is strictly larger than the quantum set [6].

Finally, there is the set of classical correlations, which may be established by sharing classically correlated data, denoted λ. These correlations may be written in the form

$$P_C = \left\{\sum_\lambda P(\lambda) D_{A_1}(a_1|\lambda, x_1) \cdots D_{A_N}(a_N|\lambda, x_N)\right\}, \quad (3)$$

where $\{D_{A_i}\}$ are deterministic functions specifying the local results of party i as a function of the corresponding measurement and the shared classical data λ. The celebrated Bell theorem implies that the set of quantum correlations is strictly larger than the classical one [7].

In this work, we provide a unified framework for all these sets of correlations in terms of local quantum observables. Indeed, we show that each of these sets of correlations can be written in the form

$$P_O = \{\mathrm{tr}(O M_{a_1}^{x_1} \otimes \cdots \otimes M_{a_N}^{x_N})\}, \tag{4}$$

where the operators $M_{a_i}^{x_i}$ correspond to local quantum measurements. By modifying the properties of O, it is possible to generate the different sets of correlations. Requiring that proper probabilities be derived from all possible local quantum measurements imposes that the operator O be positive on all product states. Namely, it must be an entanglement witness, $O = W$ [8]. We then show that while the corresponding set of correlations, denoted P_W, is equivalent to the quantum set for one and two parties, a gap appears for $N > 2$. An implication of this result is that the extension of Gleason's Theorem to local quantum measurements does not lead to quantum correlations for an arbitrary number of parties.

Nonsignalling correlations.—Let us start by showing how to write any nonsignalling probability distribution in the form of Eq. (4) with a *particular* fixed set of measurements. This is the content of the following theorem.

Theorem 1.—An N-partite probability distribution $P(a_1, \ldots, a_N | x_1, \ldots, x_N)$ is nonsignalling if, and only if, there exist local quantum measurements $M_{a_i}^{x_i}$ and a Hermitian operator O of unit trace such that Eq. (4) holds.

Note that the operator O need not give positive probabilities for other measurements.

Proof.—The "if" part is trivial, since the marginal distributions $\sum_{a_1,\ldots,a_k} \mathrm{tr}(O M_{a_1}^{x_1} \otimes \cdots \otimes M_{a_N}^{x_N})$ are clearly independent of $x_1, \ldots, x_k$. For the "only if" part, we show how to obtain O and $M_{a_i}^{x_i}$ for each nonsignalling distribution $P(a_1, \ldots, a_N | x_1, \ldots, x_N)$.

We start by constructing the local measurements, which may be taken, without loss of generality, to be the same for each of the N parties. First, we take $m(r-1)$ vectors $|\alpha_a^x\rangle \in \mathbb{C}^d$, with $a = 1, \ldots, r-1$ and $x = 1, \ldots, m$ such that the matrices $|\alpha_a^x\rangle\langle\alpha_a^x|$ and the identity $\mathbb{1}$ are linearly independent, as elements of the space of $d \times d$ Hermitian matrices. This is always possible by taking a large enough value of the dimension d, e.g., $d = \max(r, m)$. Now we choose a set of positive numbers $z_a^x > 0$ such that, for each value of x, the matrix defined as

$$M_r^x = I - \sum_{a=1}^{r-1} z_a^x |\alpha_a^x\rangle\langle\alpha_a^x| \tag{5}$$

is positive semidefinite. This can always be achieved by choosing sufficiently small z_a^x. For each value of x, the matrices $M_a^x = z_a^x |\alpha_a^x\rangle\langle\alpha_a^x|$ (for $a < r$) and M_r^x (5) constitute a local measurement.

The set of $m(r-1)+1$ linearly independent matrices $\{\mathbb{1}, M_a^x : a = 1, \ldots r-1; x = 1, \ldots m\} = \{M_1, M_2, \ldots\}$ has a dual set $\{\tilde{M}_1, \tilde{M}_2, \ldots\}$, such that $\mathrm{tr}(M_i \tilde{M}_j) = \delta_{ij}$. Then, the explicit construction of the operator O for the case $N = 2$ is

$$\begin{aligned} O = &\sum_{a_1,a_2=1}^{r-1} \sum_{x_1,x_2=1}^{m} P(a_1, a_2 | x_1, x_2) \tilde{M}_{a_1}^{x_1} \otimes \tilde{M}_{a_2}^{x_2} \\ &+ \sum_{a_1=1}^{r-1} \sum_{x_1=1}^{m} P(a_1 | x_1) \tilde{M}_{a_1}^{x_1} \otimes \tilde{\mathbb{1}} \\ &+ \sum_{a_2=1}^{r-1} \sum_{x_2=1}^{m} P(a_2 | x_2) \tilde{\mathbb{1}} \otimes \tilde{M}_{a_2}^{x_2} + \tilde{\mathbb{1}} \otimes \tilde{\mathbb{1}}. \end{aligned} \tag{6}$$

The marginal probabilities $P(a_1|x_1)$ and $P(a_2|x_2)$ are well defined because $P(a_1, a_2 | x_1, x_2)$ is nonsignalling. Note that, since the dual matrices $\tilde{M}_a^x$ are, in general, not positively defined, neither is O. After some simple algebra, one can see that this operator and the previous local measurements reproduce the initial probability distribution according to Eq. (4). It also follows directly from the construction that O is Hermitian and $\mathrm{tr}(O) = 1$.

The generalization to higher N is based on the fact that nonsignalling distributions are characterized by the numbers $P(a_1, \ldots, a_N | x_1, \ldots, x_N)$ for $a_i < r$, together with all the $(N-1)$-party marginals [e.g., $P(a_2, \ldots, a_N | x_2, \ldots, x_N)$]. These marginals, being themselves nonsignalling distributions, are also characterized by the entries with $a_i < r$, plus all the $(N-2)$-party marginals. Recursively, one arrives at the single-party marginals, which by normalization, are characterized by the entries with $a_i < r$ too. □

As an illustration of the formalism, we give the explicit form of the operator O and local measurements reproducing the Popescu–Rohrlich correlations (or "PR-box" [6]). This represents the best known example of nonsignalling correlations not attainable by quantum means, in which the algebraic maximum of the Clauser–Horne–Shimony–Holt inequality [9] is achieved. This distribution is defined to be $P_{\mathrm{PR}}(a, b | x, y) = 1/2$ if $xy = a + b \bmod 2$, and 0 otherwise, where a, b, x, y are now bits. In this case, the required operator, which is surely not an entanglement witness, reads $O = \alpha^+ \Phi^+ + \alpha^- \Phi^-$, where $\Phi^\pm$ are the projectors onto the Bell states $|\Phi^\pm\rangle = (1/\sqrt{2})(|00\rangle \pm |11\rangle)$, and $\alpha^\pm = (1 \pm \sqrt{2})/2$; and the local observables are $\{\sigma_x, \sigma_y\}$ for Alice, and $\{(\sigma_x - \sigma_y)/\sqrt{2}, (\sigma_x + \sigma_y)/\sqrt{2}\}$ for Bob.

This theorem induces a hierarchical structure for the different sets of correlations. By constraining the form of O, it is possible to generate the sets of quantum and classical correlations. Indeed, one can encapsulate all the previous sets of correlations in the following statement.

The distribution $P(a_1, \ldots, a_N | x_1, \ldots, x_N)$ is (i) *Nonsignalling* if, and only if, it may be written in the form of Eq. (4); (ii) *Quantum* whenever the operator O is positive; (iii) *Local* if, and only if, O corresponds to a separable quantum state [10].

Gleason's Theorem for local quantum observables.—Consider now a theory such that all possible local *quantum* measurements are allowed. In this case, the operator O is required to be positive on all product states, implying that it must be an *entanglement witness* W with $\mathrm{tr}(W) = 1$. Thus, the corresponding set of correlations reads

$$P_W = \{\mathrm{tr}(W M_{a_1}^{x_1} \otimes \cdots \otimes M_{a_N}^{x_N})\}. \quad (7)$$

Since W need not be positive, the set of correlations (7) could be larger than the quantum set.

Interestingly, these correlations have already appeared in several works studying the extension of Gleason's Theorem to local observables in N independent Hilbert spaces. In what follows, we name the set of correlations defined by Eq. (7) as *Gleason correlations*. Recall that Gleason's Theorem is a celebrated result in quantum mechanics proving that any map from generalized measurements to probability distributions can be written as the trace rule with the appropriate quantum state [11]. More precisely: Let $\mathcal{P}(\mathcal{H})$ be the set of operators M acting on $\mathcal{H}$ such that $0 \leq M \leq \mathbb{1}$. For any map v: $\mathcal{P}(\mathcal{H}) \rightarrow [0, 1]$ such that $\sum_i v(M_i) = 1$ when $\sum_i M_i = \mathbb{1}$, there is a positive operator ρ such that $v(M) = \mathrm{tr}(\rho M)$. A simple proof of this theorem can be found in Ref. [12]. This theorem has been generalized to the case of local observables acting on bipartite [13] and general multipartite [14] systems. In the same fashion, as the original theorem, the goal is now to characterize those maps from N measurements—one for each party—to joint nonsignalling probability distributions. It has been shown in these works that for each of these maps, there is a witness operator W such that $v(M_1, \ldots, M_N) = \mathrm{tr}(W M_1 \otimes \ldots \otimes M_N)$.

Theorem 2.—There exist Gleason correlations $P_{W'} = \mathrm{tr}(W' \Pi_{a_1}^{x_1} \otimes \Pi_{a_2}^{x_2} \otimes \Pi_{a_3}^{x_3})$ (i.e., obtained by applying local projective measurements $\Pi_{a_i}^{x_i}$ on a normalized entanglement witness W') that do not have a quantum realization, i.e., such that there exist no quantum state ρ and local measurements, $M_{a_i}^{x_i}$, in an arbitrary tripartite Hilbert space satisfying $P_{W'} = \mathrm{tr}(\rho M_{a_1}^{x_1} \otimes M_{a_2}^{x_2} \otimes M_{a_3}^{x_3})$.

Proof.—To show that the set of Gleason correlations is strictly larger than the quantum set, we construct a witness and local measurements which lead to a violation of a Bell inequality higher than the quantum one.

The Bell inequality we consider has been introduced in Ref. [15] for the tripartite scenario in which the parties apply two measurements each with two possible outcomes. We label the choice of measurements and the obtained results by bits. The inequality reads

$$\begin{aligned}\beta = {} & p(000|000) + p(110|011) + p(011|101) \\ & + p(101|110) \\ & \leq 1. \end{aligned} \quad (8)$$

One can indeed prove that the values achievable through classical and quantum means are at most unity; that is, the inequality is not violated by quantum theory [15].

Moving to the definition of the operator W', we consider the witness which detects the three-qubit bound entangled state of Ref. [16] based on unextendable product bases (UPB). Recall that an unextendable product basis in a composite Hilbert space of total dimension d is defined by a set of $n < d$ orthogonal product states which cannot be completed into a full product basis, as there is no other product state orthogonal to them. In Ref. [16], an example of such a set of product states for three qubits was constructed. It consists of the following four states:

$$|000\rangle, \quad |1e^{\perp}e\rangle, \quad |e1e^{\perp}\rangle, \quad |e^{\perp}e1\rangle \quad (9)$$

where $\{|e\rangle, |e^{\perp}\rangle\}$ is an arbitrary basis different from the computational one. We denote by Π_{UPB} the projector onto the subspace spanned by these states. One knows that the state $\rho_{\mathrm{UPB}} = (\mathbb{1} - \Pi_{\mathrm{UPB}})/4$ is bound entangled. A normalized witness W' detecting this state is given by

$$W' = \frac{1}{4 - 8\epsilon}(\Pi_{\mathrm{UPB}} - \epsilon \mathbb{1}), \quad (10)$$

where $\epsilon = \min_{|\alpha\beta\gamma\rangle} \langle \alpha\beta\gamma | \Pi_{\mathrm{UPB}} | \alpha\beta\gamma \rangle$. One immediately confirms that here $0 < \epsilon < 1/2$. Clearly, W' is positive on all product states and detects ρ_{UPB} since $\mathrm{tr}(W' \rho_{\mathrm{UPB}}) = -\epsilon/4(1 - 2\epsilon)$ which is negative for any $\epsilon < 1/2$.

Now, one can see that the witness W' when measured in the local bases in the definition of the UPB (9) leads to correlations such that

$$\beta = \frac{1 - \epsilon}{1 - 2\epsilon}, \quad (11)$$

which is larger than unity for $0 < \epsilon < 1/2$. □

This theorem, then, proves that the set of Gleason correlations is strictly larger than the quantum set for $N > 2$. The equivalence of these two sets in the bipartite scenario $N = 2$ has recently been shown in [17]. For the sake of completeness, we present here a slightly simpler proof of this result.

The Choi–Jamiołkowski (CJ) isomorphism implies that any witness W can be written as $(\mathbb{1}_{A_1} \otimes \Upsilon_{A_2})(\Phi)$, where Υ is a positive map and Φ is the projector onto the maximally entangled state. Using the same techniques as in Ref. [18], one can prove that any witness can also be written as $(\mathbb{1}_{A_1} \otimes \Lambda_{A_2})(\Psi)$, where Λ is now positive and trace preserving and Ψ is a projector onto a pure bipartite state. Denoting by Λ^* the dual [19] of Λ, we have

$$\begin{aligned}\mathrm{tr}(W M_{a_1}^{x_1} \otimes M_{a_2}^{x_2}) &= \mathrm{tr}[(\mathbb{1} \otimes \Lambda)(\Psi) M_{a_1}^{x_1} \otimes M_{a_2}^{x_2}] \\ &= \mathrm{tr}[\Psi M_{a_1}^{x_1} \otimes \Lambda^*(M_{a_2}^{x_2})] \\ &= \mathrm{tr}(\Psi M_{a_1}^{x_1} \otimes \tilde{M}_{a_2}^{x_2}), \end{aligned} \quad (12)$$

where $\tilde{M}_{a_2}^{x_2} = \Lambda^*(M_{a_2}^{x_2})$ defines a valid quantum measurement because the dual of a positive trace-preserving map is positive and unital, i.e. , $\Lambda^*(\mathbb{1}) = \mathbb{1}$.

Discussion.—There is an ongoing effort to understand the gap between quantum and nonsignalling correlations. As stated above, there exist correlations which, despite being compatible with the no-signalling principle, cannot be attained by local measurements on a quantum state. The natural question is then to study why these supra-quantum correlations do not seem to be observed in nature. Of course, a trivial answer to this question is that there exist no positive operator and projective measurements in a Hilbert space reproducing these supra-quantum correlations via the Born trace rule. However, one would wish for a set of "natural" principles with which to exclude these supra-quantum correlations. These principles would provide a better, or ideally the exact, characterization of quantum correlations.

Most of the principles proposed so far to rule out supra-quantum correlations have an information theoretic motivation. The idea is that the existence of these correlations would imply an important change in the way information is processed and transmitted. It has been shown, for instance, that communication complexity would become trivial if the PR-box, or some noisy version of it, were available [20], that some of these supra-quantum correlations violate a new information principle called *information causality* [21], or that they would lead to the violation of macroscopic locality [22]. Unfortunately, none of these principles has been proven to be able to single out the set of quantum correlations [23].

In this work, we introduce a unified mathematical formalism for nonsignalling and quantum correlations in terms of local quantum observables. We expect this formalism to be useful when tackling all such questions. It may be easier using our construction to study how new constraints may be added to the nonsignalling principle in order to derive the quantum correlations. The methods developed here may also be useful to study the degree of nonlocality of quantum states, witnesses, and O-operators.

We have, then, considered Gleason correlations, defined by nonsignalling theories in which all possible local quantum measurements are possible. We have shown the presence of a gap between this set and the quantum set of correlations for $N > 2$ parties. Thus, while the hypothesis in Gleason's Theorem for local observables completely characterizes the set of bipartite quantum correlations [17], the result does not extend to the multipartite scenario. Clearly, the proof of equivalence in the bipartite case exploits the existence of the CJ isomorphism. Actually, it is easy to see that the equivalence holds for those N-party entanglement witnesses that can be written

$$W = \sum_k p_k[\Lambda^k_{A_1} \otimes \cdots \otimes \Lambda^k_{A_N}](\rho_k), \tag{13}$$

where ρ_k are N-party quantum states, p_k some probability distribution, and $\Lambda^k_{A_i}$ are positive, trace-preserving maps and the number of terms in the sum is arbitrary. Our results imply that this decomposition is not possible for all N-party entanglement witnesses. It would be interesting to better understand why the theorem fails in the multipartite scenario and identify additional requirements able to close the gap.

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