Unlimited Ion Acceleration by Radiation Pressure

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The energy of ions accelerated by an intense electromagnetic wave in the radiation pressure dominated regime can be greatly enhanced due to a transverse expansion of a thin target. The expansion decreases the number of accelerated ions in the irradiated region resulting in an increase in the ion energy and in the ion longitudinal velocity. In the relativistic limit, the ions become phase locked with respect to the electromagnetic wave resulting in unlimited ion energy gain.

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The radiation pressure of a superintense electromagnetic pulse on a thin quasineutral plasma slab has been proposed in Ref. [1] as an acceleration mechanism able to provide ultrarelativistic ion beams. In this radiation pressure dominant acceleration (RPDA) regime (also called the "laser piston" or the "light sail"), the ions move forward with almost the same velocity as the electrons, thus acquiring much greater kinetic energy than the electrons. This acceleration process is highly efficient, with the ion energy per nucleon being proportional in the ultrarelativistic limit to the electromagnetic pulse energy. The idea of transferring momentum from light to macroscopic objects goes back to [2]. In the mid 1950s ion acceleration was suggested using the radiation pressure of a high intensity electromagnetic wave acting on an electron cloud which drags a small portion of ions by a collective electric field [3].

Recently the RPDA regime of laser ion acceleration has attracted great attention. In Refs. [4,5] the stability of the accelerated foil has been analyzed. References [6,7] are devoted to extending its range of operation towards lower laser intensities. The interaction of a high intensity laser pulse with extended plasmas in the RPDA regime has been simulated in [8]. In Refs. [1,9] the effects of foil transparency are considered.

An indication of the effect of radiation pressure on bulk target ions is obtained in experimental studies of plasma jets ejected from the rear side of thin solid targets irradiated by ultraintense laser pulses [10] and in the experiments reported in Ref. [11].

While Ref. [6] develops regimes of energy enhancement of accelerated ions by using laser pulse polarization and target structuring [12], in this Letter we propose to increase the energy of accelerated ions using transversally expanding targets. The transverse expansion of the target is provided by the action of the ponderomotive force of a laser pulse with a finite waist. It can also occur as a result of the instability described in Refs. [4,5].

The nonlinear dynamics of a laser accelerated thin target is described within the framework of the thin shell approximation [13]. The target is modeled as an ideally reflecting shell. The equation of motion of the surface element of the shell is $\delta N d\mathbf{p}/dt = \mathcal{P}\delta\mathbf{s}$, where $\mathcal{P}$ is the light pressure, and $\mathbf{p}$ is the momentum of the surface element, with the area given by the oriented vector surface area element $\delta\mathbf{s}$, carrying $\delta N = nl|\delta\mathbf{s}|$ particles which is constant in time. Here n and l are the shell density and thickness.

In order to describe the surface evolution, we introduce the Lagrange coordinates η and ζ playing the role of the markers of the shell surface element. The shell surface is parametrized by the vector $\mathbf{M}(\eta, \zeta, t) = \{x(\eta, \zeta, t), y(\eta, \zeta, t), z(\eta, \zeta, t)\}$. At a regular point, the oriented vector surface area element is $\delta\mathbf{s} = \partial_\eta \mathbf{M} \times \partial_\zeta \mathbf{M} d\eta d\zeta$ [14]. The shell initially is in the plane $x = 0$, $\mathbf{M}(\eta, \zeta, 0) = \{0, \eta, \zeta\}$, which gives $\delta N = n_0 l_0 d\eta d\zeta$, where n_0 and l_0 are the shell density and thickness at $t = 0$. The particle number conservation implies $nl = n_0 l_0/|\partial_\eta \mathbf{M} \times \partial_\zeta \mathbf{M}|$. The equations of the shell motion are

$$n_0 l_0 \partial_t p_i = \mathcal{P}\varepsilon_{ijk}\partial_\eta x_j \partial_\zeta x_k, \tag{1}$$

$$\partial_t x_i = c p_i/(m_\alpha^2 c^2 + p_k p_k)^{1/2}, \tag{2}$$

where m_α is the ion mass, ε_{ijk} is the unit antisymmetric tensor, $i = 1, 2, 3$, and summation over repeated indices is assumed. The radiation pressure exerted on the ideally reflecting shell by an electromagnetic wave propagating along the x axis with amplitude $E = E(t - x/c)$ is $\mathcal{P} = (E^2/2\pi)(1 - \beta)/(1 + \beta)$, where $\beta = c p_x/(m_\alpha^2 c^2 + p_x^2)^{1/2}$.

We consider the case when the accelerated shell moves in the longitudinal direction with an ultrarelativistic velocity, i.e., $p_x/m_\alpha c \gg 1$, while its transverse momentum is relatively small compared to the longitudinal one. We look for solutions of Eqs. (1) and (2) assuming a local dependence of the transverse coordinates, $y = \Lambda_y(t)\eta$, $z = \Lambda_z(t)\zeta$, corresponding to the surface local extrema in the direction of the x axis, when $x(\eta, \zeta, t) = x(t)$ is locally independent of η and ζ. In this case the right-hand (rhs) side of Eq. (1) for the momentum transverse components vanishes, so that the shell expands ballistically: $p_y = \pi_y^0 \eta$, $p_z = \pi_z^0 \zeta$, where π_y^0 and π_z^0 are constants, with $\Lambda_y(t) = 1 + (\pi_y^0/m_\alpha)\int_0^t dt'/\gamma(t')$ and similar formula for Λ_z. Here $\gamma(t)$ is Lorentz factor, given by $\gamma = [1 + (p_x/m_\alpha c)^2]^{1/2}$ within the framework of our approximation. Inserting these expressions into Eq. (1), we obtain the equation for the momentum longitudinal component p_x,

$$\frac{dp_x}{dt} = \frac{m_\alpha v_E^2}{l_0}\frac{1-\beta}{1+\beta}\Lambda_y\Lambda_z, \tag{3}$$

where $v_E^2 = E^2/2\pi n_0 m_\alpha$. The shell surface density decreases as $nl = n_0 l_0/\Lambda_y\Lambda_z$. In the case of no expansion of the shell we have $\pi_y^0 = \pi_z^0 = 0$ and $\Lambda_y = \Lambda_z = 1$, and Eq. (3) gives (for a constant amplitude pulse) the following asymptotic for the shell momentum [1]:

$$p_x(t) = m_\alpha c(t/\tau_{1/3})^{1/3}, \qquad t \to \infty, \tag{4}$$

where $\tau_{1/3} = 4l_0 c/3v_E^2$. In the case of transverse expansion along the y axis ($\pi_y^0 > 0$, $\pi_z^0 = 0$),

$$p_x(t) = m_\alpha c(t/\tau_{1/2})^{1/2}, \qquad t \to \infty, \tag{5}$$

with $\tau_{1/2} = (l_0 m_\alpha c/v_E^2 \pi_y^0)^{1/2}$. The shell surface density decreases as $nl \propto t^{-1/2}$. As found in Ref. [5], in the case of transverse expansion along both the y and z axes ($\pi_y^0 > 0$, $\pi_z^0 > 0$), Eq. (3) yields

$$p_x(t) = m_\alpha c(t/\tau_{3/5})^{3/5}, \qquad t \to \infty, \tag{6}$$

with $\tau_{3/5} = (48 l_0 m_\alpha^2 c/125 v_E^2 \pi_y^0 \pi_z^0)^{1/3}$. The shell surface density decreases as $nl \propto t^{-4/5}$. We see that the momentum of an expanding shell grows faster than that of a nonexpanding shell.

The wave phase is

$$\psi = \omega_0(t - x/c) = \omega_0 \int_0^t (1 - \beta(t'))dt'. \tag{7}$$

Substituting a power dependence of the ion momentum on time, $p_x(t) = m_\alpha c(t/\tau_k)^k$, into Eq. (7), we obtain

$$\frac{\psi}{\omega_0 \tau_k} = \frac{t}{\tau_k} - \frac{(t/\tau_k)^{k+1}}{k+1}\,{}_2F_1\left(\frac{k+1}{2k}, \frac{1}{2}, \frac{3k+1}{2k}; -\left(\frac{t}{\tau_k}\right)^{2k}\right), \tag{8}$$

where ${}_2F_1(\alpha, \beta, \gamma; z)$ is the Gauss hypergeometric function [14]. Asymptotically, expression (8) yields for $t \to \infty$

$$\frac{\psi}{\omega_0 \tau_k} \to \frac{(t/\tau_k)^{1-2k}}{2-4k} + \frac{1}{\pi^{1/2}}\Gamma\left(\frac{2k-1}{2k}\right)\Gamma\left(\frac{k+1}{2k}\right), \tag{9}$$

where $\Gamma(z)$ is the Euler gamma function [14]. If the power index k is larger than $1/2$, the first term in the rhs of Eq. (9) tends to zero for $t \to \infty$ and the wave phase seen by the shell becomes frozen, $\psi \to \psi_*$. For $0 < k \leq 1/2$, Eq. (9) gives unbounded phase. For the ion momentum dependence on time given by Eq. (6) $k = 3/5$, $\psi_* \approx 2.804\omega_0 \tau_{3/5}$. We see that in the case of a nonexpanding shell, when the momentum dependence on time is given by Eq. (4), or if the shell expands only in one transverse dimension as in Eq. (5), the wave phase diverges, while for a shell expanding in two transverse dimensions the phase shift between the laser pulse and the accelerated ions remains finite. For a long enough laser pulse the ions at the pulse axis become trapped inside the pulse with formally unlimited energy growth, at the expense of particle number decrease. We note that unlimited electron acceleration regimes are well known for electrons accelerated by the electromagnetic wave in the cyclotron autoresonance regime [15], for the electrons trapped by an electrostatic wave propagating perpendicularly to the magnetic field [16], and in inhomogeneous plasmas with a downgrading density [17].

A formally unlimited ion energy growth predicted by the ideally reflecting thin shell model becomes limited when we take into account the shell transparency [1,9]. Two effects compete in determining the transparency of the accelerated and expanding shell: as the longitudinal momentum increases, in the proper frame of reference of the shell the laser frequency decreases proportionally to $1/2\gamma$ (in the ultrarelativistic regime), while the shell surface density decreases. The shell remains opaque while the laser dimensionless amplitude is below the threshold determined by the ratio between the shell surface density and the laser frequency [18]

$$a_0 \leq (\varepsilon_p/\Lambda_y\Lambda_z)[(1+\beta)/(1-\beta)]^{1/2}, \tag{10}$$

where $\varepsilon_p = 2\pi n_0 l_0 e^2/m_e \omega_0 c$. For the shell expanding in two transverse dimensions, this threshold tends to zero as $t^{-1/5}$, making the foil transparent for the laser pulse. For the shell expanding in only one transverse dimension, the shell can always be opaque for the incident laser pulse.

Controlling the laser pulse shape one can set an optimal condition for an unlimited acceleration. We look for an optimal laser pulse shape represented by the function $E(\psi)$ in terms of the wave phase ψ. Changing the independent variable from t to ψ, we rewrite Eq. (3):

$$\frac{dp_x}{d\psi} = \frac{m_\alpha^2 \omega_0 c}{\pi_\perp^0} h^2(\psi)\frac{\Lambda^2}{1+\beta}, \tag{11}$$

where we introduced a normalized intensity of the laser pulse $h^2(\psi) = E^2(\psi)\pi_\perp^0/2\pi n_0 l_0 m_\alpha^2 \omega_0^2 c$, $\Lambda = \Lambda_y = \Lambda_z$, $\pi_\perp^0 = \pi_y^0 = \pi_z^0$, and

$$\frac{d\Lambda}{d\psi} = \frac{\pi_\perp^0}{m_\alpha \omega_0}\left(\frac{1+\beta}{1-\beta}\right)^{1/2}. \tag{12}$$

The system of Eqs. (11) and (12) can be cast in the form

$$\frac{d^2\Lambda}{d\psi^2} = h^2(\psi)\Lambda^2. \tag{13}$$

For a rectangular laser pulse, $E(\psi) = E_0$ for $0 < \psi < \omega_0 t_{\rm las}$ and $E(\psi) = 0$ elsewhere, the laser normalized in-

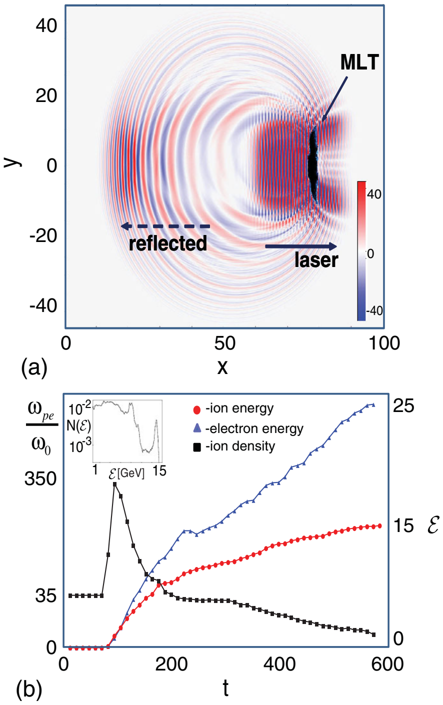

FIG. 1 (color). (a) Laser pulse, reflected radiation, and MLT shown as a superposition of the ion density and the electric field z component in the (x, y) plane at $t = 112.5(2\pi/\omega)$. (b) Electron and ion energy and the normalized Langmuir frequency corresponding to the ion density versus time. Inset: Ion energy spectrum at $t = 600(2\pi/\omega)$.

tensity is $h_0^2 = E_0^2\pi_\perp^0/2\pi n_0 l_0 m_\alpha^2\omega_0^2 c$. In this case the solution to Eq. (13) can be expressed in terms of the Weierstrass elliptic function, $\wp(u;\{g_2, g_3\})$ [14], for $g_2 = 0$:

$$\Lambda(\psi) = (6/h_0^2)\wp(\psi_* - \psi;\{0, g_3\}), \tag{14}$$

where $g_3 = (h_0^4/36)(2h_0^2\Lambda_0^3/3 - \Lambda_0'^2)$ is constant determined by initial conditions. The phase ψ_* should be determined by the smallest positive solution to $\wp(\psi_* - \psi;\{0, g_3\}) = h_0^2\Lambda_0/6$. At $\psi \to \psi_*$, Eq. (14) yields $\Lambda(\psi) = 6h_0^{-2}(\psi_* - \psi)^{-2} + 3g_3(\psi_* - \psi)^4/14h_0^2 + \cdots$.

The solution to Eq. (11) gives a dependence of the momentum on the phase:

$$\gamma + \frac{p_x}{m_\alpha c} = 1 + \frac{6m_\alpha\omega_0}{h_0^2\pi_\perp^0}[\wp'(\psi_*;\{0, g_3\}) - \wp'(\psi_* - \psi;\{0, g_3\})], \tag{15}$$

where $\wp'(u;\{g_2, g_3\})$ is the derivative of the Weierstrass elliptic function with respect to the first argument. We see that $p_x \propto (\psi_* - \psi)^{-3}$ at $\psi \to \psi_*$. From Eq. (7) we find the dependence of the phase on time, $(\psi_* - \psi) \propto t^{-1/5}$, in the limit $\psi \to \psi_*$, which corresponds to the time dependence of the momentum given by Eq. (6).

The ion acceleration can be effectively optimized by tailoring the laser pulse shape. Assuming the laser pulse shape $h(\psi) = h_0(\psi_* - \psi)^m$, corresponding to the pulse electric field profile $E(x, t) = E_0[\psi_* - \omega_0(t - x/c)]^m$ being of the same class as plotted in Fig. 1 of Ref. [4], we obtain the exact solution of Eq. (13) $\Lambda(\psi) = 2(1 + m)(3 + 2m)h_0^{-2}(\psi_* - \psi)^{-2(1+m)}$. Using this dependence and integrating Eq. (11), we obtain the dependence of the momentum on the phase

$$\gamma + \frac{p_x}{m_\alpha c} = 1 + \frac{4m_\alpha\omega_0}{h_0^2\pi_\perp^0}[(1 + m)^2(3 + 2m)] \times [(\psi_* - \psi)^{-3-2m} - \psi_*^{-3-2m}]. \tag{16}$$

Integration of Eq. (7) yields the time dependence of the phase $\psi(t)$. It reads

$$\psi = \psi_* - \left[\left(\frac{m_\alpha\omega_0}{h_0^2\pi_\perp^0}\right)^2 \frac{8(1 + m)^4(3 + 2m)^2}{(5 + 4m)\omega_0 t}\right]^{1/(5+4m)} + \cdots. \tag{17}$$

This results in the momentum time dependence

$$p_x = m_\alpha c(t/\tau_k)^k + \cdots, \tag{18}$$

where the power $k = (3 + 2m)/(5 + 4m)$. We note that it satisfies a condition $k > 1/2$ for $m > -5/4$. The characteristic acceleration time τ_k is equal to

$$\tau_k = \frac{2}{(5 + 4m)\omega_0}\left[2(1 + m)^2(3 + 2m)\frac{m_\alpha\omega_0}{h_0^2\pi_\perp^0}\right]^{1/(3+2m)}. \tag{19}$$

The momentum per unit surface of the shell asymptotically depends on time as $p_x nl \propto t^{-(1+2m)/(5+4m)}$. If the power $-5/4 < m < -1/2$, the momentum per unit surface grows in time with the same asymptotic as the rhs of the opaqueness condition Eq. (10). Our model predicts a relatively slow dependence of the accelerated ion number on their energy: $nl \propto p_x^{-(4+4m)/(3+2m)}$.

In the nonrelativistic limit, when $p_i = m_\alpha\dot{x}_i$, Eqs. (1) and (2) admit the exact solution $x = v_E^2\tau_{ex}^2[(1 + t/\tau_{ex})^4 - 4t/\tau_{ex} - 1]/12l_0$, with $y = \Lambda(t)\eta$, $z = \Lambda(t)\zeta$, and $\Lambda(t) = 1 + t/\tau_{ex}$, where $\tau_{ex} = m_\alpha/\pi_\perp^0$ is an expansion time. Asymptotically for $t \to \infty$ the ion kinetic energy grows as $\mathcal{E}_\alpha \approx m_\alpha v_E^4 t^6/18l_0^2\tau_{ex}^4$ and the ion surface density decreases as $nl = n_0 l_0(\tau_{ex}/t)^2$. Assuming the acceleration time is the laser pulse duration, $t = t_{las}$, and writing the laser pulse fluence as $w_{las} = cE^2 t_{las}/4\pi = It_{las}$, where I is the laser intensity, we find for the ion acceleration efficiency

$$\mathcal{K}_{eff} = \frac{2It_{las}}{9m_\alpha c^2 n_0 l_0}\left(\frac{t_{las}}{\tau_{ex}}\right)^2. \tag{20}$$

This efficiency is by the factor $(t_{las}/3\tau_{ex})^2$ higher than in the case of nonexpanding foil [1], which points towards a way for enhancing the efficiency of the fast ion generation required within the framework of the concept of fast ignition with laser accelerated ions [19]. The efficiency enhancement requires the laser pulse duration to be larger than the foil expansion time. Assuming the expansion time to be of the order of the inverse growth rate of the Raleigh-Taylor instability, $(q_\perp v_E^2/l_0)^{1/2}$ [4], with the wavelength of transverse perturbations equal to the inverse laser pulse waist, $q_\perp = 2\pi/w$, we find $t_{\rm las}/\tau_{\rm ex} = v_E t_{\rm las}(2\pi/wl_0)^{1/2}$. For $v_E = ca_0(m_e/m_\alpha)^{1/2}(\omega_0/\omega_{\rm pe})$, $a_0 = 100$, $l_0 = 0.1\lambda_0$, and $w \approx ct_{\rm las}/3$ with $t_{\rm las} \approx 100$ fs, it yields $v_E \approx 0.1c$ and the factor $(t_{\rm las}/\tau_{\rm ex})^2 \approx 10$–20.

As an illustration of the realization of the RPDA regime in the interaction of a high intensity laser pulse with an expanding plasma shell, we present the results of 2D particle-in-cell (PIC) simulations of the dynamics of a mass-limited target (MLT) in a strong laser field, Fig. 1. We use the PIC code REMP [20]. The simulation box size is $600\lambda \times 100\lambda$, with mesh resolution of 20 cells per laser wavelength. The number of quasiparticles is equal to 10^4. The target has the form of an ellipsoid in the x, y plane with horizontal and vertical semiaxes equal to 1λ and 7.5λ. It is initially localized at $x = 50\lambda$, $y = 0$. The target is made of hydrogen plasma with the ion-to-electron mass ratio equal to 1836. The electron density corresponds to the frequency ratio of $\omega_{\rm pe}/\omega = 6$. A circularly polarized laser pulse is excited at the left-hand side of the simulation box. It has a super-Gaussian shape with a length of $l_x = 25\lambda$, a width of $l_y = 25\lambda$, and with the dimensionless amplitude $a = 125$. The laser pulse compresses the MLT in the longitudinal direction. The reflected light wavelength increases due to its interaction with the MLT playing the role of a receding relativistic mirror as seen in Fig. 1(a), where a superposition of the ion density and the z component of the electric field in the x, y plane is shown for $t = 112.5(2\pi/\omega)$. The MLT expands along the transverse direction. The laser pulse interacts with an expanding thin dense shell in a regime close to one discussed above. In Fig. 1(b) we present the electron and ion energy and ion density versus time. At the initial stage the ion density increases and then tends to zero. The electron and ion energies grow and are of the same order of magnitude. At the time, when the accelerated shell approaches the right-hand side boundary of the simulation box, the protons reach the energy of 14 GeV and the electron energy is equal to 27 GeV. In the inset of Fig. 1 we show the ion energy spectrum at $t = 600(2\pi/\omega)$, which demonstrates the quasimonoenergetic peak with the width of the order of 5%. If we estimate the energy of the accelerated ions according to expression (4) for the simulation parameters assuming that no deformation of the MLT occurs, we find it to be of the order of 3 GeV. The enhancement of the fast ion energy is naturally explained by the MLT expansion.

In conclusion, the transverse expansion of a thin shell accelerated in the RPDA regime results in the increase of the ion energy and the acceleration efficiency at the expense of decreasing number of particles. In the relativistic limit, the ions become phase locked with respect to the electromagnetic wave, which is the indication of an unlimited acceleration. This effect and the use of optimal laser pulse shape provide a new approach for greatly enhancing the energy of laser accelerated ions.

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